Topological phases in Amorphous/glassy solid and other topological stories

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The Plan

- Topological phases through examples : Discuss three explicit examples of topological phases in microscopic lattice systems.
    - These examples will build on some of the issues that we have discussed in the past one week
- Will try to connect lattice physics with continuum field theory.
    - generally difficult to do systematically
- In the process stumble upon interesting ideas and issues such as explicit example of a connection between SPT and SET through the concept of gauging of symmetry
    - A Chiral spin liquid as a “gauged” Bosonic Integer quantum Hall effect.

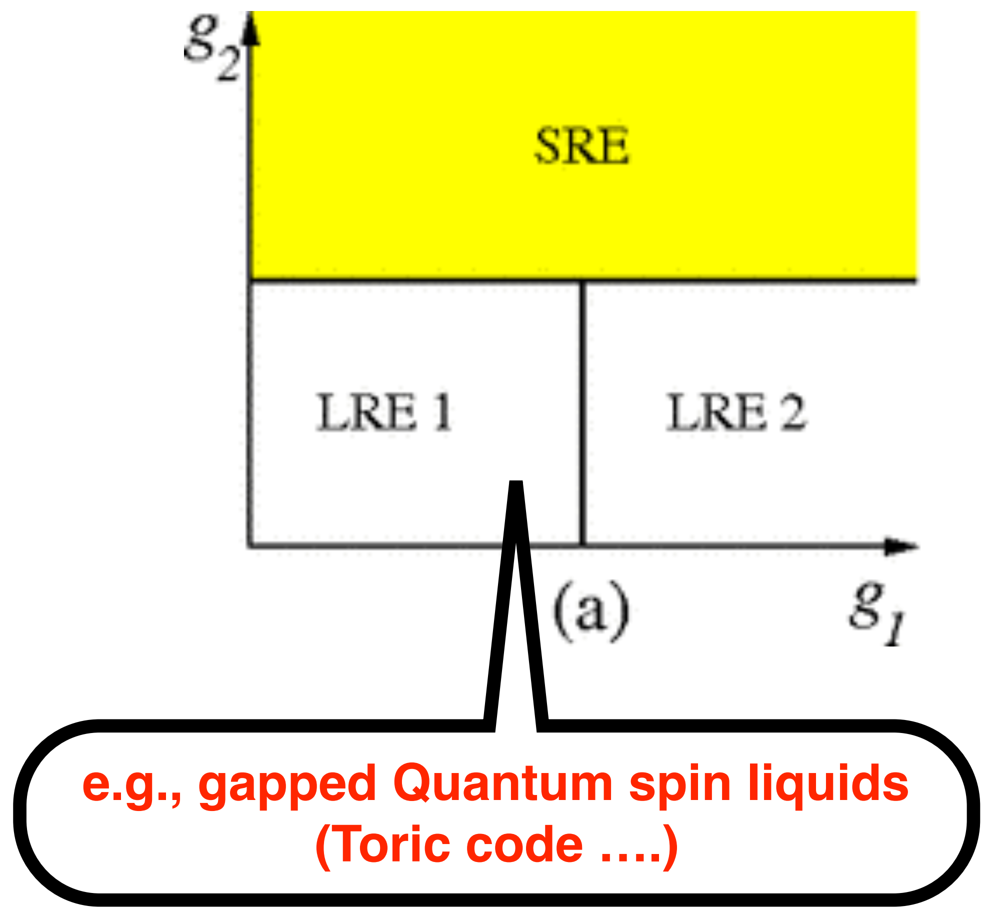
Outline

- Introduction
- Three stories :
    - 1. Free fermion Z2 SPT in a three dimensional amorphous solid
        - Characterisation through electromagnetic response.
    - 2.1 : Interacting U(1) Bosonic SPT in 2 dimensions : Bosonic Integer quantum Hall effect on a lattice
    - 2.2. A chiral Spin liquid on a Kagome Lattice
        - The connection between the two through the “gauging” of U(1).
- Summary

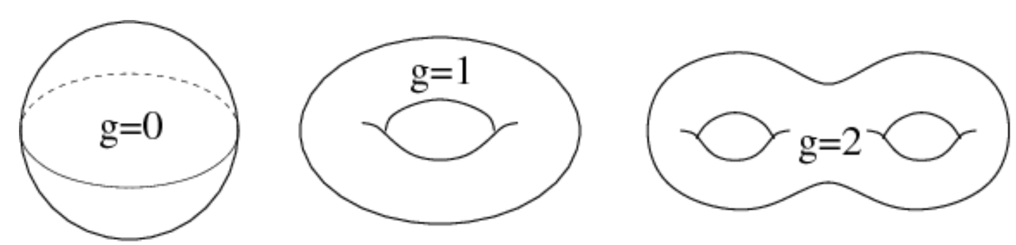
Quantum entanglement based classification of gapped phases in absence of symmetries

[Chen, Gu, Liu, Wen, PRB (2013)]

- short ranged entangled (SRE)
- long ranged entangled (LRE)



LRE :
“Intrinsic topological order”
⬇
Topological degeneracies

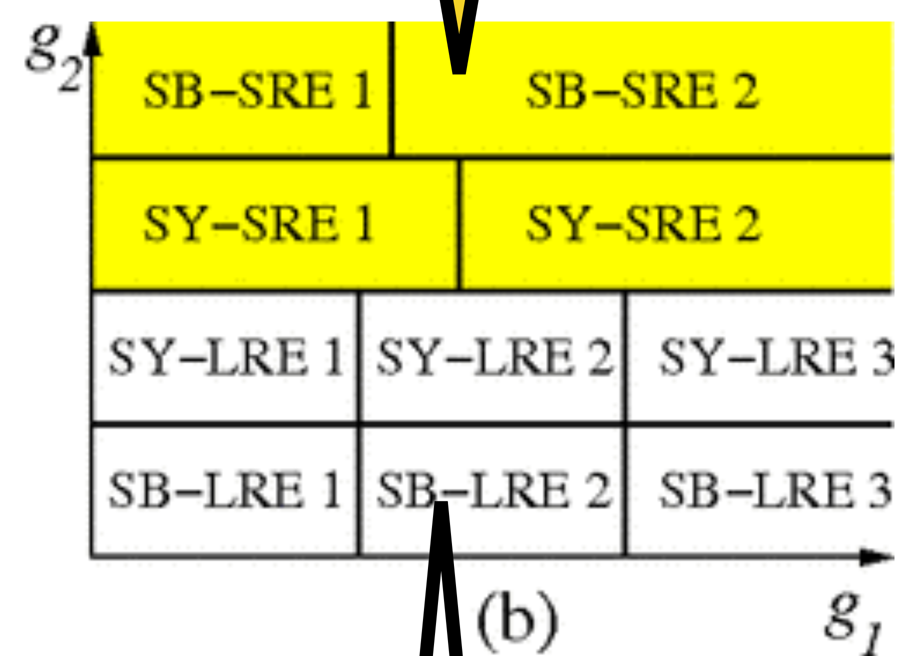


Symmetry enriched and symmetry protected topological order

[Chen, Gu, Liu, Wen, PRB (2013)]

- In presence of symmetries SRE can be of different types (even without symmetry breaking).
  - The distinction of these phases are based on topological invariants which are “protected” by symmetry ➜ Symmetry protected topological phases (SPT)
- LRE can be further classified according to various patterns of fractionalisation of the symmetries ➜ Symmetry enriched topological phases

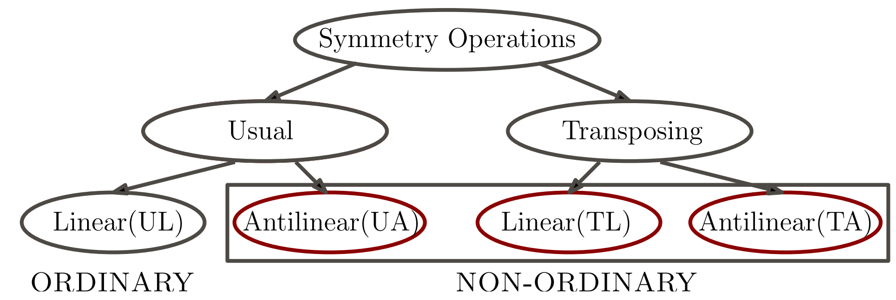
e.g., SPTs (R. Verresen’s talk; J. Cano’s talk)



e.g., FQHE States (D. T. Son’s talk; S. Simon’s talk)

Free fermion SPTs

Usual symmetries: $\mathcal{U}\psi\mathcal{U}^{-1} \sim \psi$
Transposing symmetries: $\mathcal{U}\psi\mathcal{U}^{-1} \sim \psi^{\dagger}$
Linear symmetries: $\mathcal{U}i\mathcal{U}^{-1} \sim i$
Antilinear symmetries: $\mathcal{U}i\mathcal{U}^{-1} \sim -i$



Usual antilinear: Time Reversal ($\mathcal{T}$)
Transposing linear: Charge Conjugation ($\mathcal{C}$)
Transposing antilinear: Sublattice ($\mathcal{S}$)

Based on three symmetries – "time reversal, charge conjugation and sublattice" and spatial dimension

Class	(T,C,S)	d=0	d=1	d=2	d=3	d=4	d=5	d=6	d=7
A	(0,0,0)	Z	0	Z	0	Z	0	Z	0
AIII	(0,0,1)	0	Z	0	Z	0	Z	0	Z
AI	(+1,0,0)	Z	0	0	0	2Z	0	Z_2	Z_2
BDI	(+1,+1,1)	Z_2	Z	0	0	0	2Z	0	Z_2
D	(0,+1,0)	Z_2	Z_2	Z	0	0	0	2Z	0
DIII	(-1,+1,1)	0	Z_2	Z_2	Z	0	0	0	2Z
AII	(-1,0,0)	2Z	0	Z_2	Z_2	Z	0	0	0
CII	(-1,-1,1)	0	2Z	0	Z_2	Z_2	Z	0	0
C	(0,-1,0)	0	0	2Z	0	Z_2	Z_2	Z	0
CI	(+1,-1,1)	0	0	0	2Z	0	Z_2	Z_2	Z

Add crystalline symmetries
+
information about atomic orbitals
= Large number of Crystalline free fermion SPTs

[J. Cano's talk]

Kitaev(2009); Ryu, Ludwig, Furusaki, Schnyder(2008); Moore, Ryu, Ludwig(2012)

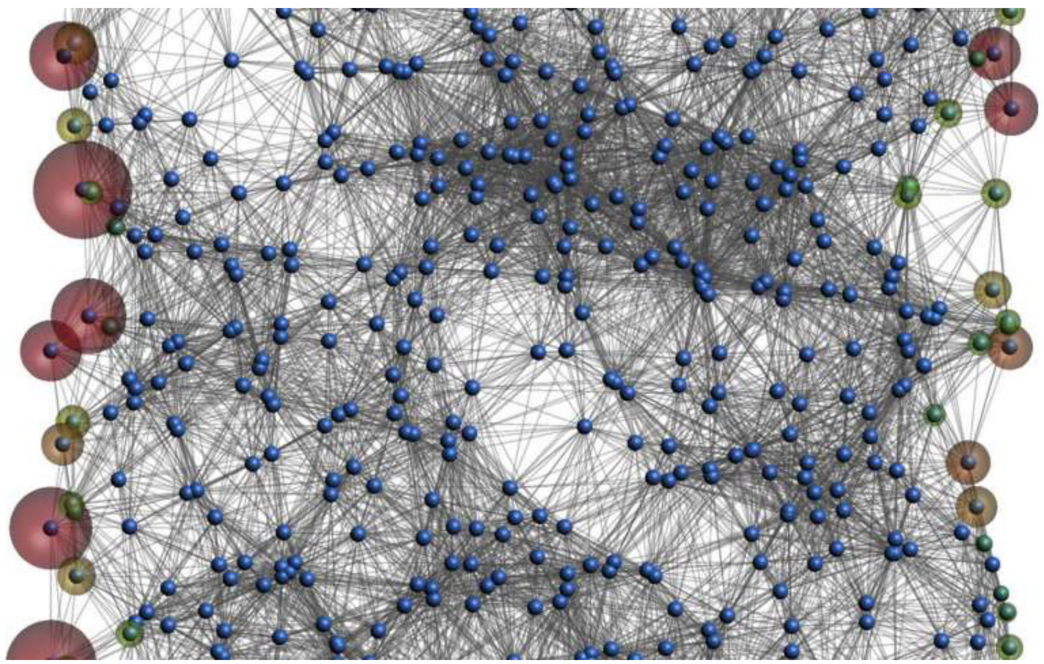
Free fermion SPTs

Class	(T,C,S)	d=0	d=1	d=2	d=3	d=4	d=5	d=6	d=7
A	(0,0,0)	Z	0	Z	0	Z	0	Z	0
AIII	(0,0,1)	0	Z	0	Z	0	Z	0	Z
AI	(+1,0,0)	Z	0	0	0	2Z	0	Z_2	Z_2
BDI	(+1,+1,1)	Z_2	Z	0	0	0	2Z	0	Z_2
D	(0,+1,0)	Z_2	Z_2	Z	0	0	0	2Z	0
DIII	(-1,+1,1)	0	Z_2	Z_2	Z	0	0	0	2Z
AII	(-1,0,0)	2Z	0	Z_2	Z_2	Z	0	0	0
CII	(-1,-1,1)	0	2Z	0	Z_2	Z_2	Z	0	0
C	(0,-1,0)	0	0	2Z	0	Z_2	Z_2	Z	0
CI	(+1,-1,1)	0	0	0	2Z	0	Z_2	Z_2	Z

- For this table : presence of crystalline symmetries are NOT essential but are often very useful to calculate the topological invariants
    - Translation and inversion symmetries to calculate the Z2 invariant for 3d topological band insulator.

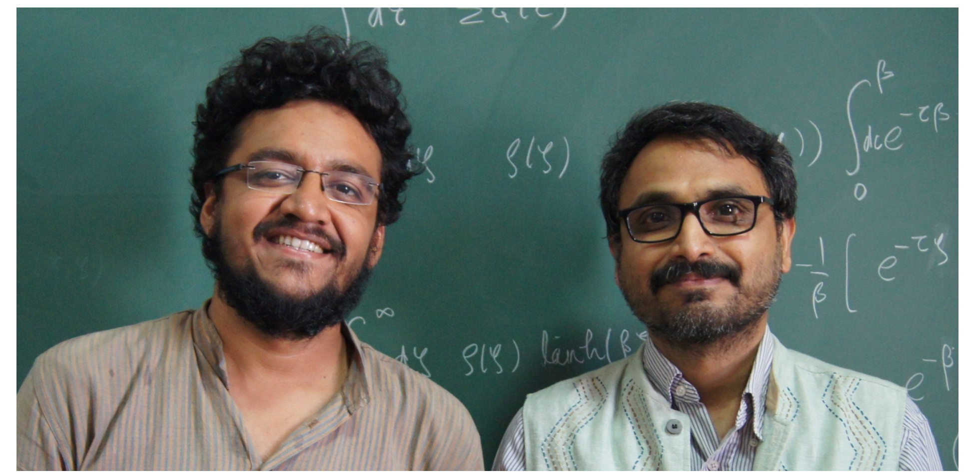
How to characterise the free fermion SPTs in absence of lattice symmetries ?

