Spectroscopy of spinons in Coulomb quantum spin liquids

Quantum spin ice

Josephson junction arrays

Interacting dipoles

Chris R. Laumann (Boston University)

Work with:
Siddhardh Morampudi
Frank Wilczek

Primary Reference:
Morampudi, Wilczek, CRL arXiv:1906.01628

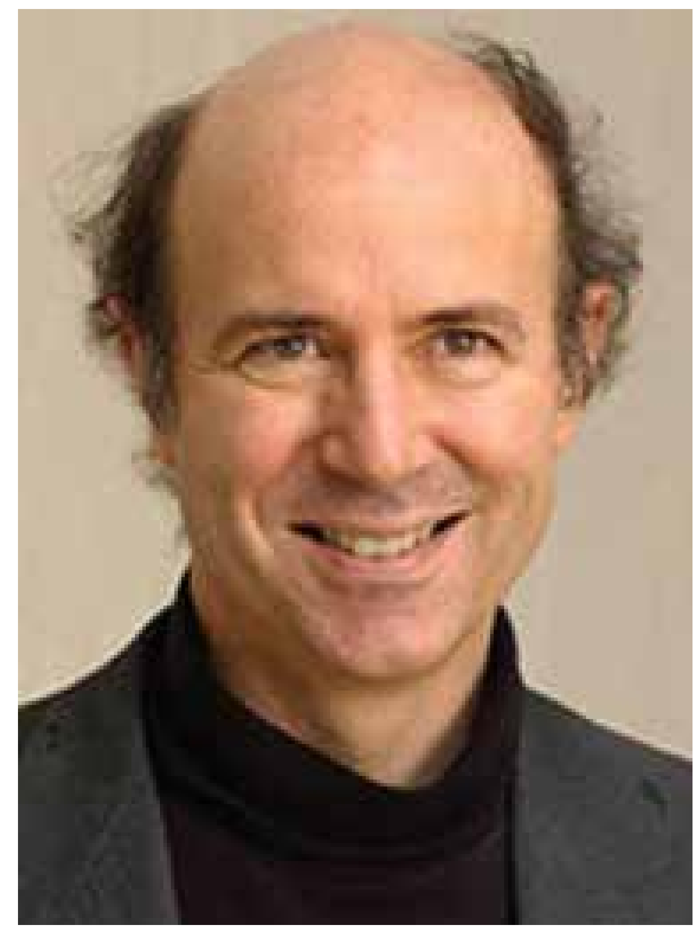
Les Houches School: Topology Something Something

September 5, 2019

Collaborators

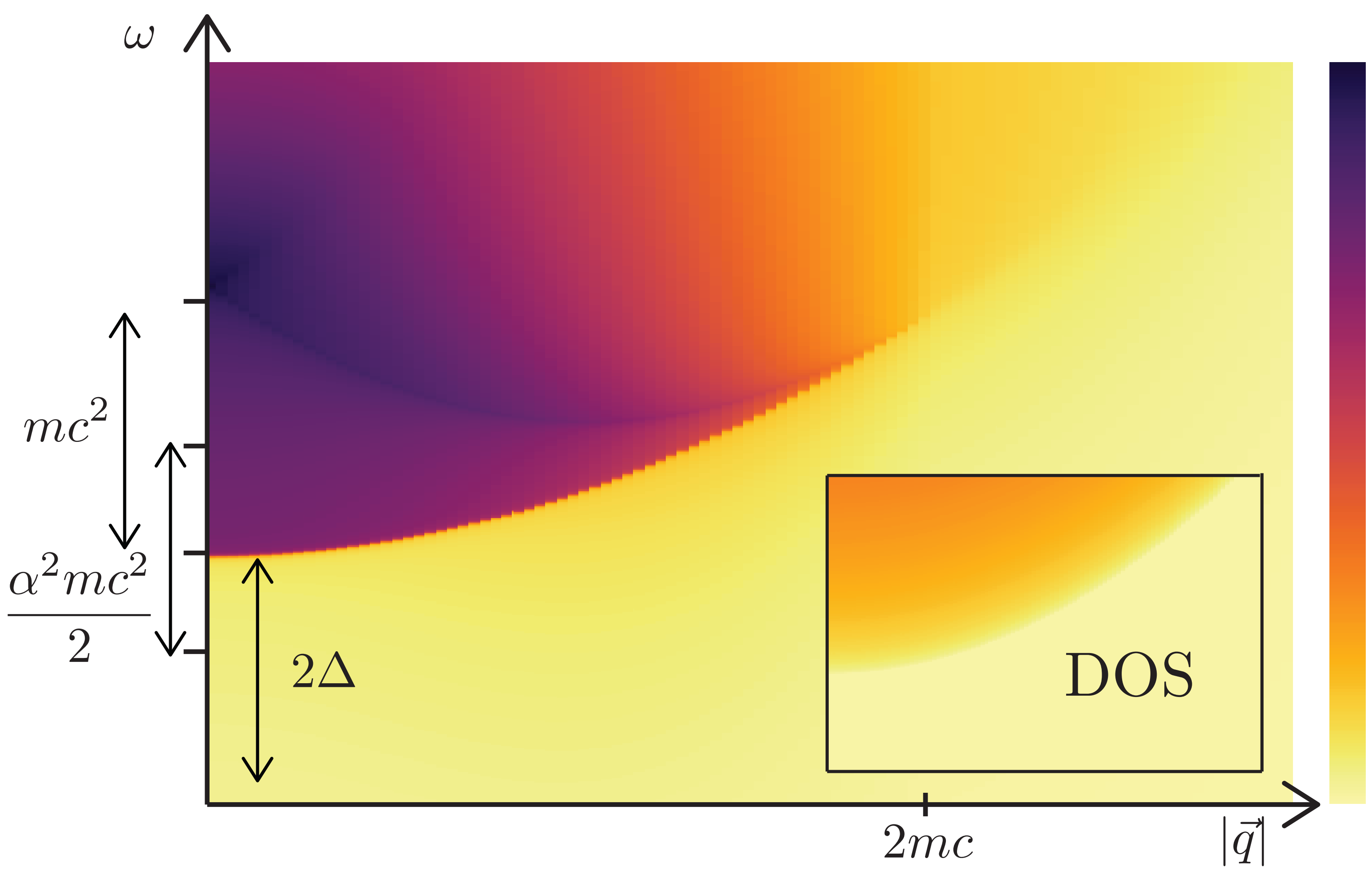
Siddhardh Morampudi

Frank Wilczek



Summary

Emergent photon in the Coulomb spin liquid leads to characteristic signatures in neutron scattering

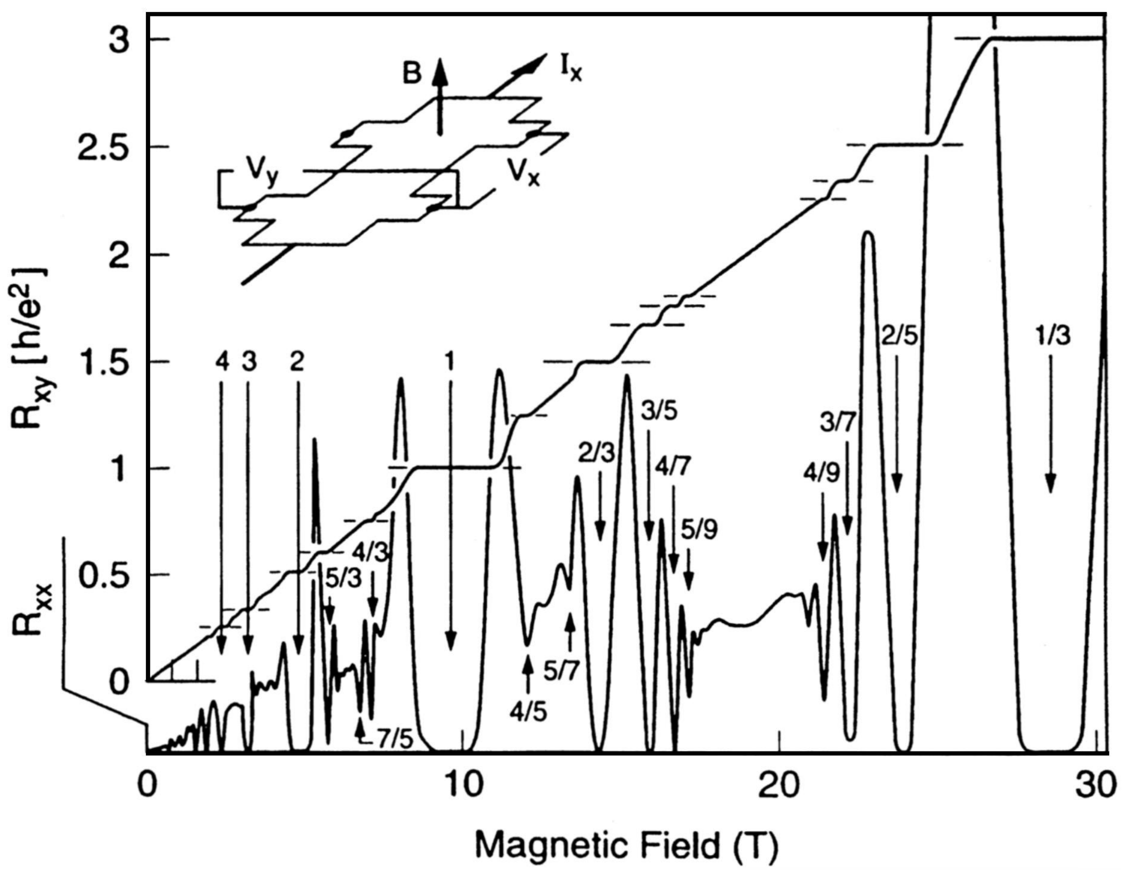


Outline

1. Introduction
    A. Emergent QED in quantum spin ice
    B. Spectroscopy
2. Results
    A. Universal enhancement
    B. Cerenkov radiation
    C. Comparison to numerics and experiments
3. Summary