Free fermion SPTs in Amorphous/glassy solids



[PRL 118, 236402 (2017)]

How to characterise the free fermion SPTs in absence of lattice symmetries ?



Adhip Agarwala & Vijay Shenoy, IISc

Provided a recipe to engineer several free fermion SPTs explicitly in two dimensional amorphous solids

How to characterise them ?

In particular, How to characterise a 3D amorphous Z2 TI ?

Electromagnetic response : Witten Effect

Witten Effect in topological insulators $\mathcal{L}_\theta = \frac{\theta}{4\pi^2}\mathbf{E}\cdot\mathbf{B} \qquad \theta \in (0, 2\pi)$

[Witten, 1979; Qi et. al., 2009; Essin et. al. 2009]

Under TR : $\theta \to -\theta \qquad \Rightarrow \theta = n\pi$

$$\mathcal{L}_\theta = \frac{\theta}{4\pi^2}\mathbf{E}\cdot\mathbf{B} = -\frac{\theta}{4\pi^2}\nabla\phi\cdot\mathbf{B} = \frac{\theta}{4\pi^2}\phi\nabla\cdot\mathbf{B}$$

$$\nabla\cdot\mathbf{B} = 4\pi m \qquad \mathcal{L}_\theta = \frac{\theta}{2\pi}m\phi$$

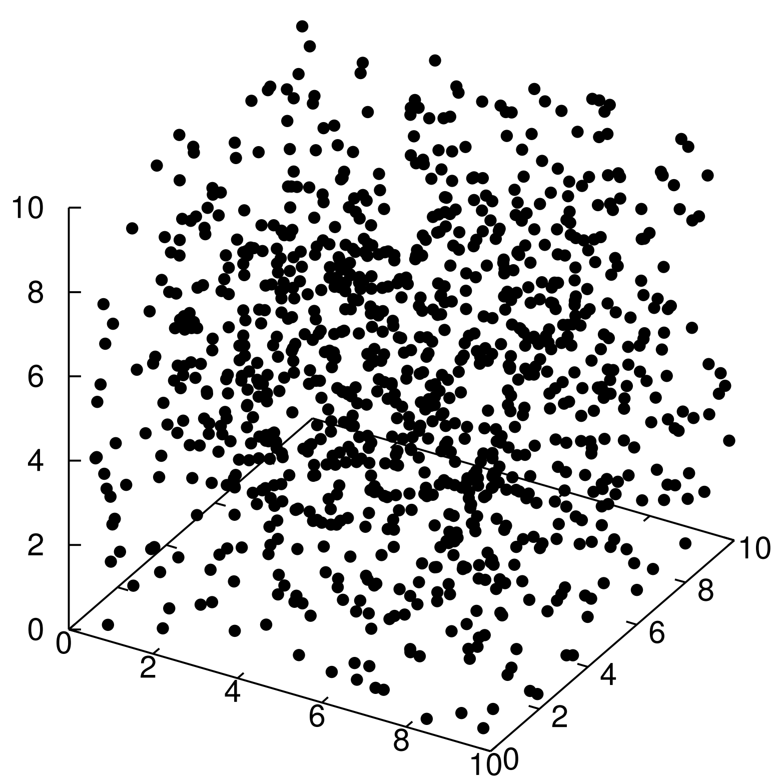
Unit magnetic monopole carries electric charge of $\frac{\theta}{2\pi}$

Trivial/topological Insulator : $\theta = 0/\pi$

Can this be measured ?

Can ask the same question for crystalline case, but in that case, other ways to calculate the bulk invariant. [Rosenberg et. al, 2010]

Hoping model of 3 dimensional Amorphous solid



Every site : Two pairs of Kramer's doublet

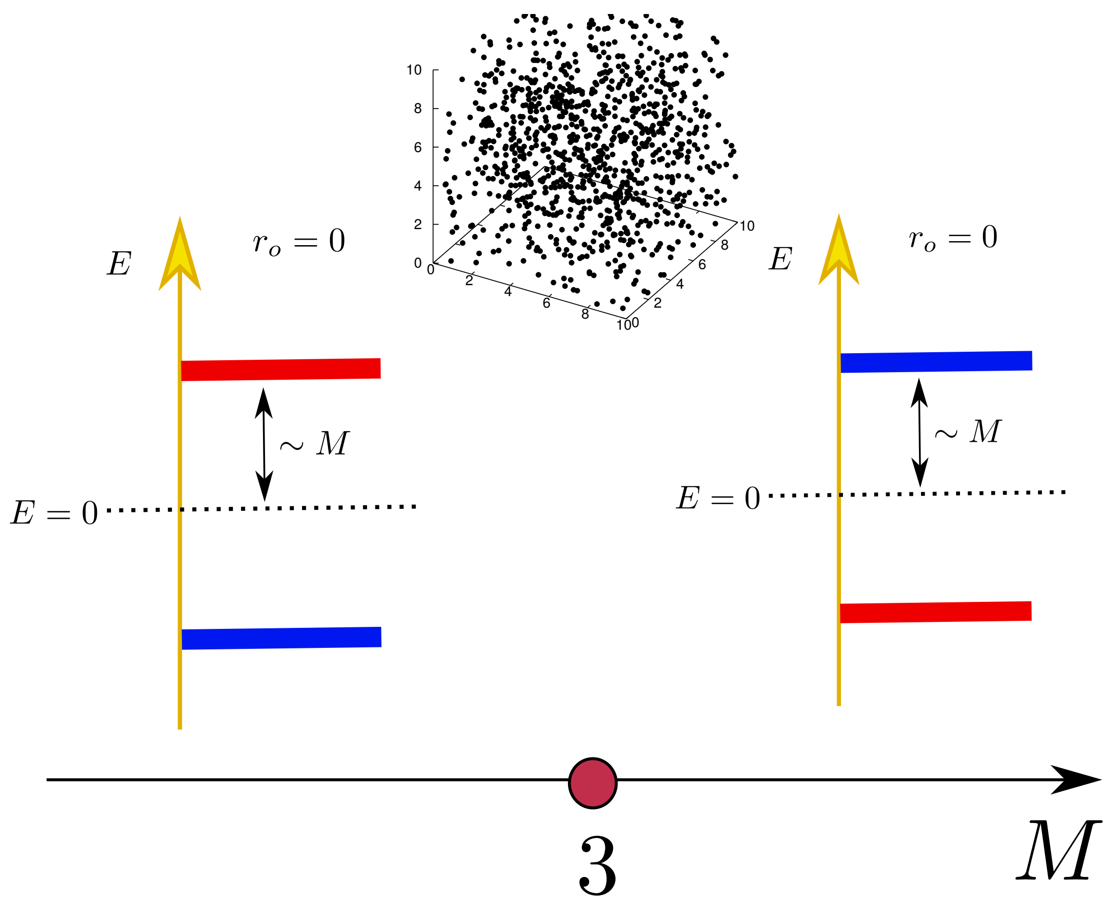
$$C_{I\alpha}$$

$$\alpha = s_{\uparrow}, s_{\downarrow}, p_{\uparrow}, p_{\downarrow}$$

$$H = \sum_{I\alpha} \sum_{J\beta} t_{\alpha\beta}(\overrightarrow{r}_{IJ}) C^{\dagger}_{I,\alpha} C_{J,\beta}$$

At half filling

$$t_{\alpha\beta}(0) = \begin{pmatrix} -3+M & 0 & 0 & 0 \\ 0 & -3+M & 0 & 0 \\ 0 & 0 & 3-M & 0 \\ 0 & 0 & 0 & 3-M \end{pmatrix}$$

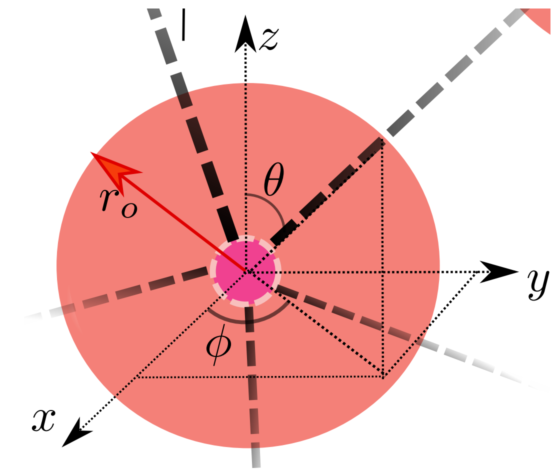


Trivial Atomic insulator when the sites are decoupled

Hopping

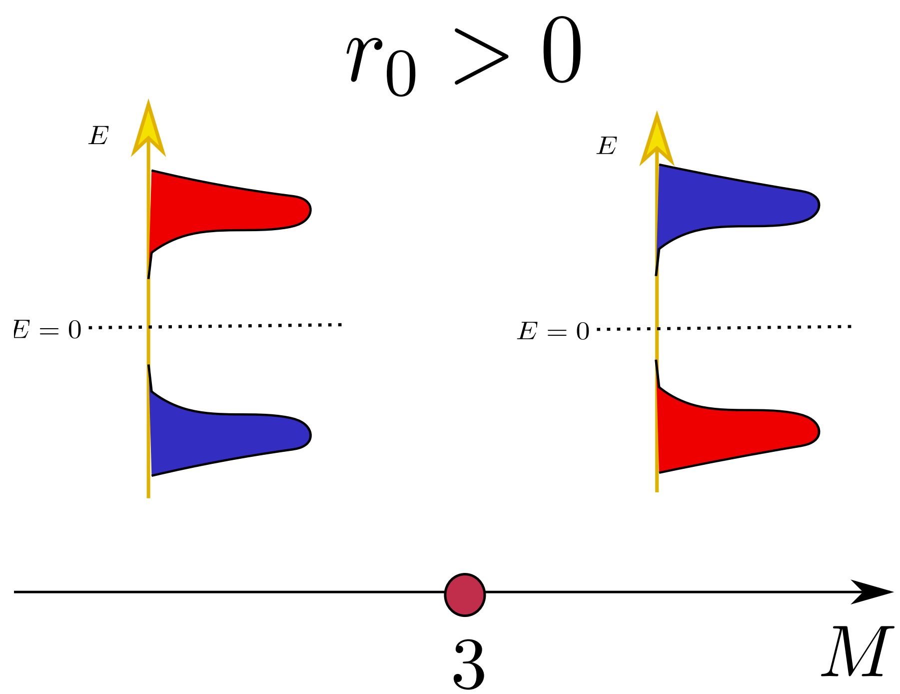
$$t_{\alpha\beta}(\overrightarrow{r}) = t(r)T_{\alpha\beta}(\hat{r})$$

$$t(r) = \Theta(R - r)e^{-r/r_0}$$



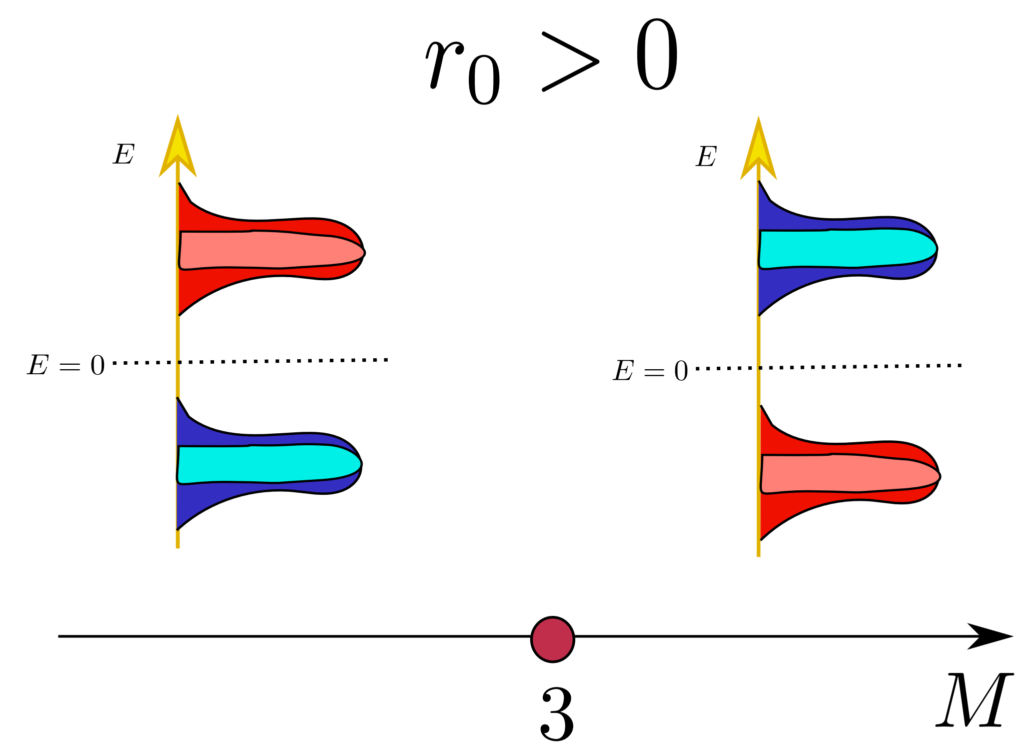
$$T_{\alpha\beta}(\hat{r}) = \begin{pmatrix} 1 & 0 & -i\cos\theta & -ie^{-i\phi}\sin\theta \\ 0 & 1 & -ie^{i\phi}\sin\theta & i\cos\theta \\ -i\cos\theta & -ie^{-i\phi}\sin\theta & -1 & 0 \\ -ie^{i\phi}\sin\theta & i\cos\theta & 0 & -1 \end{pmatrix}$$