New phases beyond broken symmetry paradigm

Fractional Quantum Hall Effect



D.C. Tsui; H.L. Stormer; A.C. Gossard - Phys. Rev. Lett. (1982)

Quantum Spin Liquids

Balents - Nature (2010)

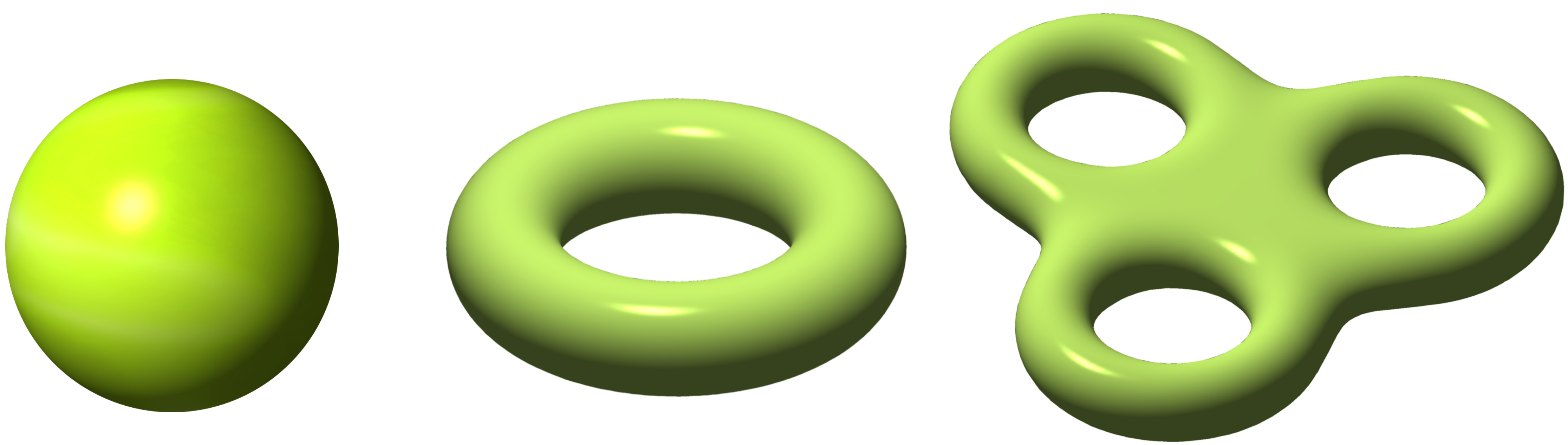
Savary et al.- Rep. Prog. Phys (2017)

Knolle et al. - Ann. Rev. Cond. Mat. (2019)

Theoretically describing quantum spin liquids

- Lack of local order parameters
- Topological ground state degeneracy
- Fractionalized excitations
- Emergent gauge fields

Interplay in this talk



How do we get a quantum spin liquid?

(Emergent gauge theory)

Local constraints + quantum fluctuations

+ Luck

Rare earth pyrochlores

$\mathrm{R_2M_2O_7}$

$\mathrm{Ho_2Ti_2O_7}$

$\mathrm{Dy_2Ti_2O_7}$

Classical spin ice

R^{3+} — 4f rare-earth

M^{4+} — Non-magnetic

$\mathrm{Yb_2Ti_2O_7}$

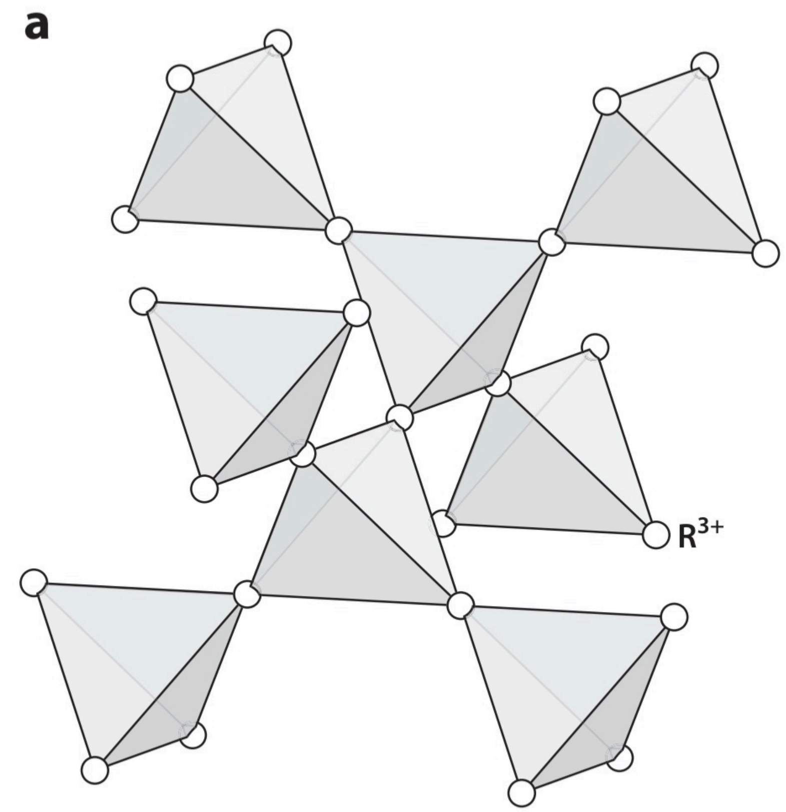
$\mathrm{Tb_2Ti_2O_7}$

$\mathrm{Pr_2Sn_2O_7}$

$\mathrm{Pr_2Zr_2O_7}$

$\mathrm{Pr_2Hf_2O_7}$

Quantum spin ice



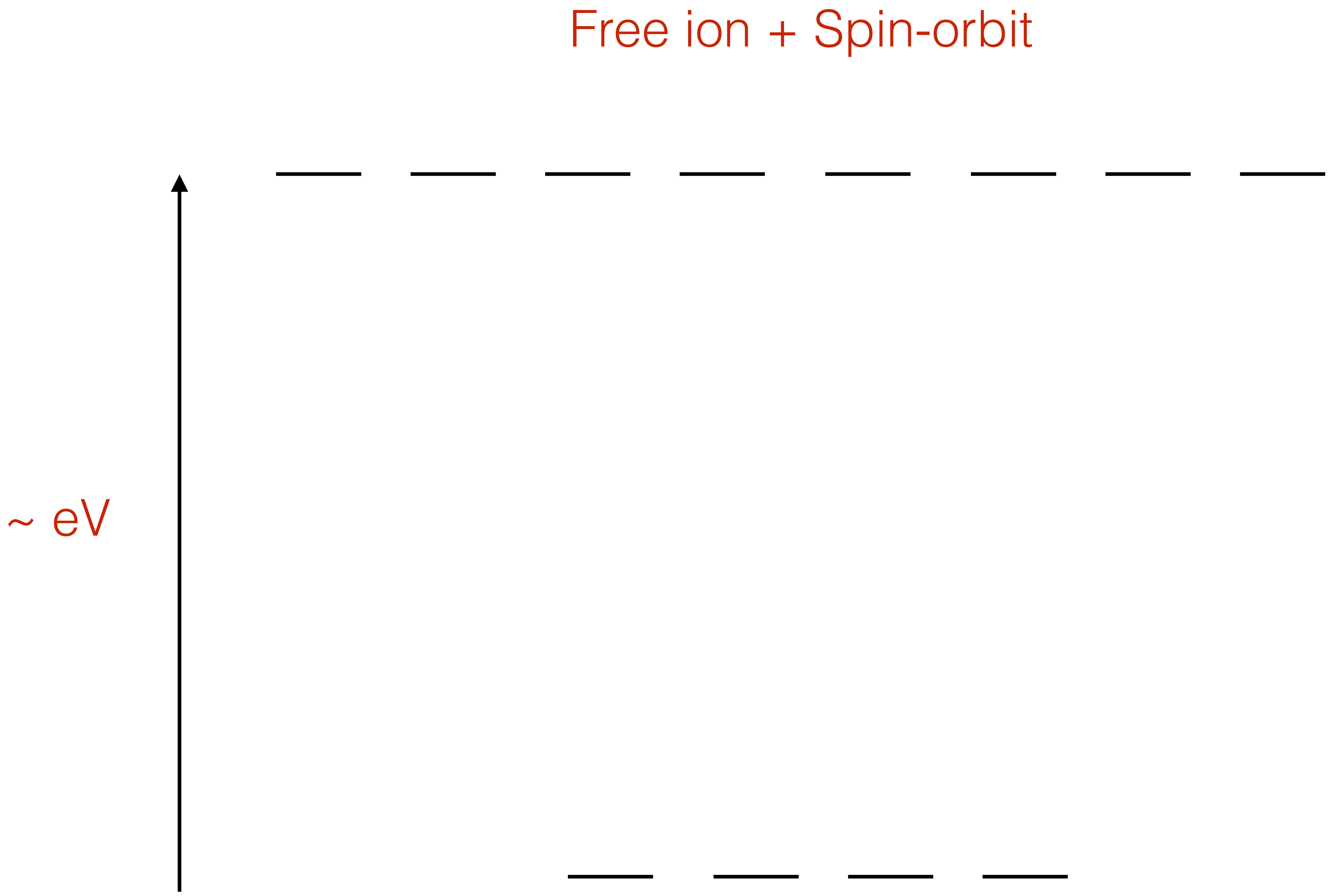
Gingras and McClarty - Rep. Prog. Phys. (2014)