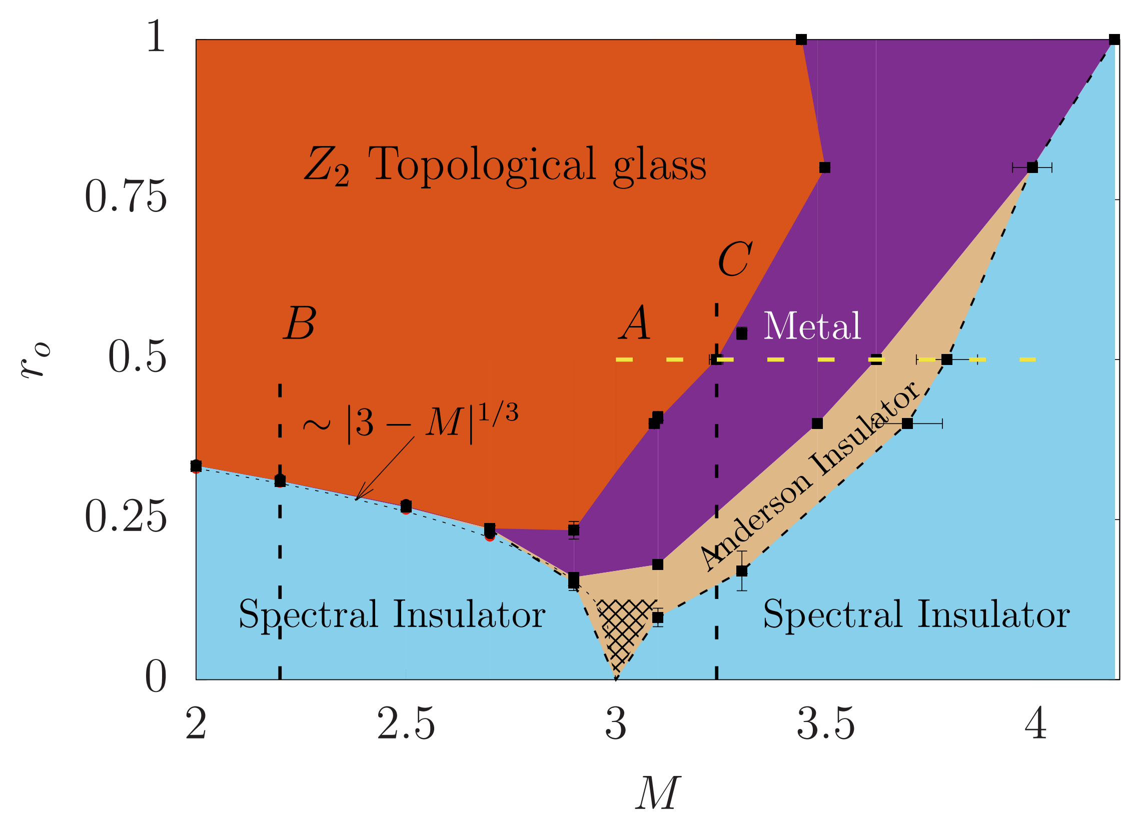
slight hopping



expectation: two bands of localized states

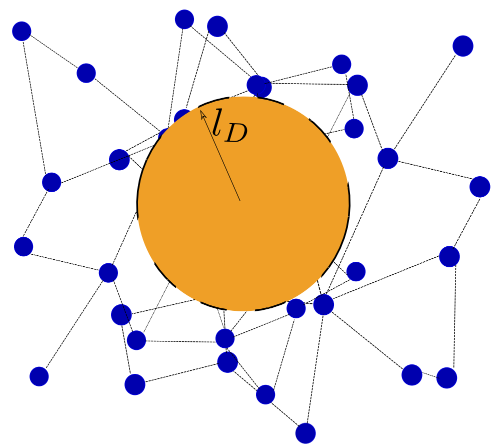
increase more

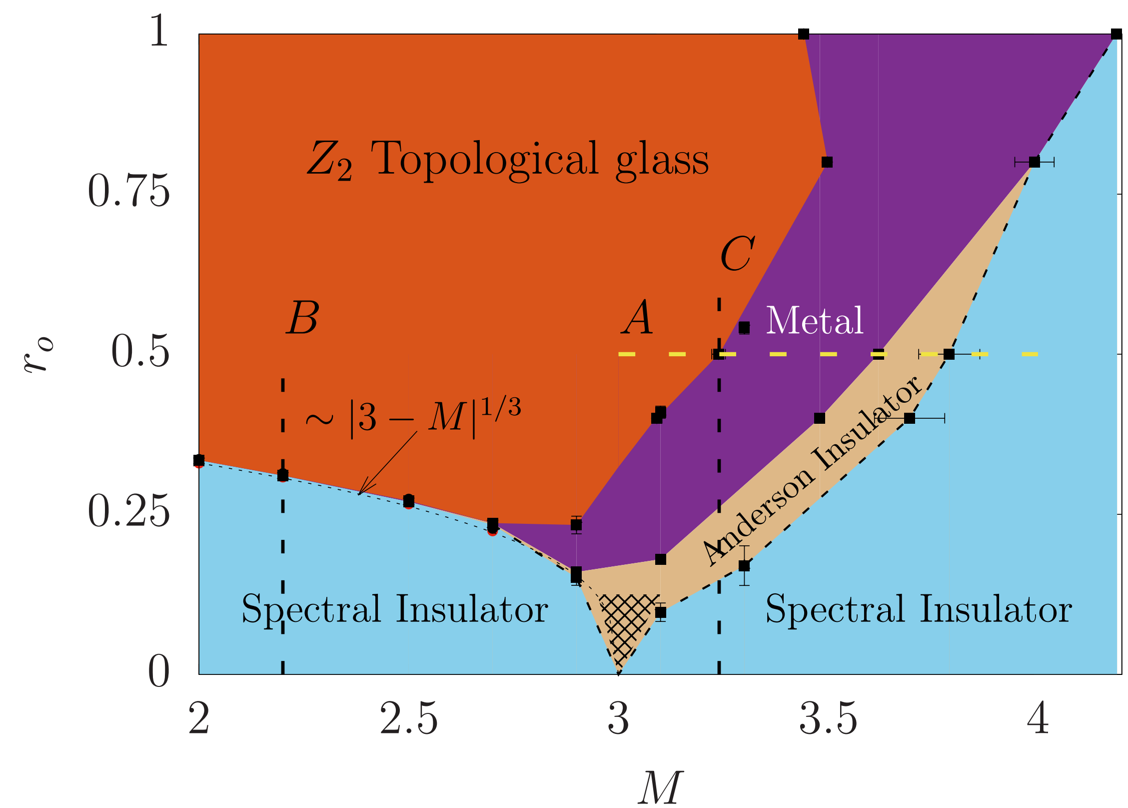


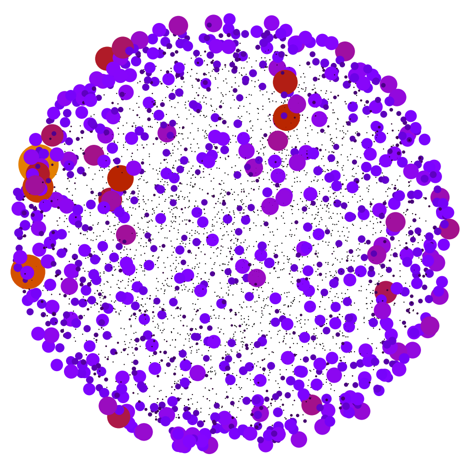


Witten Effect

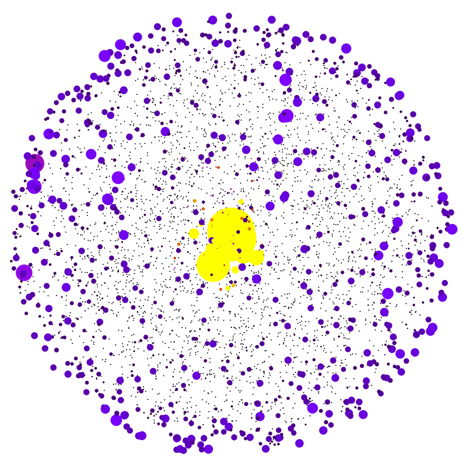
Put a magnetic monopole inside the glass!



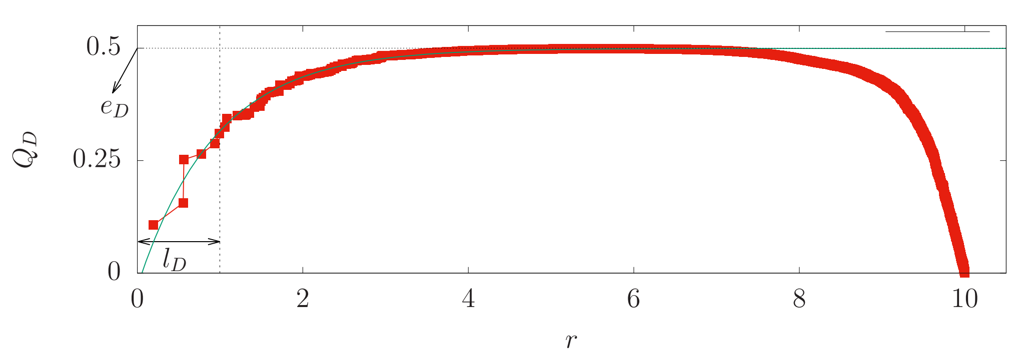


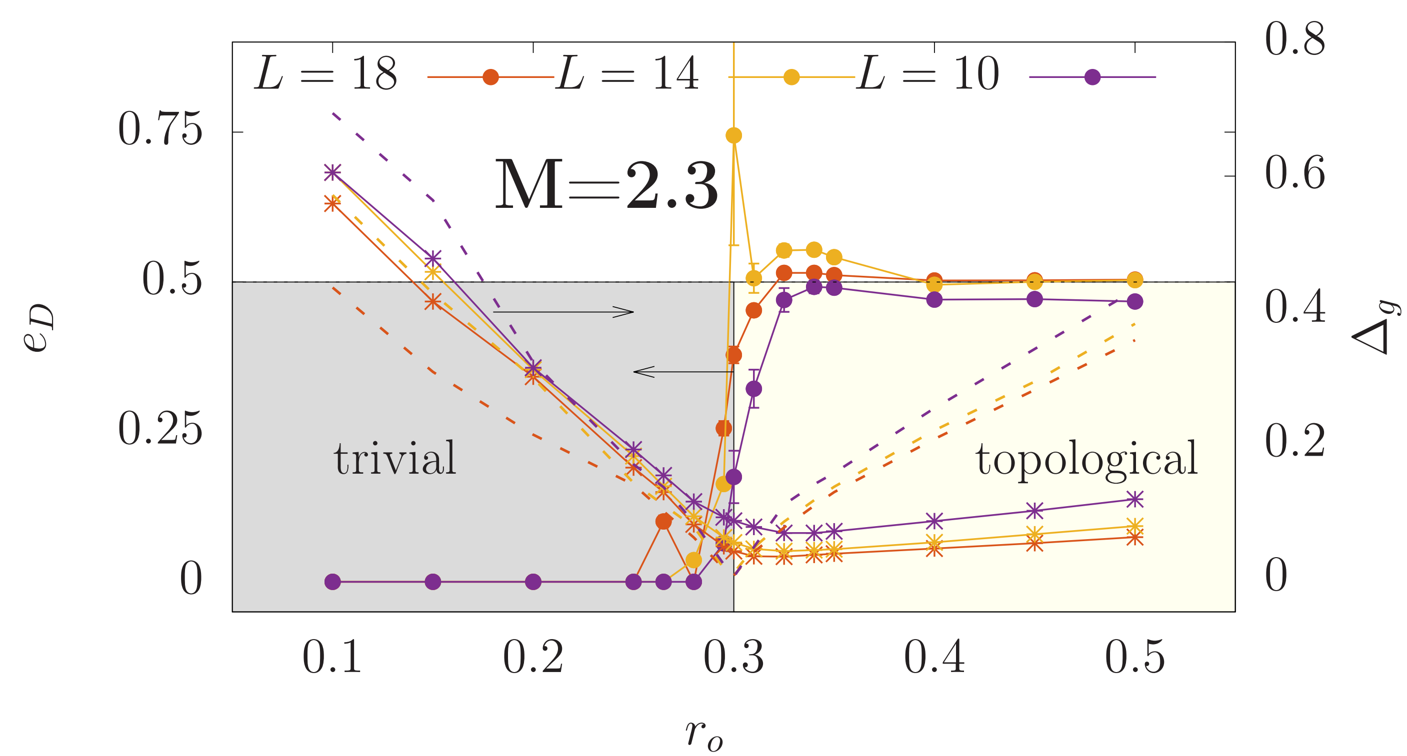


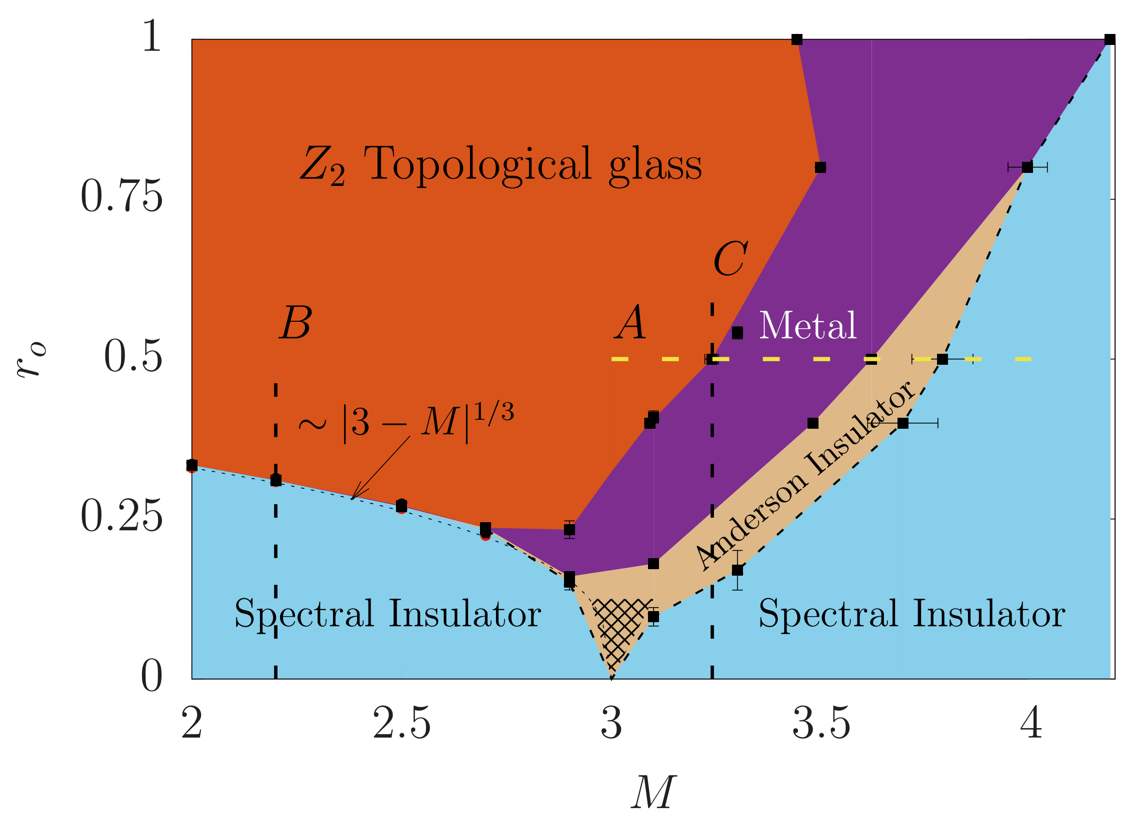
(a) Without Monopole



(b) With monopole







- General Phase diagram : Interplay of disorder an topology

Prateek Mukati **Adhip Agarwala**

arXiv: 1907.10098

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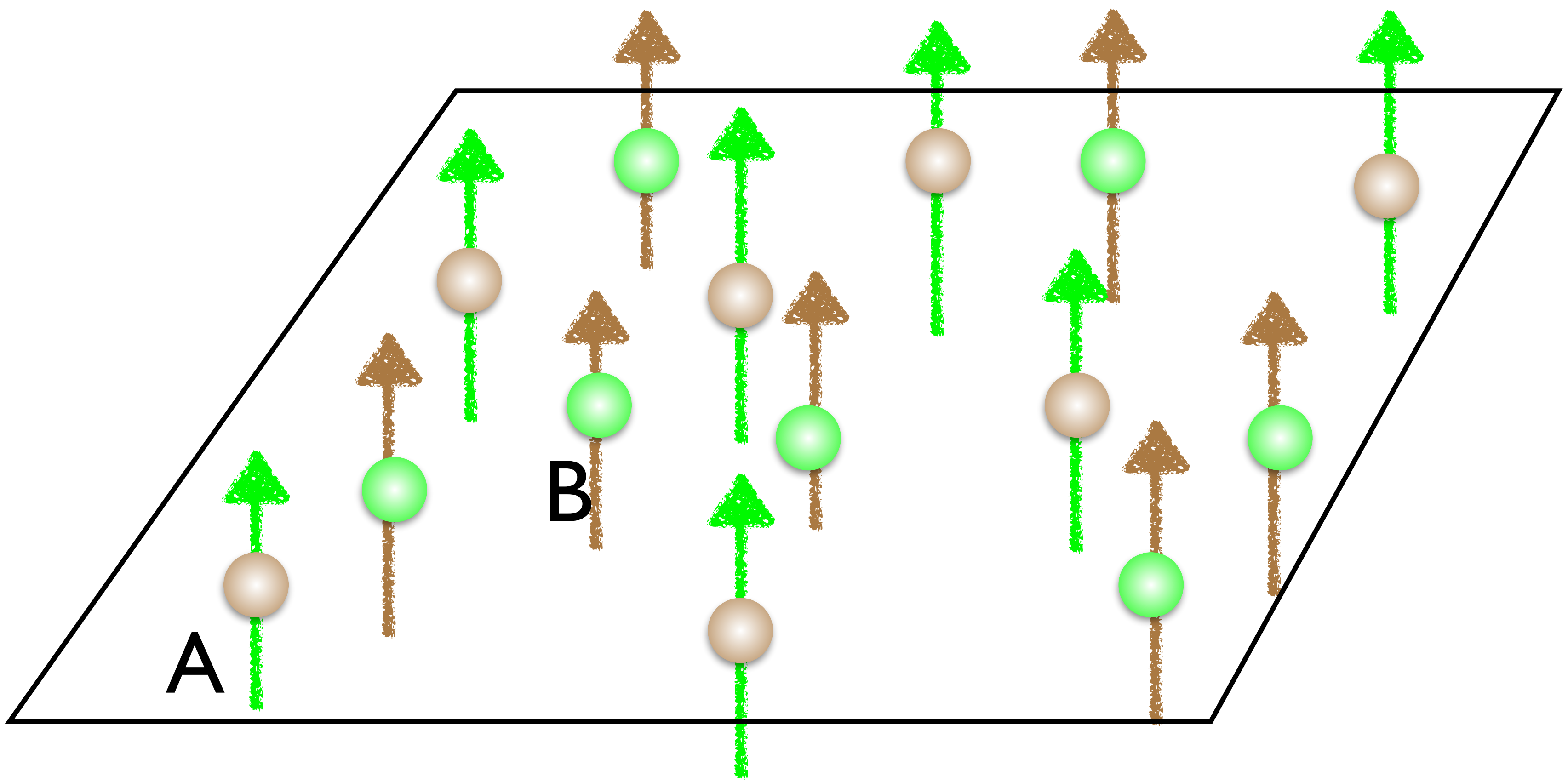
Integer quantum Hall effect of bosons

[Senthil & Levin, PRL 2013; Lu and Vishwanath, PRB 2012]



System of interacting bosons where the total boson number is conserved, U(1)

But NO time reversal symmetry

Fate of such a system

- **Topological order (in a moment)**
- **superfluid (breaks U(1) symmetry)**
- Mott insulators: Classification suggests that there can be **at least** a countably infinite number of Mott Insulators distinct from each other characterised by

$$n = 0, \pm 1, \pm 2, \pm 3, \cdots$$

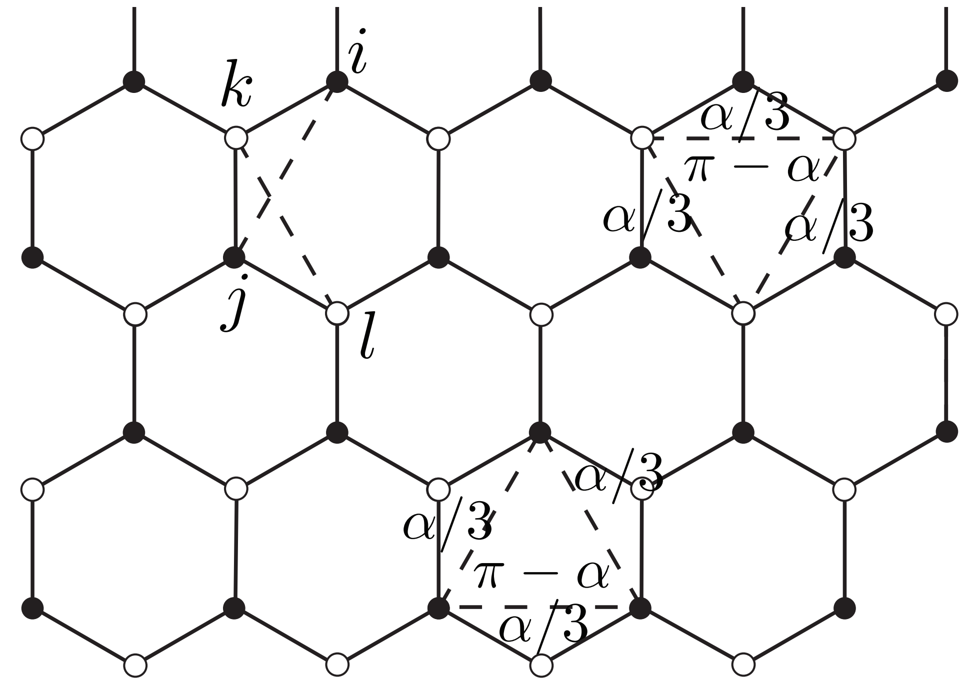
- Equal number of counter propagating chiral edge modes protected by U(1)

$$\sigma_{xy}^{charge} = 2n; \qquad \sigma_{xy}^{thermal} = 0$$

- Unique gapped ground states on a 2-tori

Lattice Model...1

- Hard-core bosons on honeycomb lattice at half filling.
- Correlated hopping in presence of a background flux preserving sub-lattice flavour.
- Last term NN hopping
- Background flux: Broken time reversal symmetry



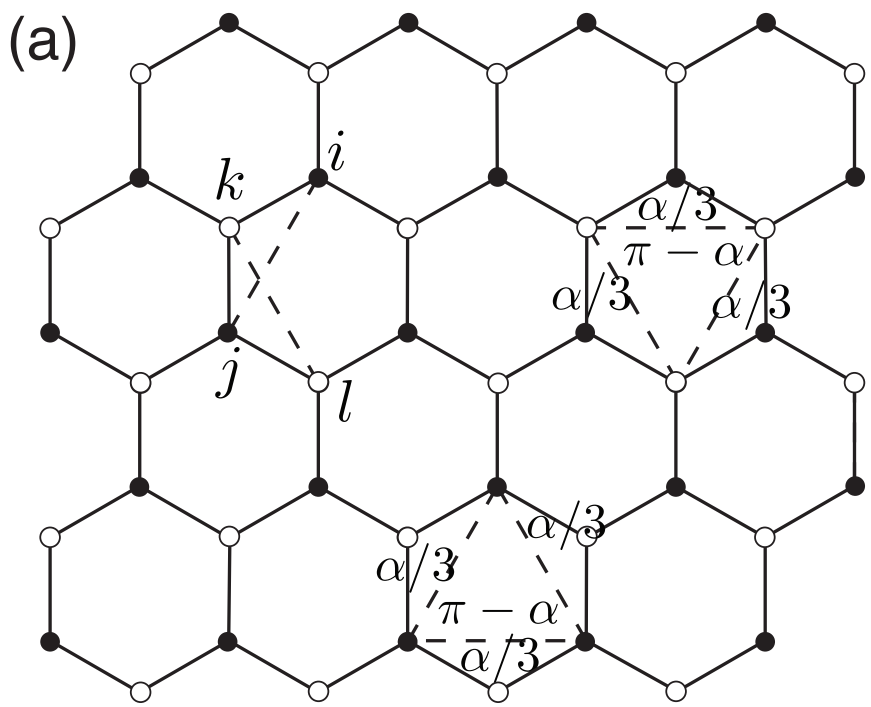
$$H = \sum_{\langle\langle ij\rangle\rangle}\left[e^{i\mathcal{A}_{ij}}(2n_k^b-1)a_i^\dagger a_j + h.c.\right]$$
$$+\sum_{\langle\langle kl\rangle\rangle}\left[e^{i\mathcal{A}_{kl}}(2n_j^a-1)b_k^\dagger b_l + h.c.\right]$$
$$+\lambda\sum_{\langle kj\rangle}(e^{i\mathcal{A}_{kj}}a_k^\dagger b_j + h.c.)$$

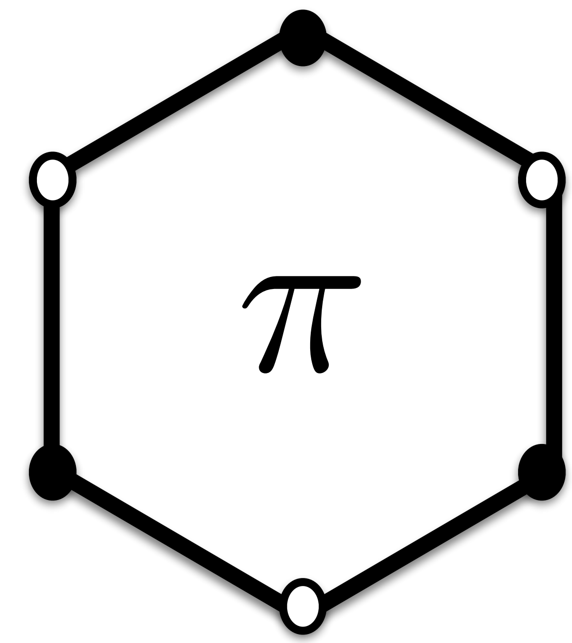
$$\lambda = 0 :\rightarrow U(1)\times U(1)$$
$$\neq 0 :\rightarrow U(1)$$

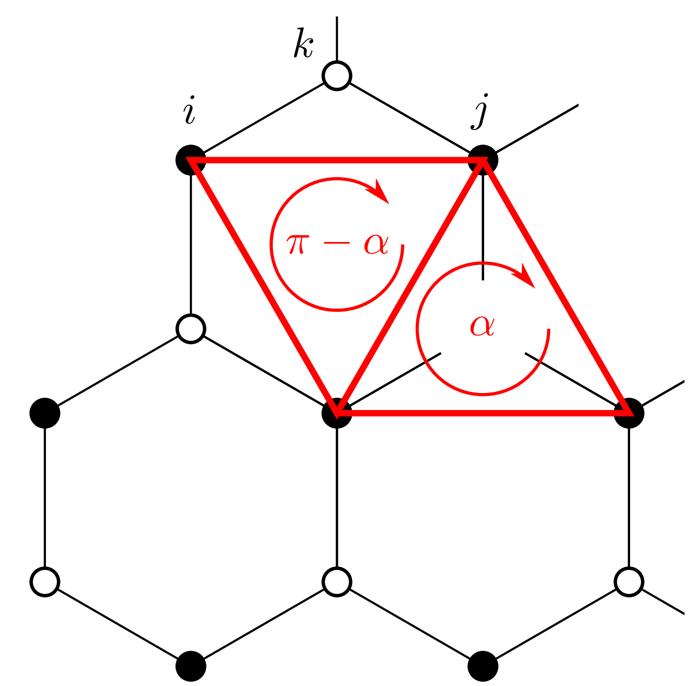
Other models : Regnault et. al., 2015

[SB et. al., 2015]