Rau and Gingras (2019)

Pseudo-spins in rare-earth pyrochlores

Free ion

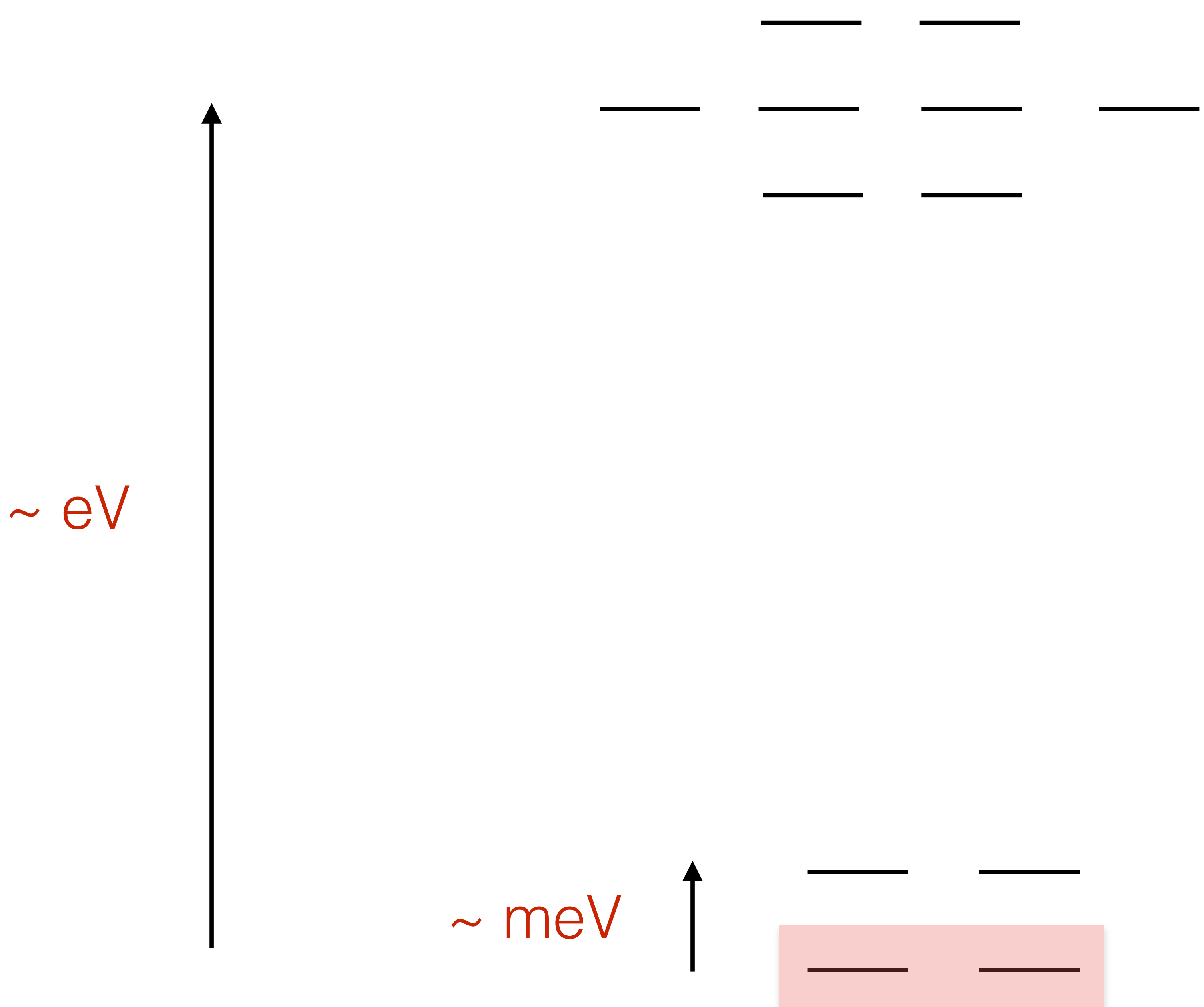
Pseudo-spins in rare-earth pyrochlores

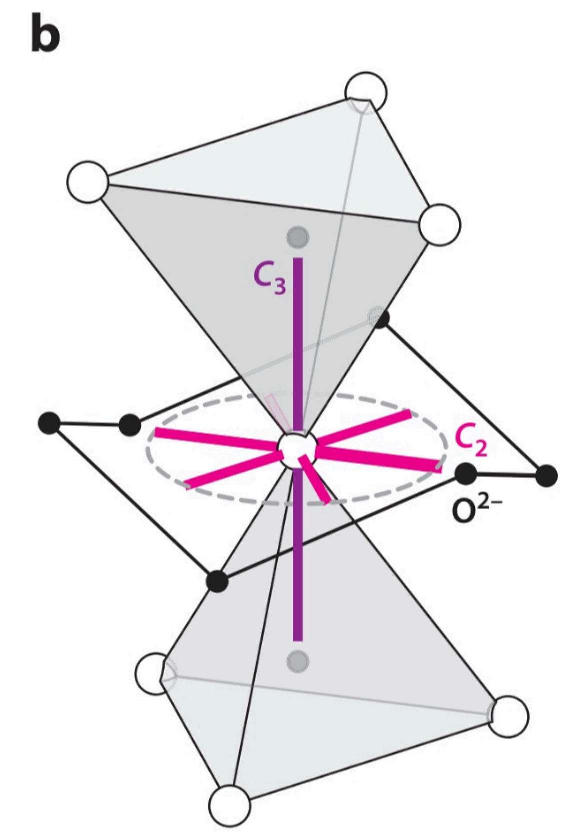


Pseudo-spins in rare-earth pyrochlores

Free ion + Spin-orbit + Crystal field

Single-ion anisotropy





Rau and Gingras (2019)

Allowed NN microscopic Hamiltonian

$$
\begin{aligned}
H = & \sum_{\langle ij \rangle} [J_{zz} \mathsf{S}_i^z \mathsf{S}_j^z - J_{\pm}(\mathsf{S}_i^+ \mathsf{S}_j^- + \mathsf{S}_i^- \mathsf{S}_j^+) \\
& + J_{\pm\pm}(\gamma_{ij} \mathsf{S}_i^+ \mathsf{S}_j^+ + \gamma_{ij}^* \mathsf{S}_i^- \mathsf{S}_j^-) \\
& + J_{z\pm}(\mathsf{S}_i^z(\zeta_{ij} \mathsf{S}_j^+ + \zeta_{ij}^* \mathsf{S}_j^-) + (\zeta_{ij} \mathsf{S}_i^+ + \zeta_{ij}^* \mathsf{S}_i^-) \mathsf{S}_j^z)]
\end{aligned}
$$

Doublet = spin-1/2 like Kramers pair

Ising + Heisenberg + Dipolar + Dzyaloshinskii-Moriya

Ross et al - Phys. Rev. Lett. 2011

Pyrochlore XXZ as a simple model

$$H = \sum_{\langle ij \rangle} [J_{zz} \mathsf{S}_i^z \mathsf{S}_j^z - J_{\pm}(\mathsf{S}_i^+ \mathsf{S}_j^- + \mathsf{S}_i^- \mathsf{S}_j^+)]$$

Local constraints Quantum fluctuations

Ring exchange model at low energies

$$J_{\pm} \ll J_{zz}$$

$$H_{\text{ring}} \sim \frac{J_{\pm}^3}{J_{zz}^2} \sum_{h \in \{\hexagon\}} S_{h,1}^{+} S_{h,2}^{-} S_{h,3}^{+} S_{h,4}^{-} S_{h,5}^{+} S_{h,6}^{-}$$

Huse et al - Phys. Rev. Lett 2003

Hermele et al - Phys. Rev. B 2004

Emergent gauge symmetry!

$$J_\pm \ll J_{zz}$$

$$H_{\text{ring}} \sim \frac{J_\pm^3}{J_{zz}^2} \sum_{h \in \{\hexagon\}} S_{h,1}^+ S_{h,2}^- S_{h,3}^+ S_{h,4}^- S_{h,5}^+ S_{h,6}^-$$

Small bandwidth

||

Slow photon

$$\prod_{i \in t} \exp(i\theta S_i^z)$$

Emergent photon?

Verified in numerics!