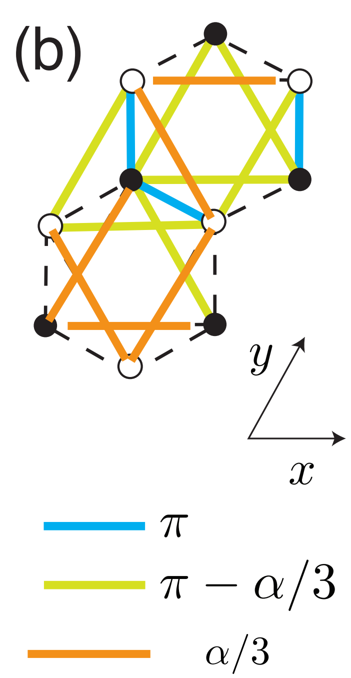
Lattice Model...2: The background flux







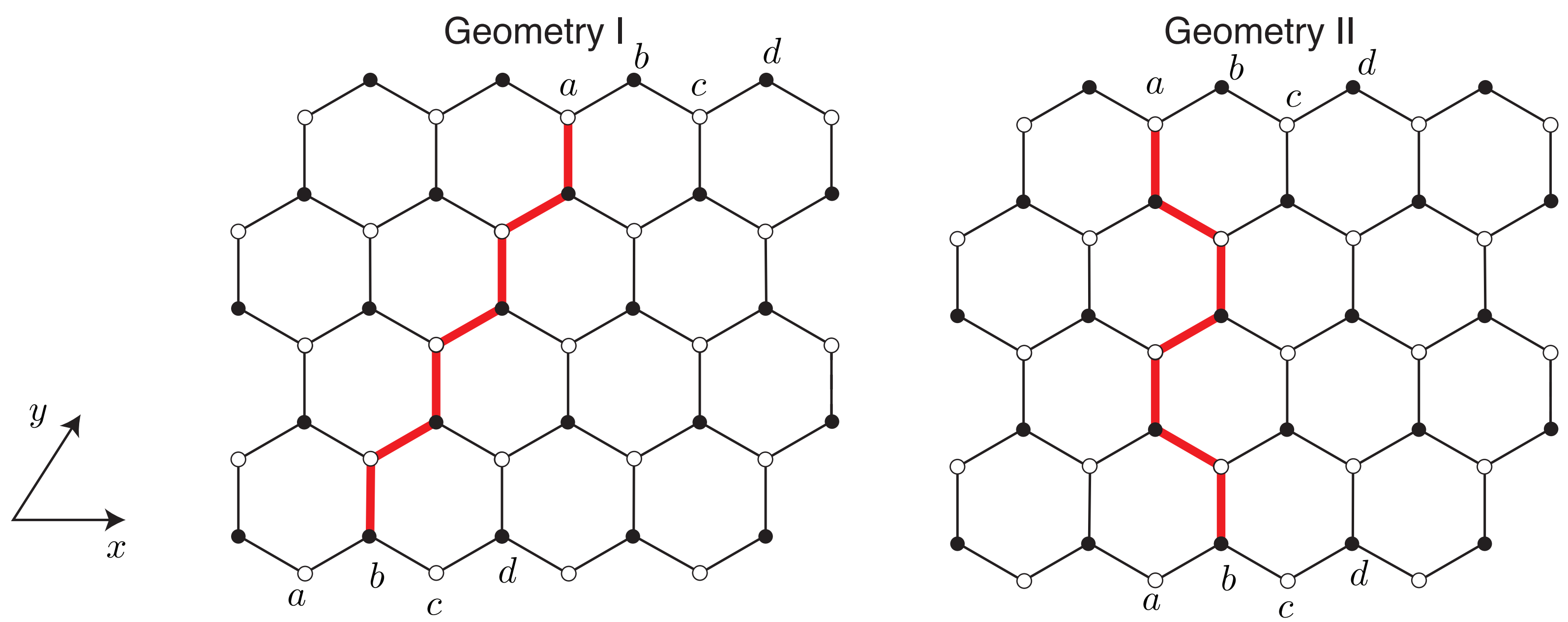
Gauge Choice



4-site unit cell

[SB et. al., 2015]

Numerical studies: Infinite DMRG on cylinders



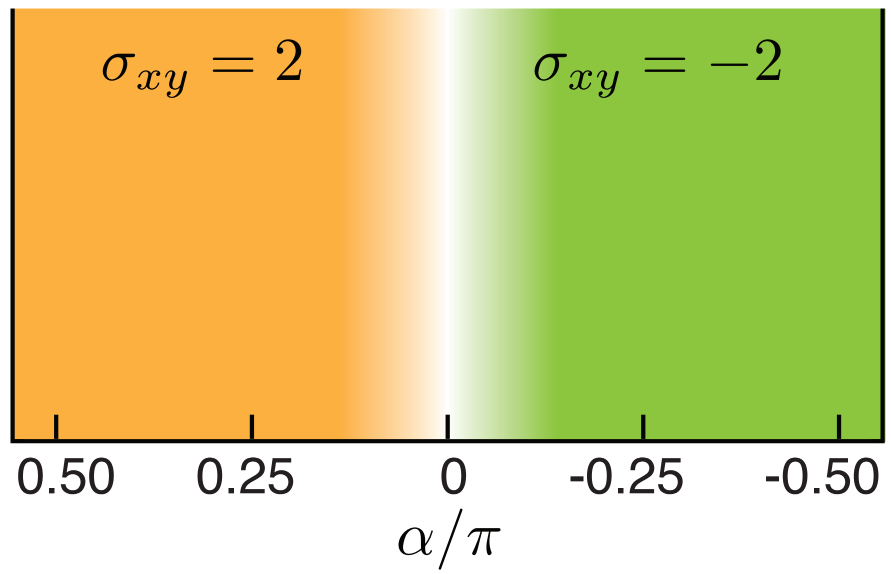
- Two geometries (no dependence of results)
- 4 site/unit cell.
- # of sites in y direction= 8, 12,16
- open boundary along x direction.

[SB et. al., 2015]

Numerical results

$\alpha \neq 0$

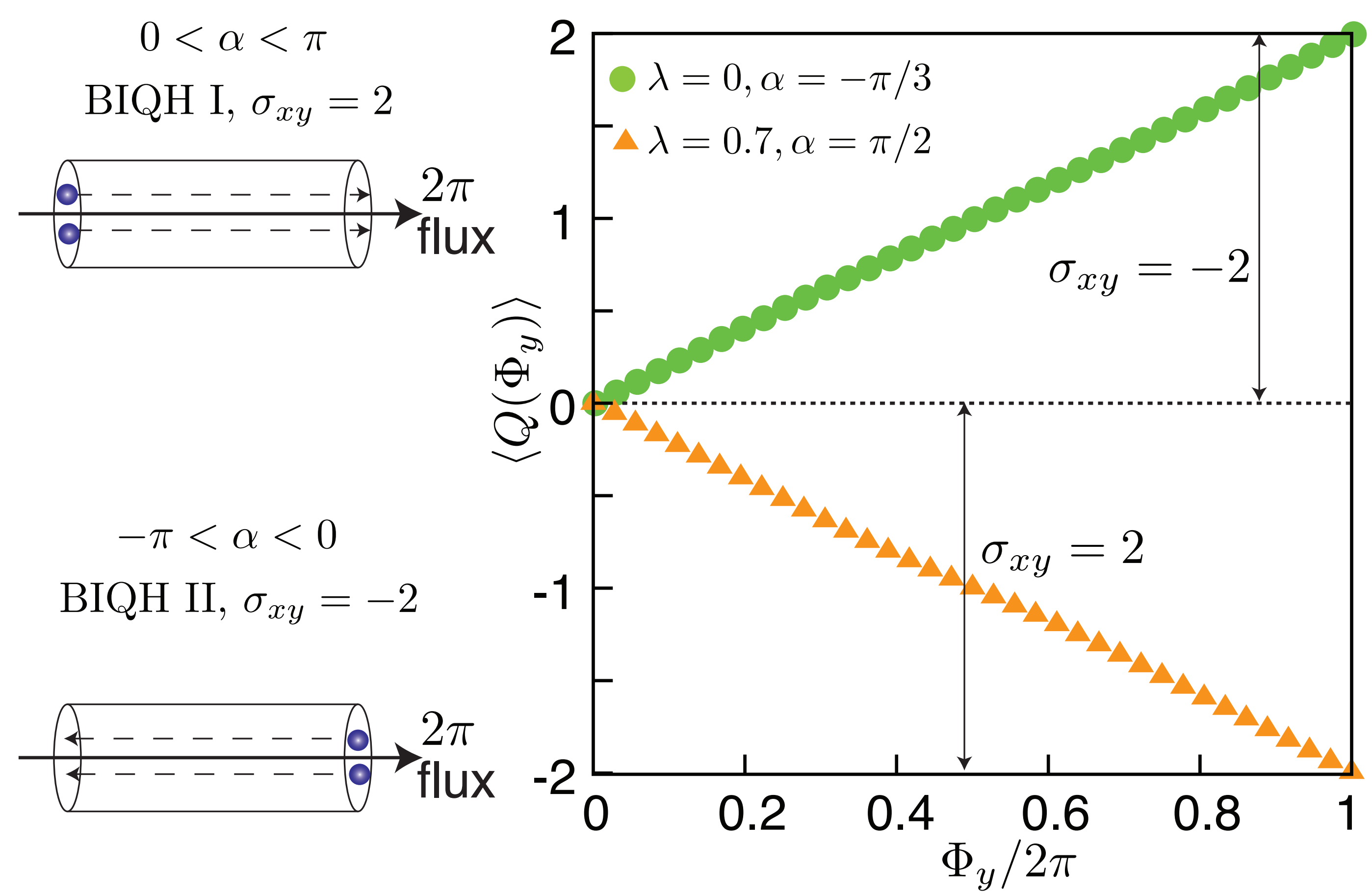
Unique ground state with a gapped excitation spectrum



- Quantized Hall response
- degeneracy of edge modes from entanglement spectra.

[SB et. al., 2015]

Numerical results…(1) : quantized (charge) Hall response



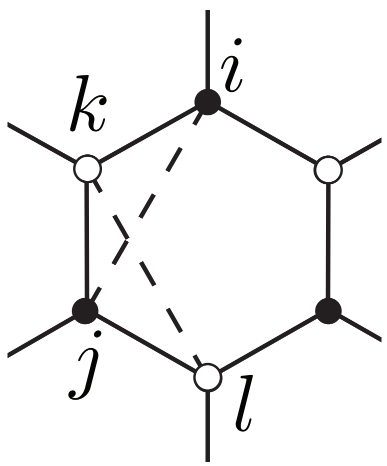
[SB et. al., 2015]

Quantization is robust for fairly large λ

Adiabatic insertion of 2π flux through the hole transfers σ_{xy} charges from one end to another

[Laughlin, 1981]

Lattice Model…3: Correlated hoping



$$e^{i\mathcal{A}_{ij}}(2n_k^b - 1)a_i^\dagger a_j$$

Amplitude of the hopping is opposite depending on presence/absence of b boson in the intermediate site

$$= -e^{i[\mathcal{A}_{ij}+\pi n_k^b]}a_i^\dagger a_j$$

The bosons of one flavour changes the flux seen by the other flavour

BIQH as a result of mutual flux binding ?

[Senthil & Levin, PRL 2013; Lu and Vishwanath, PRB 2012]

Particles in 2D as source of flux

Particle number conservation $\quad \partial_\mu J^\mu_i = 0 \quad\quad i = 1, 2$

Particles as source of flux $\quad \Rightarrow J^\mu_i = \frac{\epsilon^{\mu\nu\lambda}}{2\pi} \partial_\nu \alpha^i_\lambda$

Mutual Flux binding

Bind one quantum of flux of b with a particle of type a and vice versa

(particle of type 1 sees particle of type 2 as source of 2π flux)

$$\phi_a(x) = \int d^2x'\Theta(x-x')a^\dagger(x)a(x)$$

$$\tilde{a}(x) = e^{i\phi_b(x)}a(x) \qquad \tilde{b}(x) = e^{i\phi_a(x)}b(x)$$

At low energy, it is possible that b_i become the effective quasi particles

$$\tilde{a}(x) \text{ and } a(x) \rightarrow U_a(1) \qquad A_c = A_a + A_b$$

$$\tilde{b}(x) \text{ and } b(x) \rightarrow U_b(1) \qquad A = \frac{A_a - A_b}{2}$$

Low energy theory for the composite bosons and BIQH

[Senthil & Levin, PRL 2013]

$$\mathcal{L} = \mathcal{L}_a + \mathcal{L}_b + \mathcal{L}_{\text{int}} + \mathcal{L}_{CS}$$

$$\left.\begin{aligned}
\mathcal{L}_a =& i\tilde{a}^* \left[\partial_0 - i\left(\frac{1}{2}A_0^c + \mathcal{A}_0\right) + i\alpha_0\right]\tilde{a} \\
&- \frac{1}{2m}\left|\nabla\tilde{a} + i\left(\frac{1}{2}\vec{A}^c + \vec{\mathcal{A}}\right)\tilde{a} + i\vec{\alpha}\tilde{a}\right|^2 \\
\mathcal{L}_b =& i\tilde{b}^* \left[\partial_0 - i\left(\frac{1}{2}A_0^c - \mathcal{A}_0\right) + i\beta_0\right]\tilde{b} \\
&- \frac{1}{2m}\left|\nabla\tilde{b} + i\left(\frac{1}{2}\vec{A}^c - \vec{\mathcal{A}}\right)\tilde{b} + i\vec{\alpha}\tilde{b}\right|^2 ,
\end{aligned}\right\} \text{Action for composite bosons}$$

$$\mathcal{L}_{CS} = \frac{1}{4\pi}\epsilon^{\mu\nu\lambda}\left[\alpha_\mu\partial_\nu\beta_\lambda + \beta_\mu\partial_\nu\alpha_\lambda\right], \longrightarrow \text{Mutual Chern-Simons term (Flux attachment)}$$

Low energy theory for the composite bosons and BIQH

[Senthil & Levin, PRL 2013]

BIQH ➜ Condense composite bosons

$$\begin{aligned}
\mathcal{L}_a =& i\tilde{a}^* \left[\partial_0 - i\left(\frac{1}{2}A_0^c + \mathcal{A}_0\right) + i\alpha_0\right]\tilde{a} \\
&- \frac{1}{2m}\left|\nabla\tilde{a} + i\left(\frac{1}{2}\vec{A}^c + \vec{\mathcal{A}}\right)\tilde{a} + i\vec{\alpha}\tilde{a}\right|^2 \\
\mathcal{L}_b =& i\tilde{b}^* \left[\partial_0 - i\left(\frac{1}{2}A_0^c - \mathcal{A}_0\right) + i\beta_0\right]\tilde{b} \\
&- \frac{1}{2m}\left|\nabla\tilde{b} + i\left(\frac{1}{2}\vec{A}^c - \vec{\mathcal{A}}\right)\tilde{b} + i\vec{\alpha}\tilde{b}\right|^2,
\end{aligned}$$

Locking of gauge fields
(Anderson-Higgs Mechanism)

$$\alpha = A^c + \mathcal{A}$$

$$\beta = A^c - \mathcal{A}$$

Low energy theory for BIQH

BIQH ➜ Condense composite bosons

$$\mathcal{L}_{SPT} = -\frac{2}{4\pi}\varepsilon_{\mu\nu\lambda}\mathcal{A}^c_\mu\partial_\nu\mathcal{A}^c_\lambda + \frac{2}{4\pi}\varepsilon_{\mu\nu\lambda}\mathcal{A}_\mu\partial_\nu\mathcal{A}_\lambda$$

$$\sigma^c_{xy} = +2, \qquad \sigma^s_{xy} = -2$$

Edge :

charge mode

non-chiral

s mode

Numerical results...(1) : Degeneracy of edge modes

[Furukawa et. al., 2013]

Expectation from edge theory $\mathcal{L} = -\frac{1}{4\pi}(K_{\alpha\beta}\partial_t\phi_\alpha\partial_x\phi_\beta + V_{\alpha\beta}\partial_x\phi_\alpha\partial_x\phi_\beta)$ $K = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

Introduce charge and pseudo spin modes $\phi_{c(s)} = (\phi_a \pm \phi_b)/\sqrt{2}$