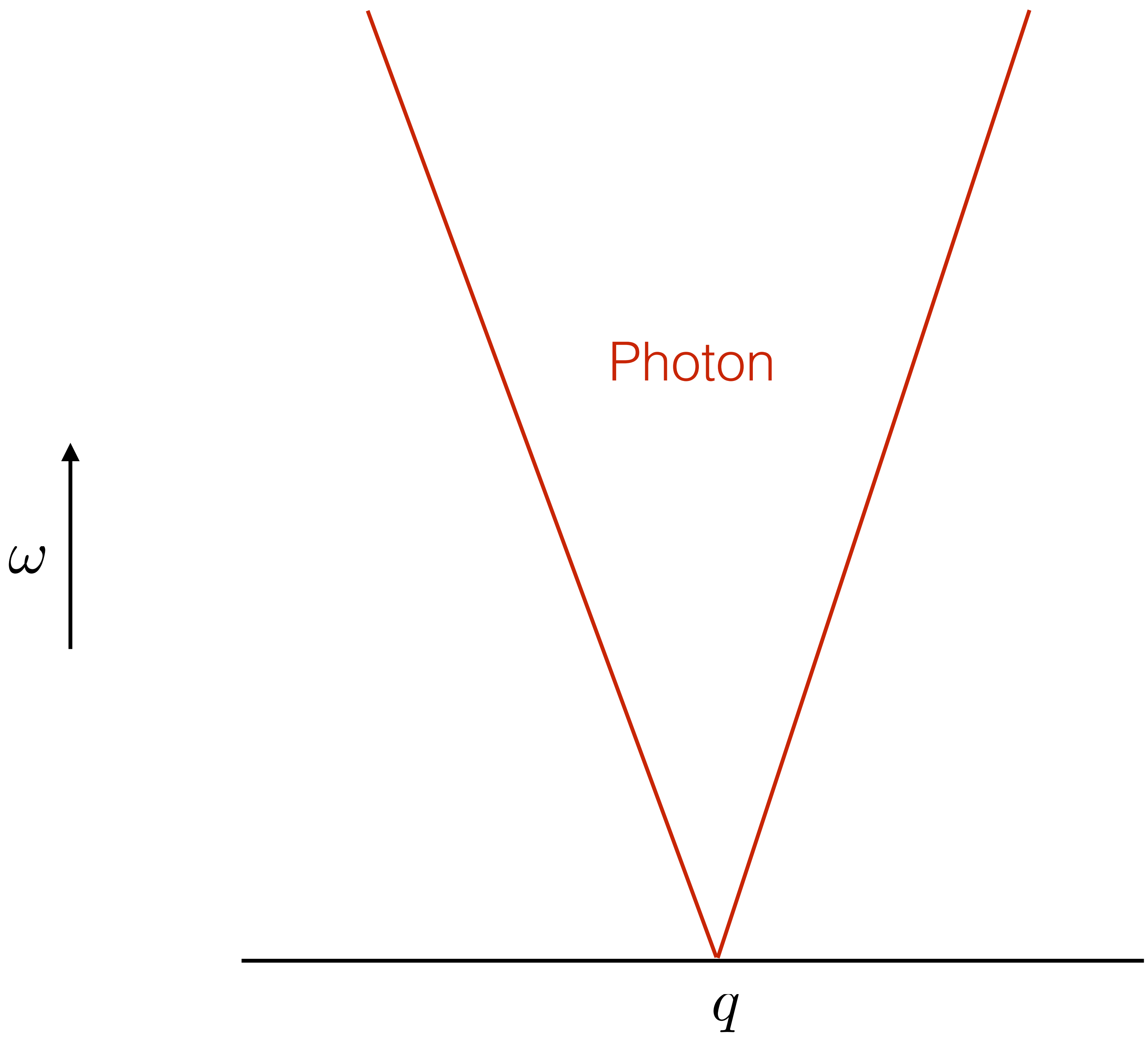
Banerjee et al - Phys. Rev. Lett. 2008

Shannon et al - Phys. Rev. Lett 2012

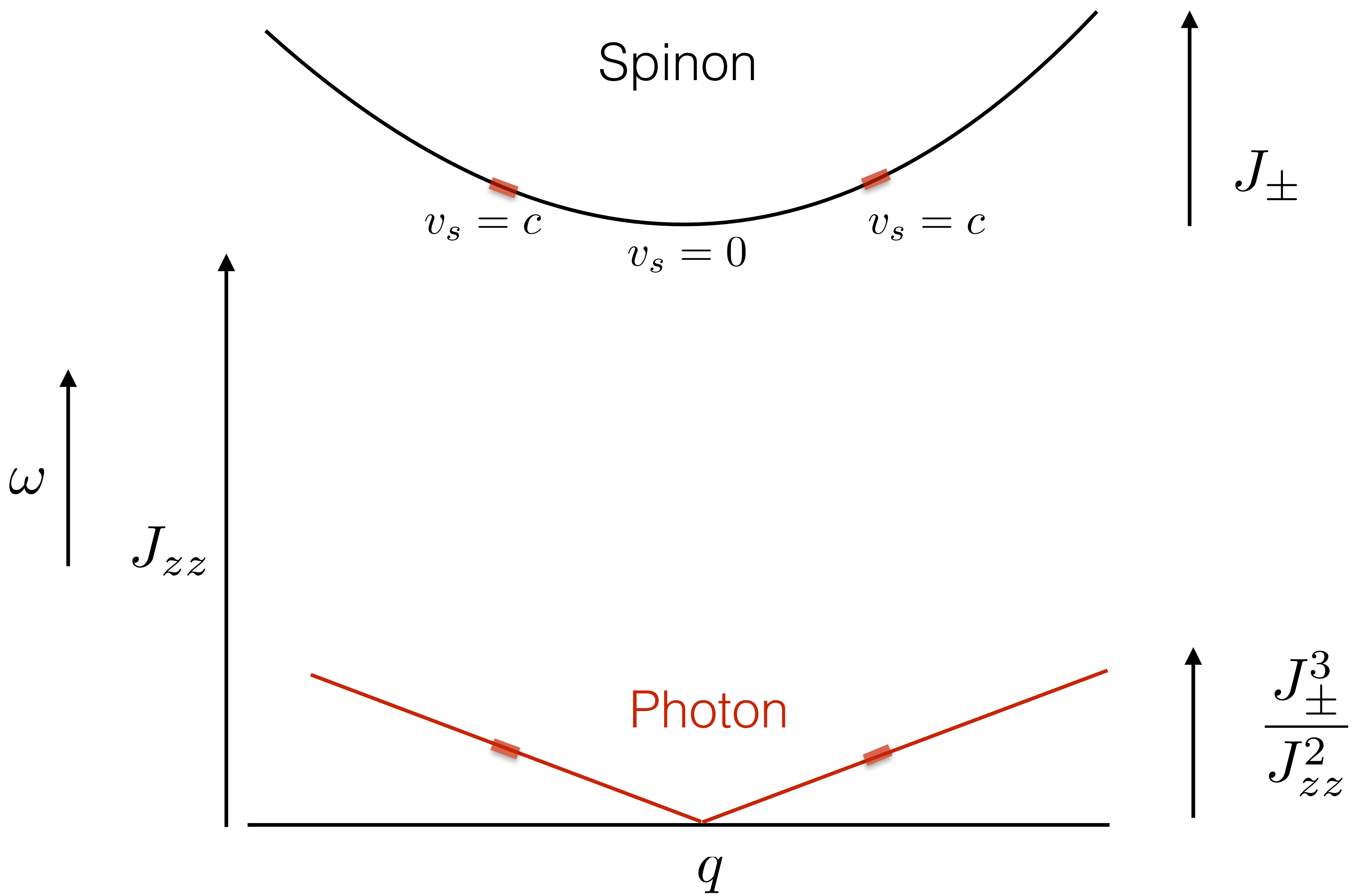
Huse et al - Phys. Rev. Lett 2003

Hermele et al - Phys. Rev. B 2004

Usual QED

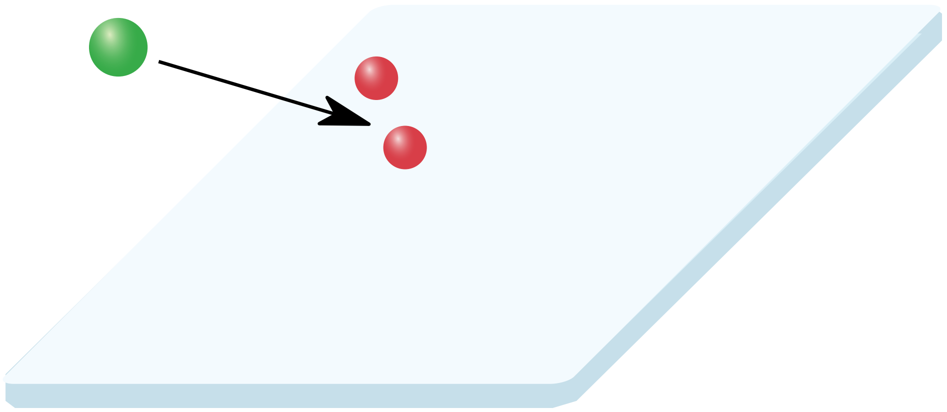


Quantum spin ice



What should we see in neutron scattering?

Neutron produces pair of spinons



Energy ω

Momentum q

Fractionalization seen in spectroscopy

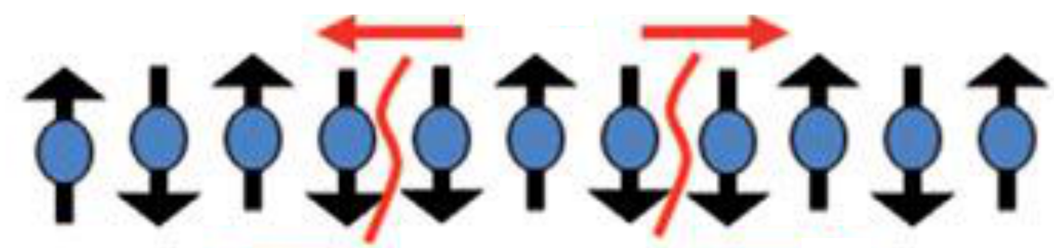
$$\vec{q},\omega(\vec{q}) \;\begin{array}{l} \nearrow \;\; \vec{k},\omega(\vec{k}) \\ \\ \searrow \;\; \vec{q}-\vec{k},\omega(\vec{q}-\vec{k}) \end{array}$$

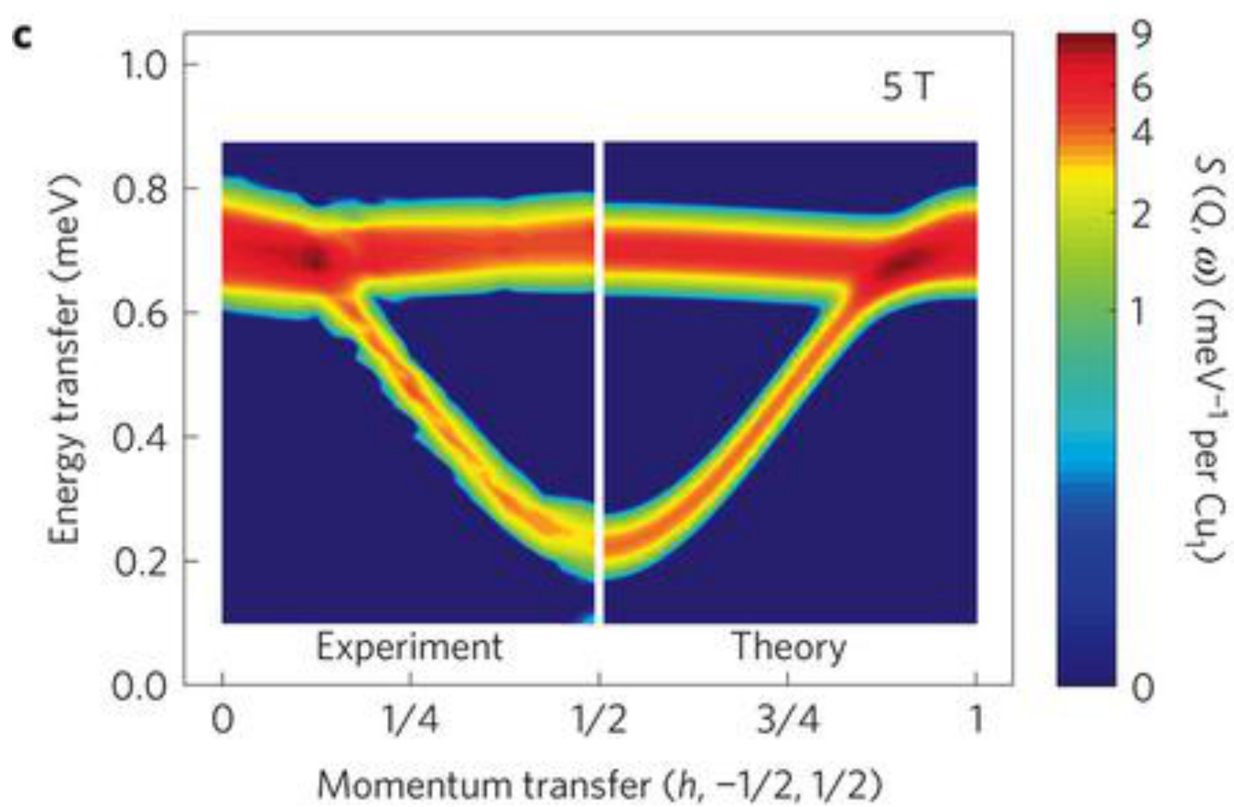
Multiple excitations $\Longrightarrow$ Continuum!