Hamiltonian

Momentum

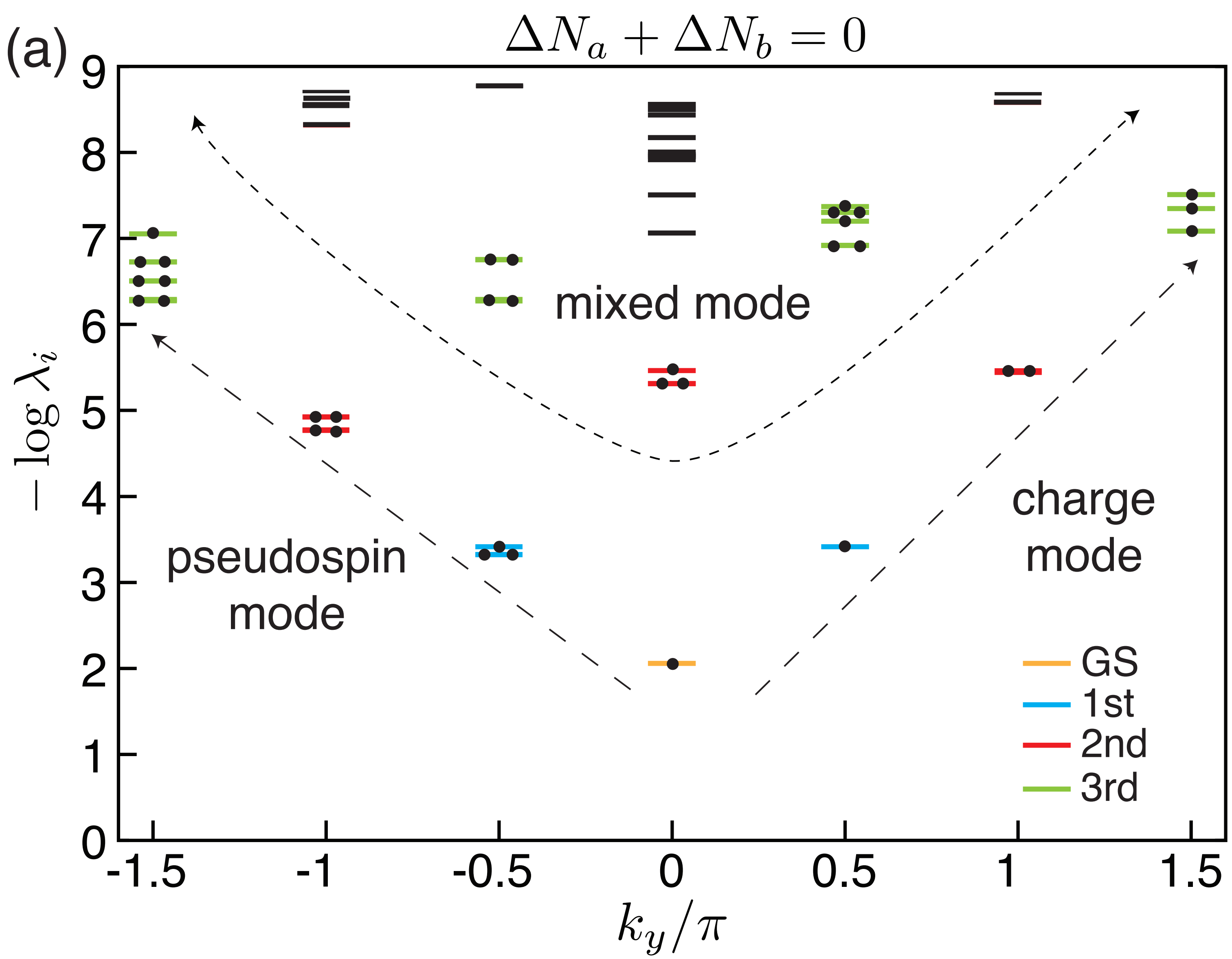
$$H = \frac{2\pi v}{L_y}(L_0^c + L_0^s), \qquad P = \frac{2\pi}{L_y}(L_0^c - L_0^s)$$

$$L_0^{c(s)} = \frac{(\Delta N_a \pm \Delta N_b)^2}{4} + \sum_{m=1}^{\infty} m n_m^{c(s)}$$

$\Delta N_{a(b)} \rightarrow$ deviation of boson number from GS

$m = 1, 2, 3 \cdots$ $\quad n_m^{c(s)} \rightarrow$ oscillator occupation no.

Numerical results...(2) : Degeneracy of edge modes



$$L_y = 4$$

Bosonic Integer Quantum Hall phase

TABLE I: The energy levels of entanglement Hamiltonian (edge mode): $\Delta N_a + \Delta N_b = 0$.

Levels	mode	k_y	L_0^c, L_0^s	ΔN_a, ΔN_b	$\{n_m^c\}$	$\{n_m^s\}$	Degeneracy
Ground state	-	0	$L_0^c = L_0^s = 0$	$\Delta N_a = \Delta N_b = 0$	$n_m^c = 0$	$n_m^s = 0$	1
1st excited state	charge	$2\pi/L_y$	$L_0^c = 1$, $L_0^s = 0$	$\Delta N_a = \Delta N_b = 0$	$\sum m n_m^c = 1$	$n_m^s = 0$	1
$L_0^c + L_0^s = 1$	spin	$-2\pi/L_y$	$L_0^c = 0$, $L_0^s = 1$	$\Delta N_a - \Delta N_b = \pm 2$	$n_m^c = 0$	$n_m^s = 0$	3
				$\Delta N_a = \Delta N_b = 0$	$n_m^c = 0$	$\sum m n_m^s = 1$	

[SB et. al., 2015]

Outline

- Introduction
- Three stories :
  - 1. Free fermion Z2 SPT in a three dimensional amorphous solid
    - Characterisation through electromagnetic response.
  - 2.1 : Interacting U(1) Bosonic SPT in 2 dimensions : Bosonic Integer quantum Hall effect on a lattice
  - 2.2. A chiral Spin liquid on a Kagome Lattice
    - The connection between the two through the “gauging” of U(1).
- Summary

Quantum spin liquids and Kagome antiferromagnets

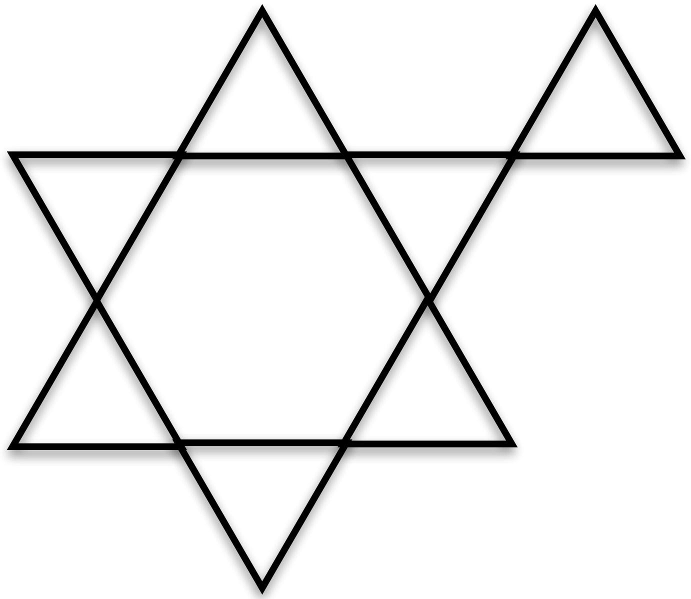
S=1/2

$$H = J \sum_{\langle ij \rangle} \mathbf{S_i} \cdot \mathbf{S_j} + \cdots$$

- Rich and intriguing possibilities
- quantum spin liquids ?
- Several candidate materials

XXZ antiferromagnets on Kagome and Chiral spin liquids

$$H = J_z \sum_{\langle ij \rangle} S_i^z S_j^z + \lambda H_{pert}$$



$$J_z, \lambda > 0 \quad \text{and} \quad J_z \gg \lambda$$

Classical problem has macroscopically degenerate ground state

Quantum fluctuation can lead to long range entangled state

[C. Laumann's talk]

Model : XXZ antiferromagnet with 3-spin interaction

[Bauer et. al., Nat. comm. 2014; Schroeter et. al. PRL, 2007]

$$H_{\text{chiral}} = J_z \sum_{\langle pq \rangle} S_p^z S_q^z + \lambda \sum_{p,q,r \in \bigtriangledown, \bigtriangleup} \vec{S}_r \cdot (\vec{S}_p \times \vec{S}_q)$$

[He, SB, Pollmann, Moessner, PRL, 2015]

Other models :

[He, Sheng & Chen, PRL, 2014; Gong et. al. Sci. Rep. 2014]

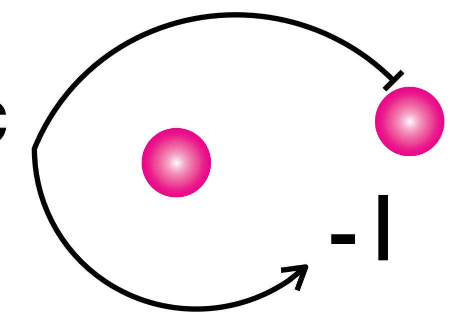
Chiral spin liquid

[Kalmeyer & Laughlin, PRL 1987]

Laughlin's state of Holstein Primakoff /Schwinger bosons

Fractional quasiparticle (spinon) 1/2 spin

Semionic statistics

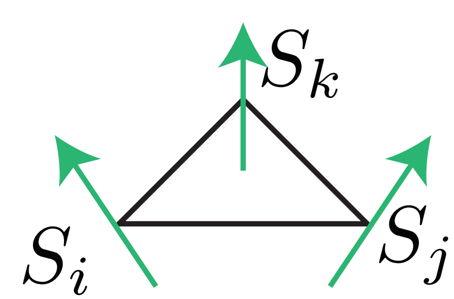


Degeneracy on a torus = 2

[Wen, Int. J. Mod. Phys. B, 1990]

Spontaneous time-reversal symmetry breaking

Scalar chirality order

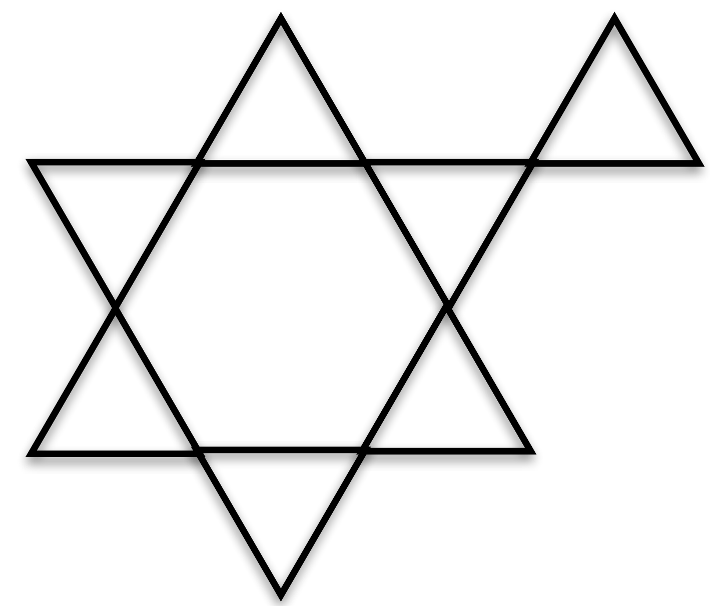


$$\vec{S}_i \cdot (\vec{S}_j \times \vec{S}_k)$$

[Wen, Wilczek & Zee, PRB 1989]

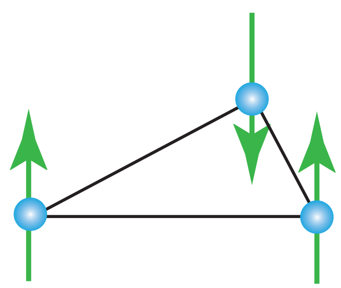
Easy axis limit : U(1) lattice gauge theory with dynamical bosonic spinons

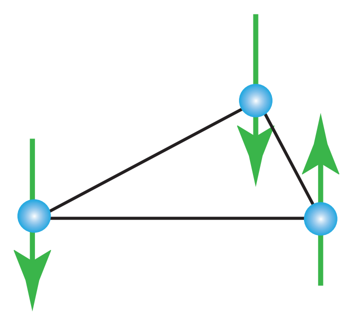
$$H = J_z \sum_{\langle ij \rangle} S_i^z S_j^z + \lambda H_{pert}$$



$$J_z, \lambda > 0 \quad \text{and} \quad J_z \gg \lambda$$

extensive classical degeneracy



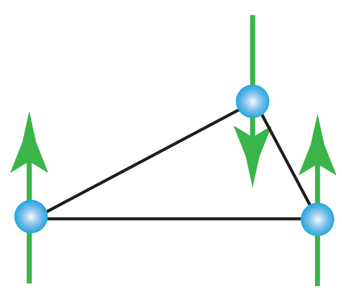


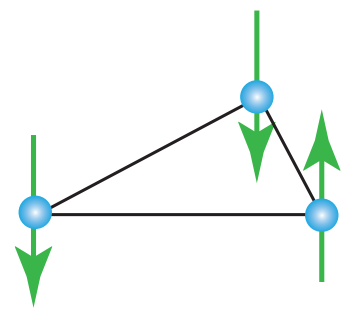
degeneracy lifted by H_{pert}

$$\sum_{p \in \triangle, \bigtriangledown} S_p^z = \pm 1/2$$

Easy axis limit : U(1) lattice gauge theory with dynamical bosonic spinons

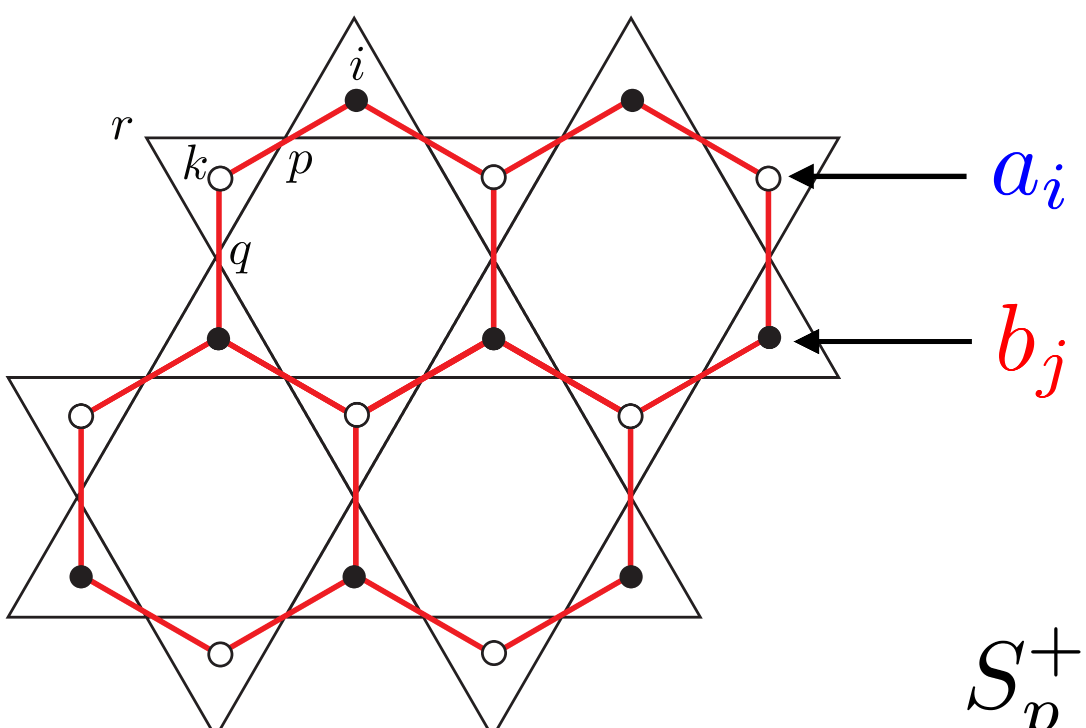
[Nikolic, Senthil, PRB, 2005]