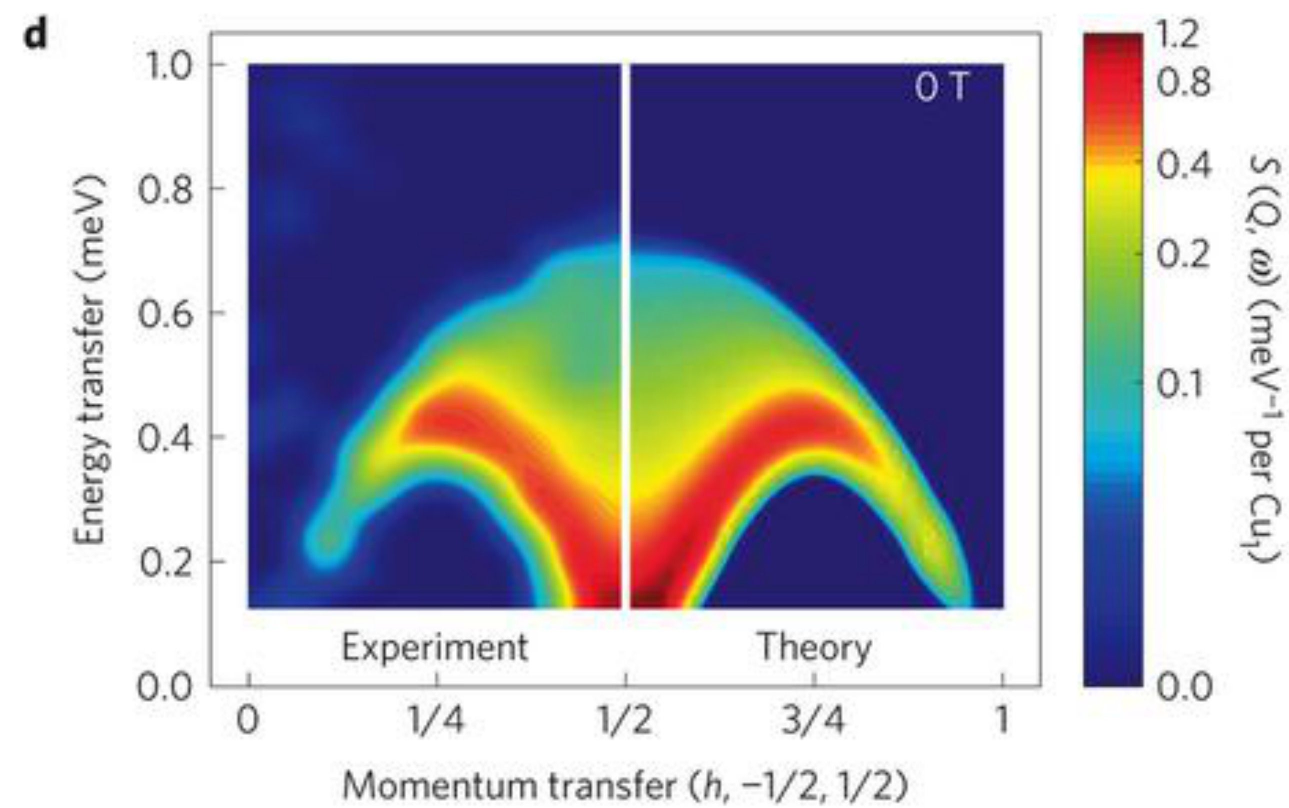
Fractionalization seen in spectroscopy

Spin waves

Fractionalized spinons

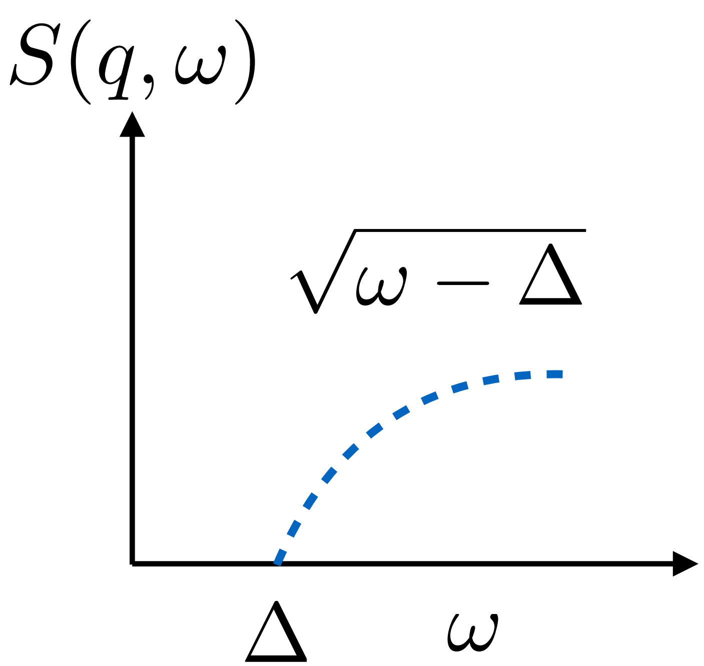


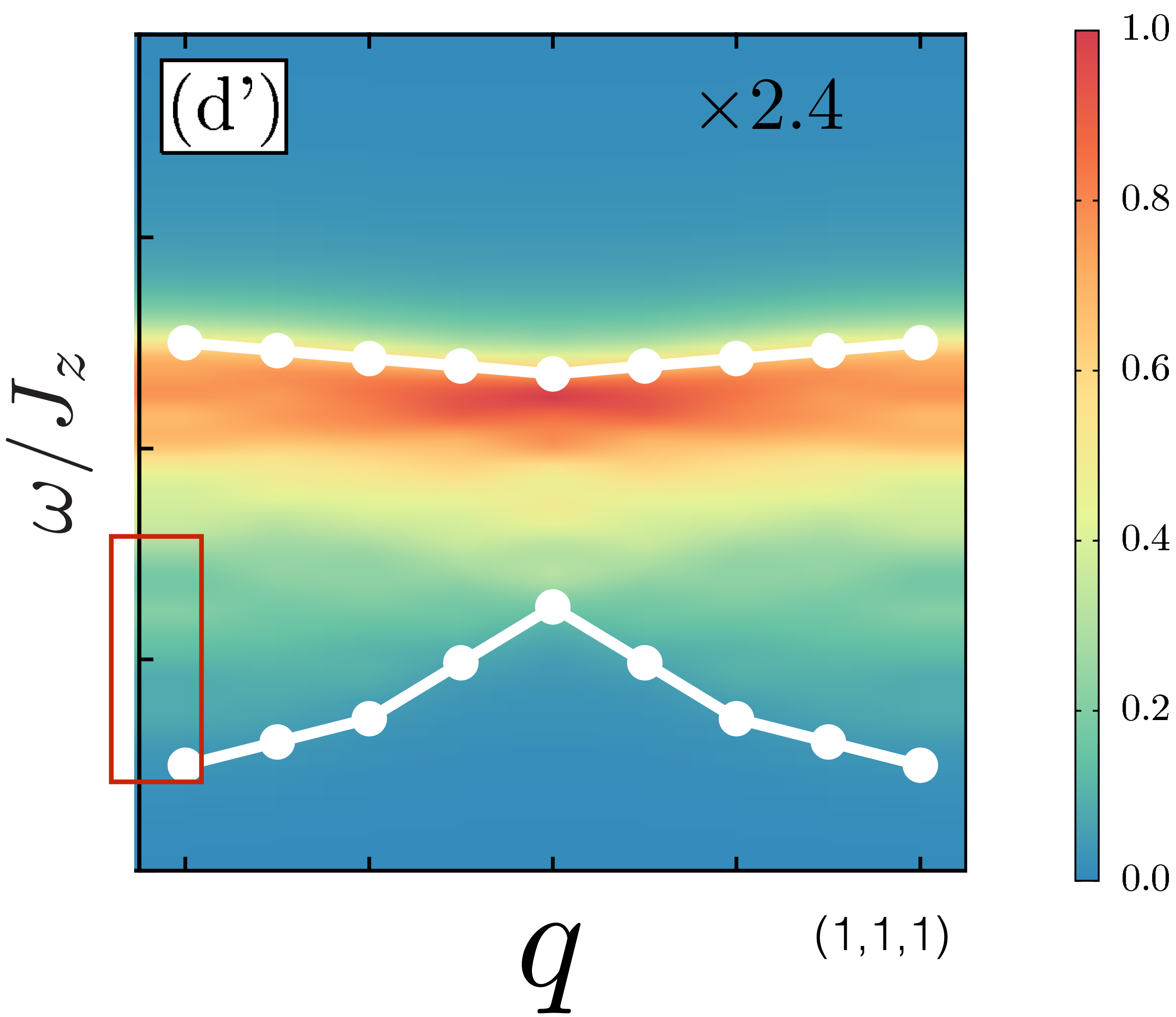




M. Mourigal et al. - Nature Physics (2013)

Density of states at mean field level

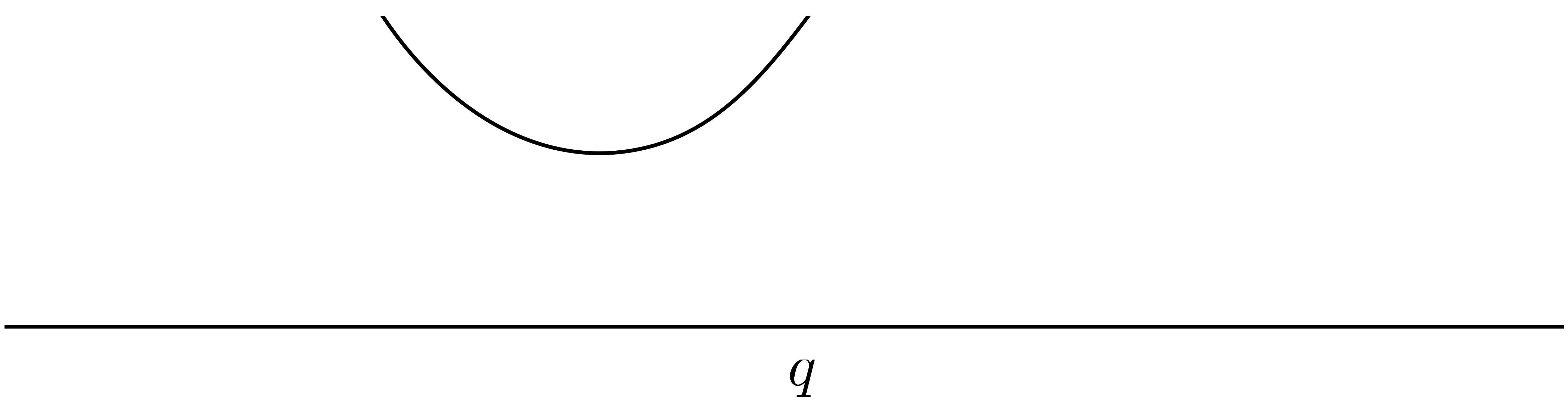




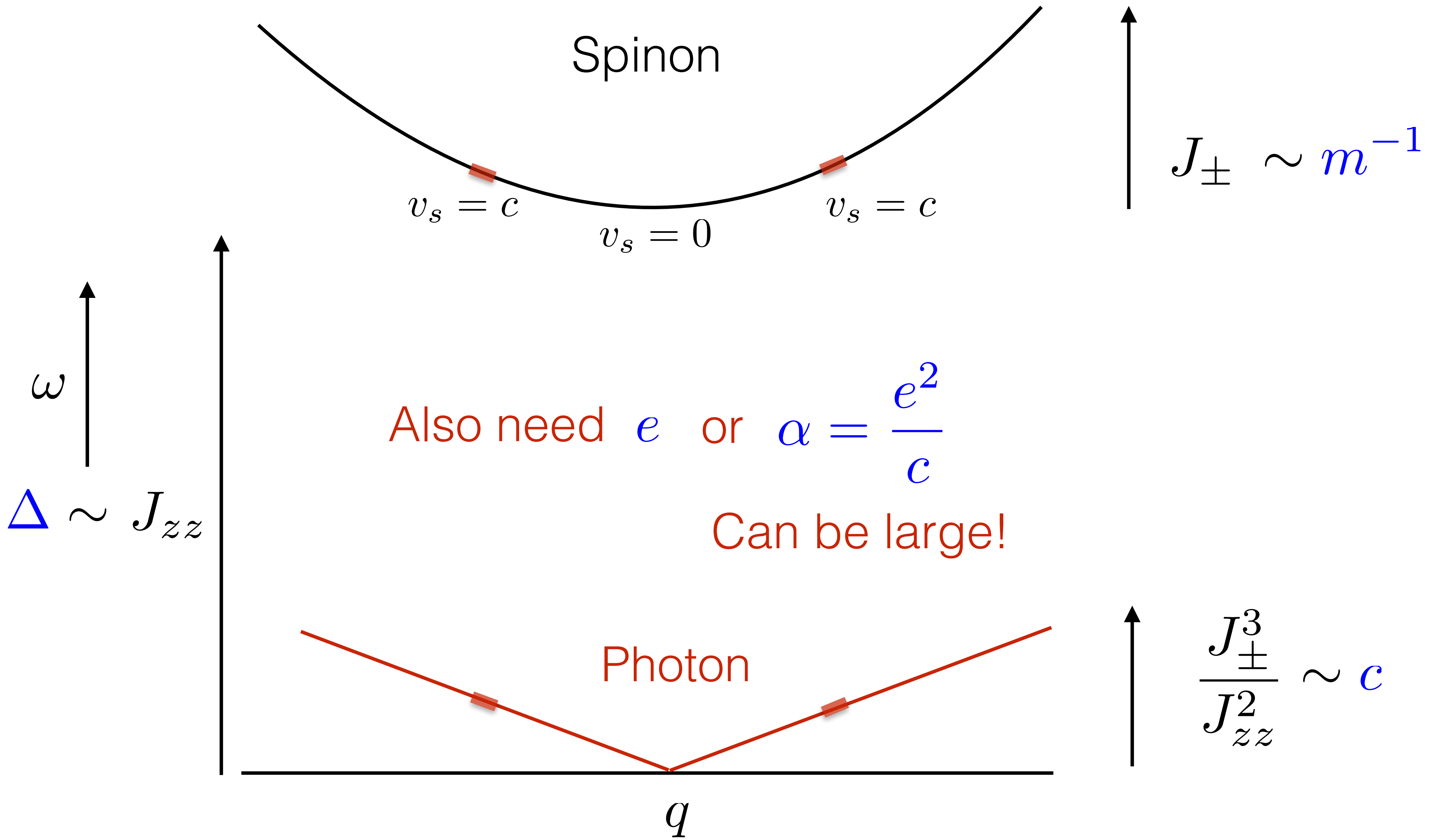
Huang et al - Phys. Rev. Lett. (2018)

But there is a gapless and slow photon!