$$\sum_{p\in\triangle,\nabla} S_p^z = \pm 1/2$$

Hard-core boson representation on medial honeycomb lattice

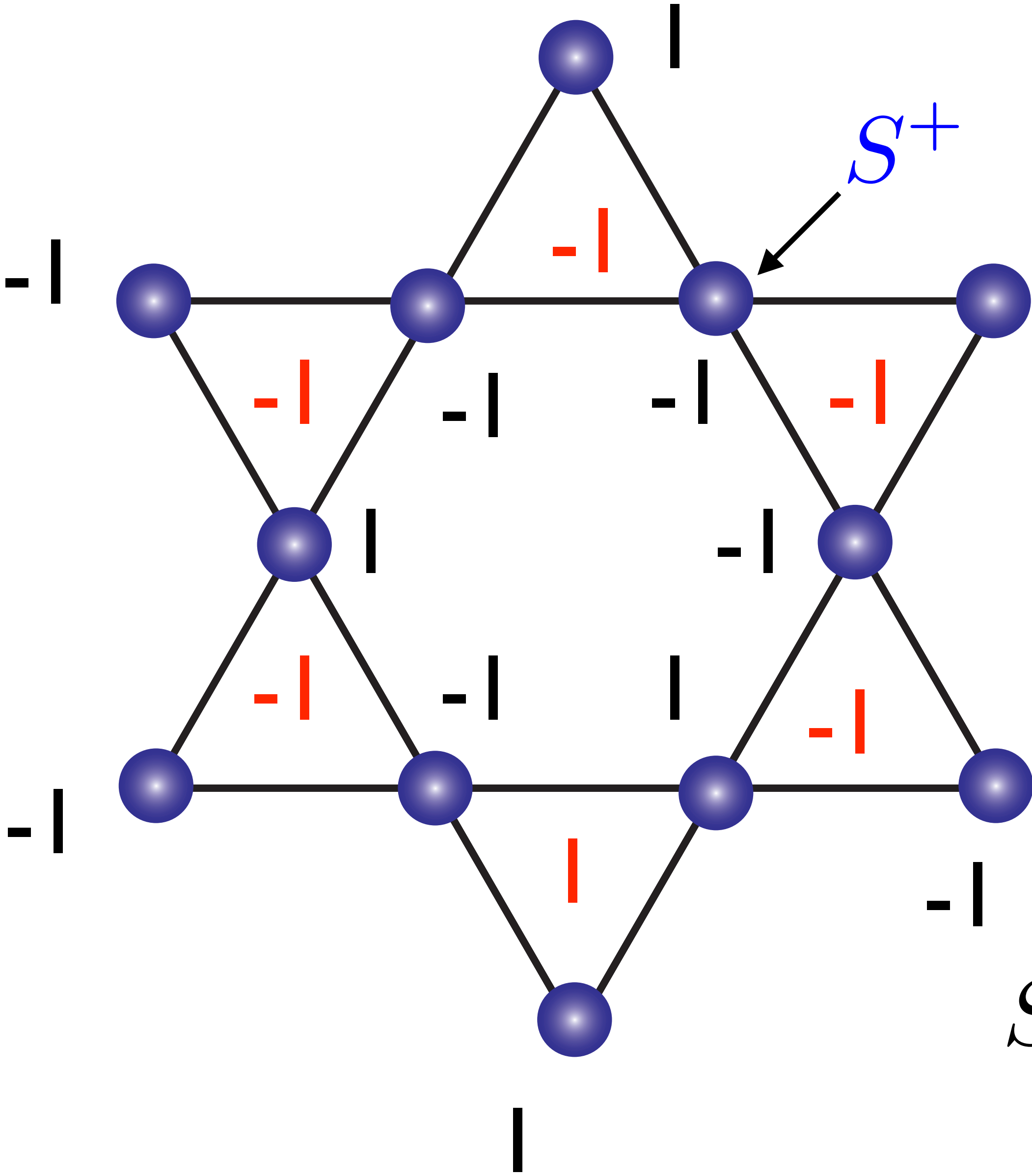


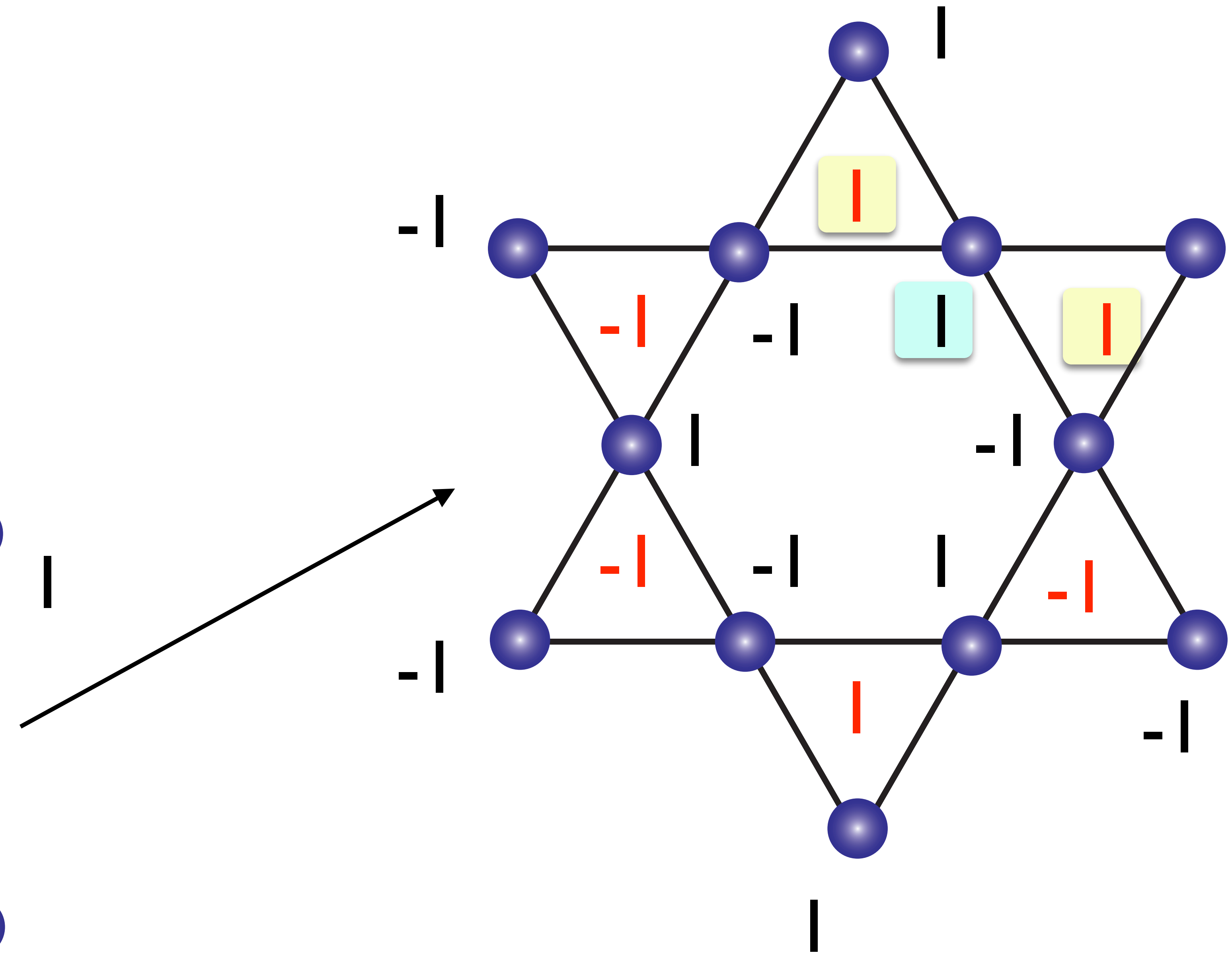
$$\sum_{p\in\triangle_i} S_p^z = a_i^\dagger a_i - \frac{1}{2},$$

$$\sum_{p\in\nabla_k} S_p^z = b_k^\dagger b_k - \frac{1}{2}$$

$$S_p^+ = e^{i\mathcal{A}_{ik}} a_i^\dagger b_k^\dagger \quad \text{and} \quad S_p^- = e^{i\mathcal{A}_{ki}} a_i b_k$$

Bosonic Spinons

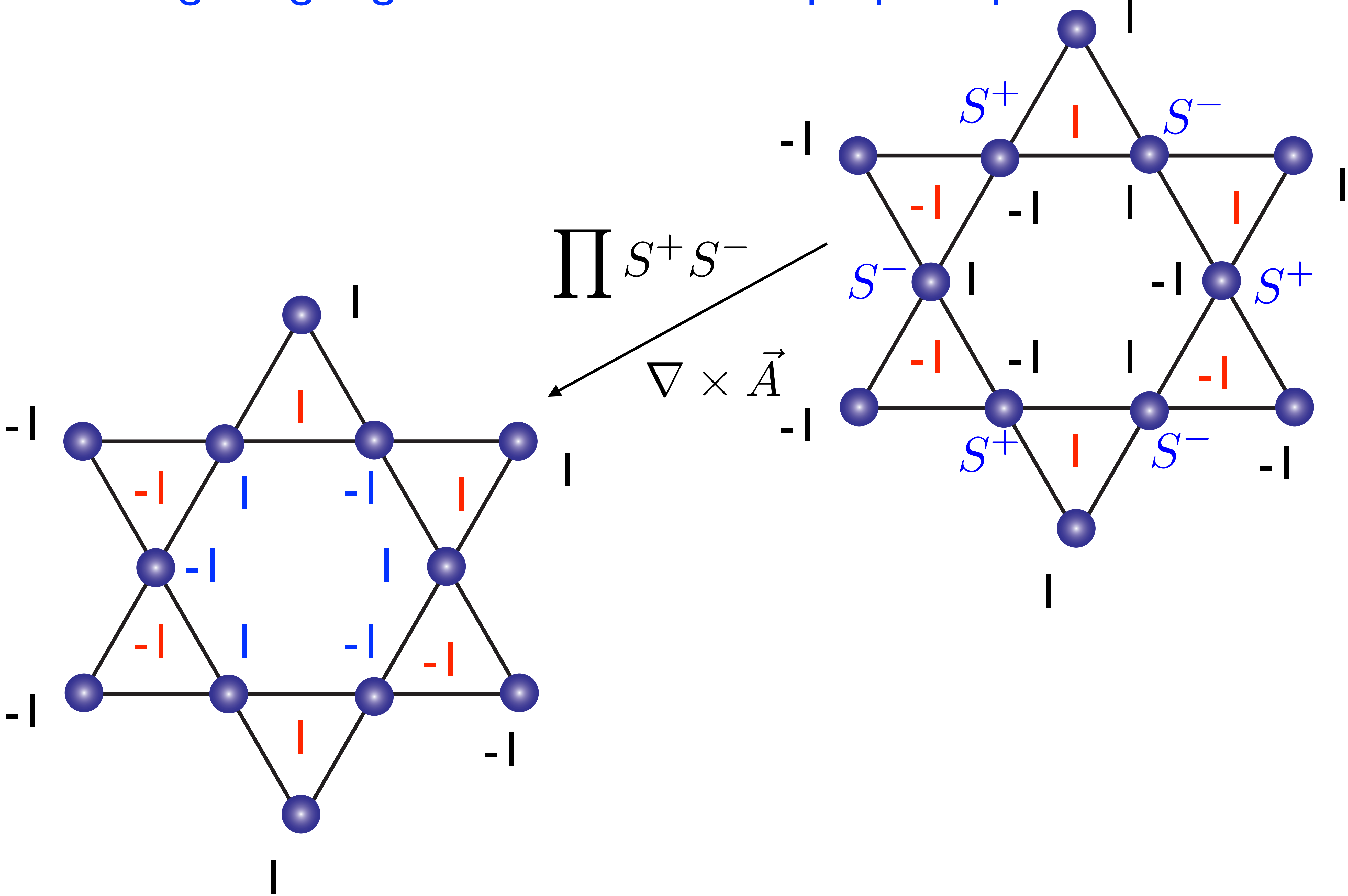




$$S_p^+ = e^{i\mathcal{A}_{ik}} a_i^\dagger b_k^\dagger \quad \text{and} \quad S_p^- = e^{i\mathcal{A}_{ki}} a_i b_k$$

$$E_{ik} = -E_{ki} = (S_p^z + 1/2)$$

emergent gauge field : Closed loop spin flips



Easy axis limit : U(1) lattice gauge theory with dynamical bosonic spinons

Recast the spin model in terms of bosonic spinons and the U(1) gauge field

Charge quantum numbers of the spinons

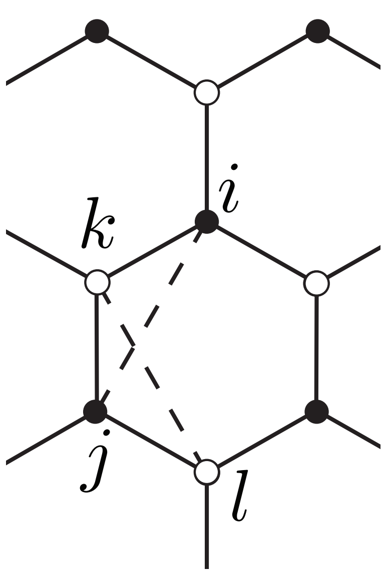
Type	S_z (U(1) global charge)	$\mathcal{A}$ (U(1) gauge charge)
a_i	$1/2$	1
b_j	$1/2$	-1