Spinons in an effective mass approximation



Quantum spin ice



Spinons in an effective mass approximation

Instantaneous Coulomb

Transverse photon

$$\mathcal{L} = \psi_\sigma^\dagger (i\partial_t \qquad\qquad)\psi_\sigma - \frac{|(-i\nabla \qquad\qquad)\psi_\sigma|^2}{2m} - \Delta|\psi_\sigma|^2$$

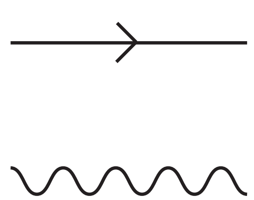
Maxwell term

Short-range interactions

$$\boldsymbol{\nabla} \cdot \vec{A} = 0$$

Diagrammatic evaluation of structure factor

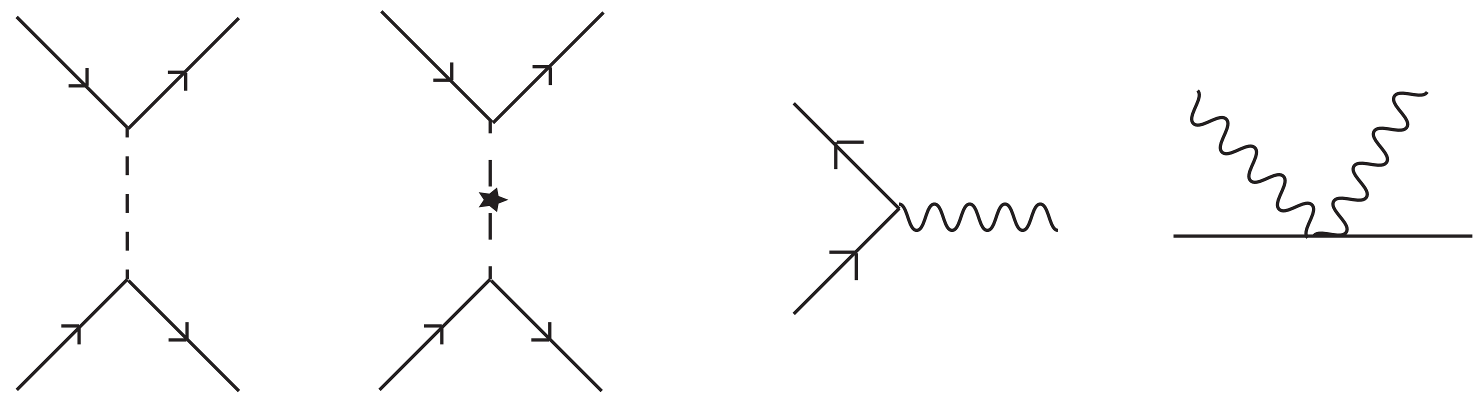
Spinon



G_0

$D_{\mu\nu}$

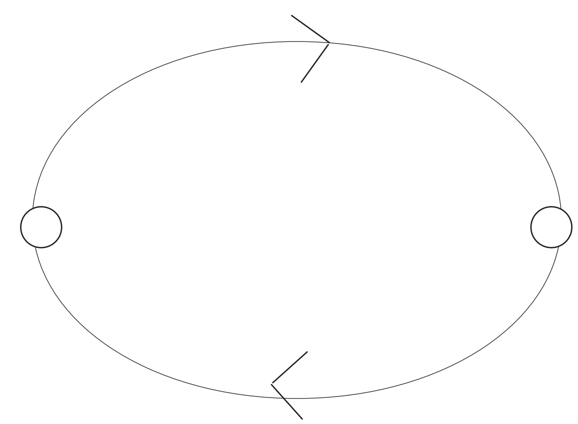
Transverse photon



$$\frac{e^2}{k^2} \qquad V_{\boldsymbol{k}} \qquad \frac{e}{m}\left(q_\mu + \frac{k_\mu}{2}\right) \qquad \frac{e^2}{2m}$$

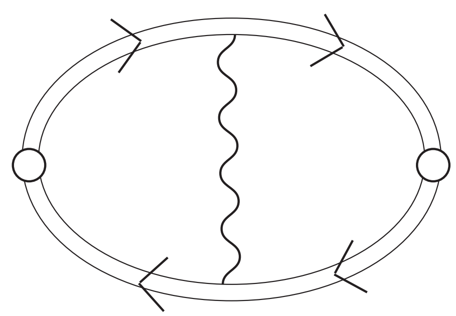
Coulomb Short-range Spinon-photon

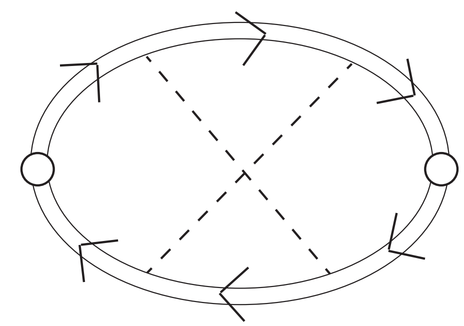
Density of states if we neglect interactions

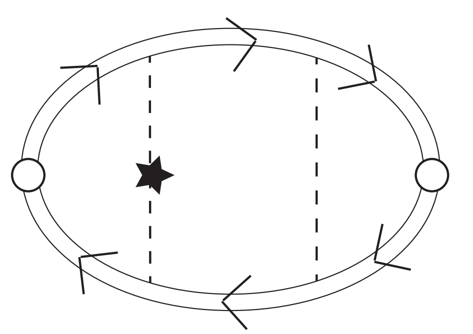


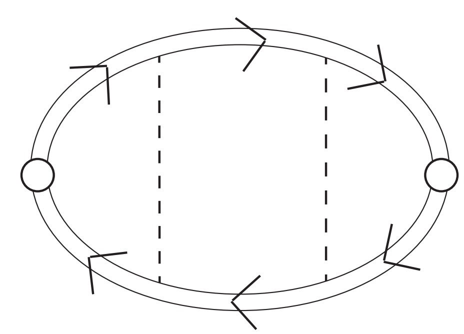
$$S^0(\vec{q}, \omega) \sim m^{3/2} \sqrt{\omega - 2\Delta - q^2/4m}$$

Plethora of additional diagrams from interactions

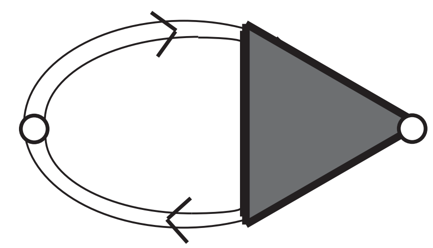


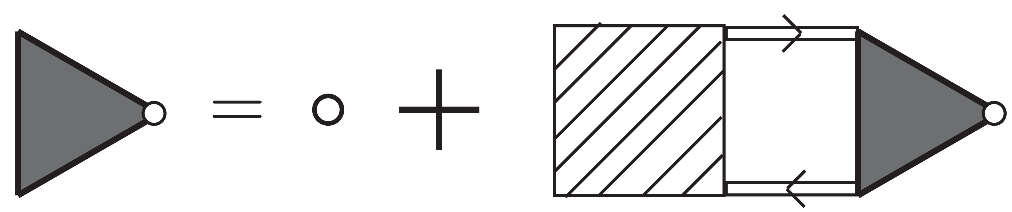


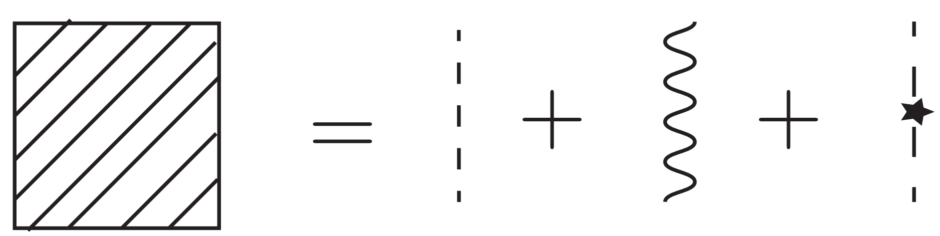




Resum dominant ladder diagrams





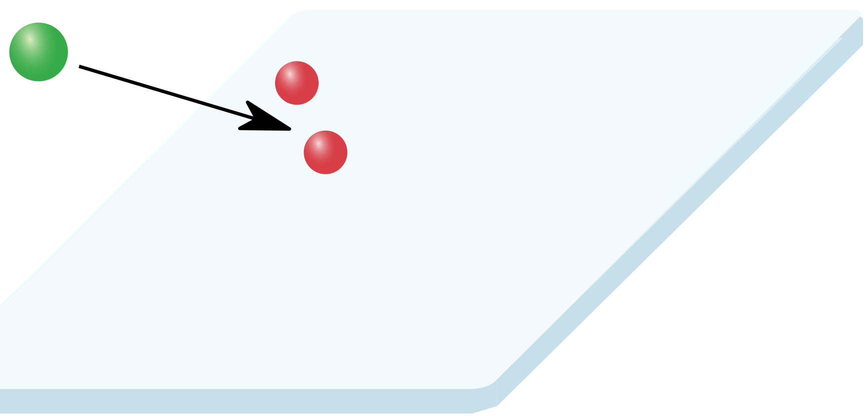


Effective Hamiltonian problem in absence of radiation

$$(i\omega - H_s - 2\Delta)W(\vec{r};\vec{q},i\omega) = \delta^3(r)$$

Response ∝ relative probability at short distance

$$S(\vec{q}, \omega) \propto |\psi(r=0)|^2$$



Threshold scattering cross-sections are "universal"