Effective Low energy Hamiltonian

$$H_{\text{chiral}} = J_z \sum_{\langle pq \rangle} S_p^z S_q^z + \lambda \sum_{p,q,r \in \bigtriangledown, \bigtriangleup} \vec{S}_r \cdot (\vec{S}_p \times \vec{S}_q)$$

$$\mathcal{P}\left[\vec{S}_p \cdot (\vec{S}_q \times \vec{S}_r)\right]\mathcal{P} = \frac{1}{4}(2n_k^b - 1)\left[e^{i(\mathcal{A}_{ij}+\pi/2)}\hat{a}_i^\dagger \hat{a}_j + \text{h.c.}\right] + \text{cyclic perm.}$$

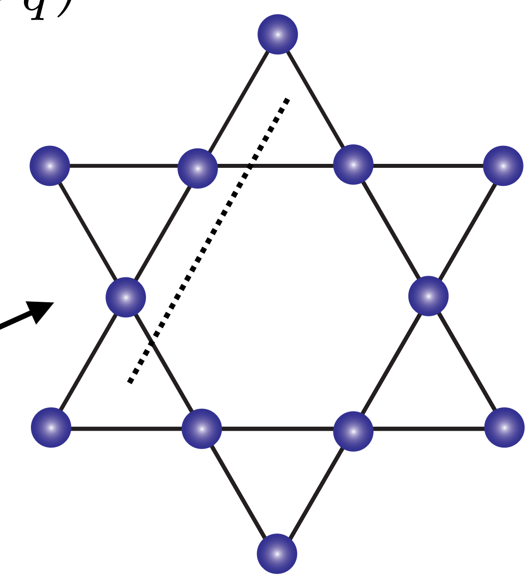
Correlated Hopping of bosons



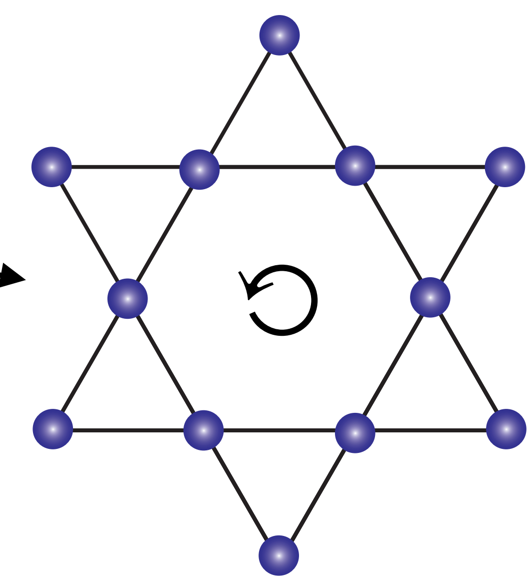
- second neighbour hopping of bosons.
- hopping amplitude changes sign depending on the occupancy of intermediate site
- Dynamic gauge field

Effective Low energy Hamiltonian

$$H_{\text{chiral}} = J_z \sum_{\langle pq \rangle} S_p^z S_q^z + \lambda \sum_{p,q,r \in \bigtriangledown,\bigtriangleup} \vec{S}_r \cdot (\vec{S}_p \times \vec{S}_q)$$



$$H_{\text{chiral}}^{\text{LGT}} = \lambda \sum_{\langle\langle ij \rangle\rangle} \left[e^{i\mathcal{A}_{ij} + i\pi/2} (2n_k^b - 1) a_i^\dagger a_j + h.c. \right]$$
$$+ \lambda \sum_{\langle\langle kl \rangle\rangle} \left[e^{i\mathcal{A}_{lk} + i\pi/2} (2n_j^a - 1) b_k^\dagger b_l + h.c. \right]$$
$$+ \frac{\lambda^3}{J_z^2} \sum_{\text{hex}} \cos \left[\nabla \times \mathcal{A} \right]$$



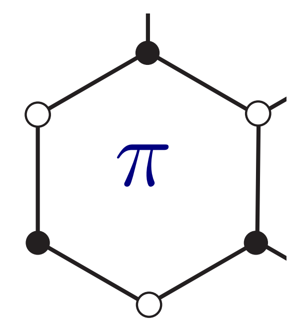
Gauge mean field theory

[Wilson, PRD, 1974; Savary & Balents, PRL, 2012]

$$H^{\mathrm{LGT}}_{\mathrm{chiral}} = \lambda \sum_{\langle\langle ij \rangle\rangle} \left[e^{i\mathcal{A}_{ij} + i\pi/2} (2n^b_k - 1) a^\dagger_i a_j + h.c. \right]$$
$$+ \lambda \sum_{\langle\langle kl \rangle\rangle} \left[e^{i\mathcal{A}_{lk} + i\pi/2} (2n^a_j - 1) b^\dagger_k b_l + h.c. \right]$$
$$+ \frac{\lambda^3}{J_z^2} \sum_{\mathrm{hex}} \cos\left[\nabla \times \mathcal{A}\right]$$

Energy minimisation for the gauge field

$\pi -$ flux through the hexagons



$$\mathcal{A}_{ij} = \mathcal{A}^0_{ij} + \mathcal{A}^s_{ij}$$

$$\mathcal{A}^0 \to \text{static background}$$

$$\nabla \times \mathcal{A}^0 = \pi$$

$$\mathcal{A}^s \to \text{ fluctuations}$$

G-MFT

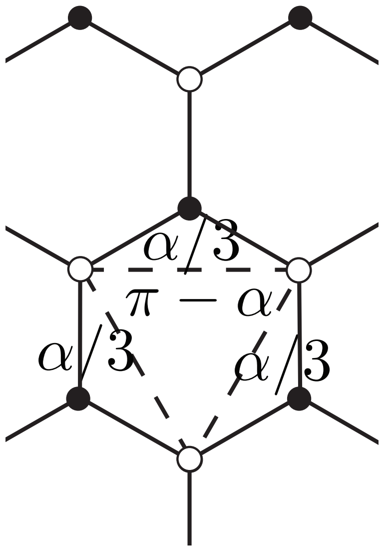
$$\mathcal{A}^s = 0$$

Dropping dynamics of the gauge field

G-MFT : U(1) SPT
(Integer quantum hall effect of bosons)

$$H^{\mathrm{GMFT}}_{\mathrm{chiral}} = \lambda \sum_{\langle\langle ij \rangle\rangle} \left[e^{i\mathcal{A}^0_{ij} + i\pi/2} (2n^b_k - 1) a^\dagger_i a_j + h.c. \right]$$
$$+ \lambda \sum_{\langle\langle kl \rangle\rangle} \left[e^{i\mathcal{A}^0_{lk} + i\pi/2} (2n^a_j - 1) b^\dagger_k b_l + h.c. \right]$$

- second neighbour hopping of bosons.
- hopping amplitude changes sign depending on the occupancy of intermediate site
- **STATIC BACKGROUND GAUGE FIELD**



$\alpha = \pi/2$

Restoring gauge fluctuations

$$\mathcal{A}_{ij} = \mathcal{A}^0_{ij} + \mathcal{A}^s_{ij}$$

G-MFT

$$\mathcal{A}^s = 0$$

- We now need to restore the gauge fluctuations
- This is done by adding a Maxwell term for $\mathcal{A}$

Restoring gauge fluctuations

$$\mathcal{L} = \mathcal{L}_a + \mathcal{L}_b + \mathcal{L}_{\rm int} + \mathcal{L}_{CS} + \boxed{\mathcal{L}_{\mathcal{A}}}$$

$$\begin{aligned}
\mathcal{L}_a =& i\tilde{a}^* \left[\partial_0 - i\left(\frac{1}{2}A_0^c + \mathcal{A}_0\right) + i\alpha_0\right]\tilde{a} \\
& - \frac{1}{2m}\left|\nabla\tilde{a} + i\left(\frac{1}{2}\vec{A}^c + \vec{\mathcal{A}}\right)\tilde{a} + i\vec{\alpha}\tilde{a}\right|^2, \\
\mathcal{L}_b =& i\tilde{b}^* \left[\partial_0 - i\left(\frac{1}{2}A_0^c - \mathcal{A}_0\right) + i\beta_0\right]\tilde{b} \\
& - \frac{1}{2m}\left|\nabla\tilde{b} + i\left(\frac{1}{2}\vec{A}^c - \vec{\mathcal{A}}\right)\tilde{b} + i\vec{\alpha}\tilde{b}\right|^2, \\
\mathcal{L}_{CS} =& \frac{1}{4\pi}\epsilon^{\mu\nu\lambda}\left[\alpha_\mu\partial_\nu\beta_\lambda + \beta_\mu\partial_\nu\alpha_\lambda\right],
\end{aligned}$$

$$\boxed{\mathcal{L}_{\mathcal{A}} = -\frac{1}{4e^2}(\partial\mu\mathcal{A}_\nu - \partial_\nu\mathcal{A}_\mu)^2.}$$

Low energy theory (after condensing the composite bosons)

$$\mathcal{L} = \frac{1}{4\pi}\epsilon^{\mu\nu\lambda}\left[\frac{1}{2}A^{\mathrm{c}}_{\mu}\partial_{\nu}A^{\mathrm{c}}_{\lambda} - 2\mathcal{A}_{\mu}\partial_{\nu}\mathcal{A}_{\lambda}\right] - \frac{1}{4e^2}(\partial\mu\mathcal{A}_{\nu} - \partial_{\nu}\mathcal{A}_{\mu})^2.$$

- Bulk : photon is massive due to chern-simons term

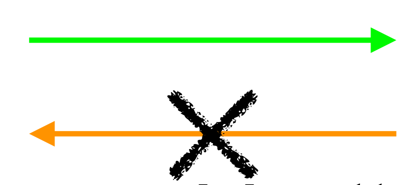
$$m_{photon} \sim e^2$$

Low energy theory
(after condensing the composite bosons)

$$\mathcal{L} = \frac{1}{4\pi}\epsilon^{\mu\nu\lambda}\left[\frac{1}{2}A^{c}_{\mu}\partial_{\nu}A^{c}_{\lambda} - 2\mathcal{A}_{\mu}\partial_{\nu}\mathcal{A}_{\lambda}\right] - \frac{1}{4e^2}(\partial\mu\mathcal{A}_{\nu} - \partial_{\nu}\mathcal{A}_{\mu})^2.$$

- Edge : s-mode at the edge gapped out

c mode (S_z)



removed by “gauging”

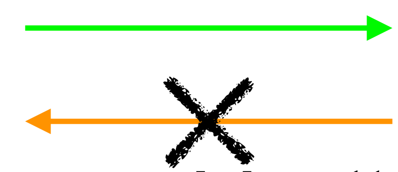
s - mode couples to gapped gauge field $\mathcal{A}$

$$\text{Integrate out } \mathcal{A} \rightarrow \text{ generate mass for s} - \text{mode}$$

Low energy theory (after condensing the composite bosons)

$$\mathcal{L} = \frac{1}{4\pi}\epsilon^{\mu\nu\lambda}\left[\frac{1}{2}A^{c}_{\mu}\partial_{\nu}A^{c}_{\lambda} - 2\mathcal{A}_{\mu}\partial_{\nu}\mathcal{A}_{\lambda}\right] - \frac{1}{4e^2}(\partial\mu\mathcal{A}_{\nu} - \partial_{\nu}\mathcal{A}_{\mu})^2.$$

c mode (S_z)

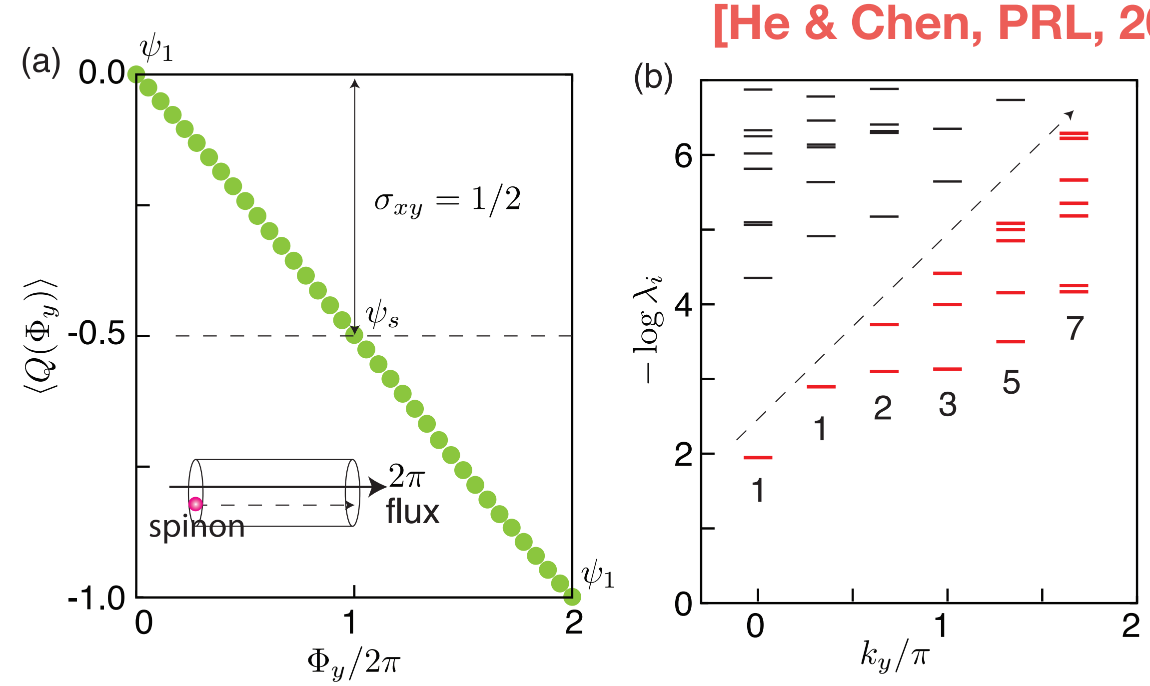


removed by “gauging”

- Chiral c-mode (S_z)

$$\sigma^{c}_{xy} = 1/2$$

[He, SB, Pollmann, Moessner, PRL, 2015]
[He & Chen, PRL, 2015]



Low energy CS Theory

$$\mathcal{L} = \frac{K_{IJ}}{4\pi} g_I d g_J$$

$$g = [\alpha, \beta, \mathcal{A}]$$

$$K = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & -1 \\ 1 & -1 & 0 \end{bmatrix}$$

- Chiral
- Degeneracy on Torus = 2

References

- Phys. Rev. Lett. 115, 116803 (2015)
- Phys. Rev. Lett. 115, 267209 (2015)
- Phys. Rev. B 93, 195143 (2016)

Collaborators

- Yin Chen He, Perimeter Inst.
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- Roderich Moessner, MPIPKS Dresden

Summary

- Three examples of topological phases
  - 3d Z2 free fermion topological insulator in an amorphous/glassy solid
  - Bosonic Integer Quantum Hall phase
  - Chiral spin liquid
- Chiral spin liquid as gauged BIQH phase

Thank You