PHYSICAL REVIEW VOLUME 73, NUMBER 9 MAY 1, 1948

On the Behavior of Cross Sections Near Thresholds

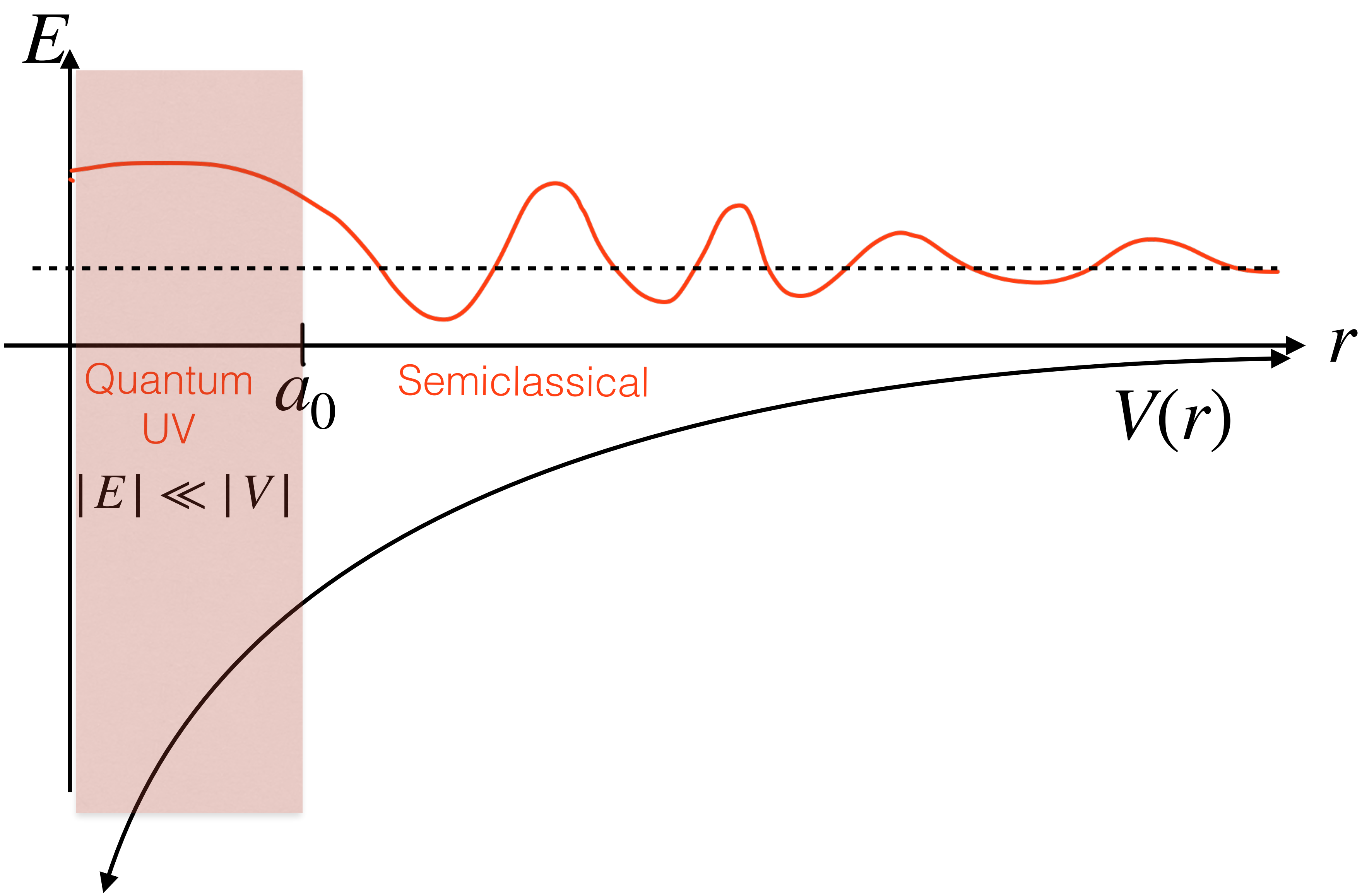
EUGENE P. WIGNER
Palmer Physical Laboratory, Princeton University, Princeton, New Jersey

(Received January 16, 1948)

The energy dependence of the cross section for the formation of a product, near the threshold energy for that formation, is considered. It is shown that the cross section is, apart from a constant, in the neighborhood of the threshold the same function of energy, no matter what the reaction mechanism is, as long as the long-range interaction of the product particles is the same. The same must hold, because of the principle of detailed balance, for the back reaction, i.e., the reaction between particles with very low relative velocities. In this case, the cross section, as function of the energy, depends only on the long-range interaction of the reacting particles. The energy dependence of the cross section is determined for three types of interactions, *viz.* no interaction, Coulomb repulsion and Coulomb attraction. The rule for a $1/r^2$ interaction can be obtained from the first case. Reasons are adduced to show that two interactions, the difference of which goes to zero at least as fast as $r^{-2-\epsilon}$ with $(\epsilon>0)$, give the same energy dependence of the cross section. Hence, long-range interaction in the above connection should mean an interaction which, at large distances of the particles, does not go to zero faster than r^{-2}. The effect of small perturbations in the long-range interaction is discussed in general.

Universality of Threshold Laws

$$H = \frac{p^2}{m} - \frac{e^2}{r} + V_{SR}(r) \qquad \text{FGR} \sim 2\pi \sum_f |\psi_f(0)|^2 \delta(\epsilon - \epsilon_f)$$



Universal enhancement from gapless photon

$$S(\vec{q},\omega) \sim \frac{m^{3/2}\sqrt{2\pi R}}{1-\exp\left(-\sqrt{\frac{2\pi R}{\omega-2\Delta-\frac{q^2}{4m}}}\right)}\theta(\omega-2\Delta-\frac{q^2}{4m})$$

Non-perturbative

Universal enhancement from gapless photon

$$S(\vec{q},\omega) \sim \frac{m^{3/2}\sqrt{2\pi R}}{1-\exp\left(-\sqrt{\frac{2\pi R}{\omega-2\Delta-\frac{q^2}{4m}}}\right)}\theta(\omega-2\Delta-\frac{q^2}{4m})$$

Universal enhancement from gapless photon

$$S(\vec{q},\omega) \sim \frac{m^{3/2}\sqrt{2\pi R}}{1-\exp\left(-\sqrt{\frac{2\pi R}{\omega-2\Delta-\frac{q^2}{4m}}}\right)}\theta(\omega-2\Delta-\frac{q^2}{4m})$$

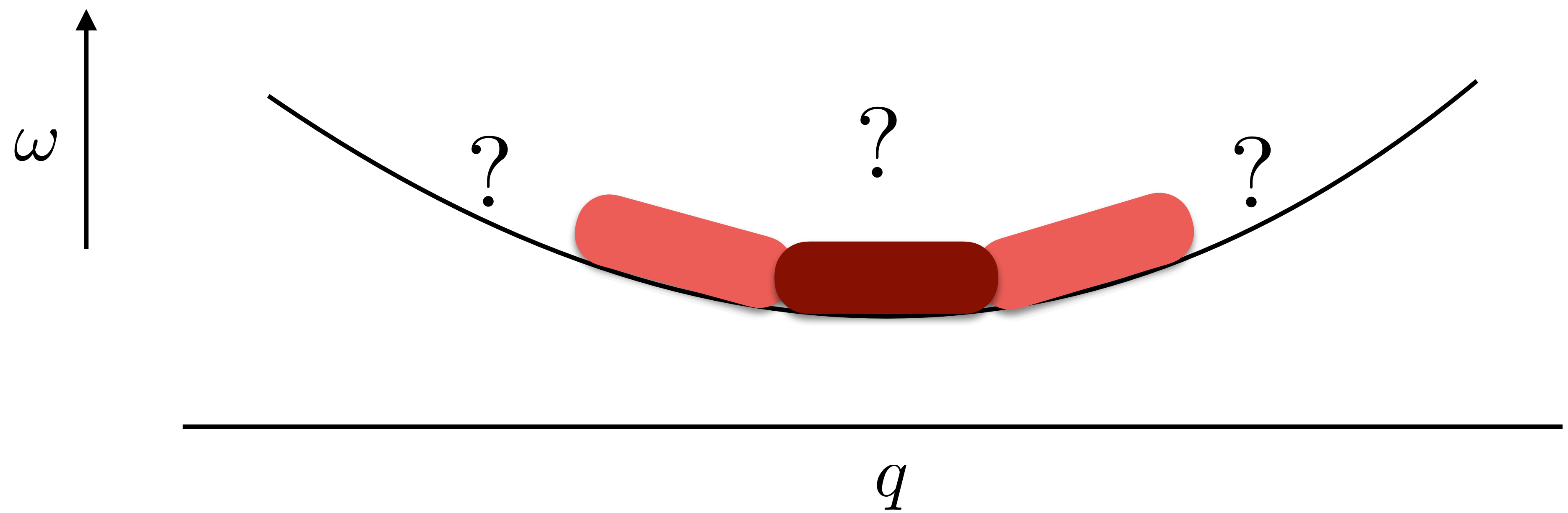
$$\downarrow \text{ Near threshold}$$

$$S(\vec{q},\omega) \sim m^2 c\alpha\left(1-\frac{1}{4}\left(\frac{q}{mc}\right)^2\right)\theta(\omega-2\Delta-\frac{q^2}{4m})$$

Jump discontinuity

Suppression at finite wavevector

Universal enhancement from gapless photon



$$S(\vec{q},\omega) \sim m^2 c\alpha \left(1 - \frac{1}{4}\left(\frac{q}{mc}\right)^2\right)\theta(\omega - 2\Delta - \frac{q^2}{4m})$$

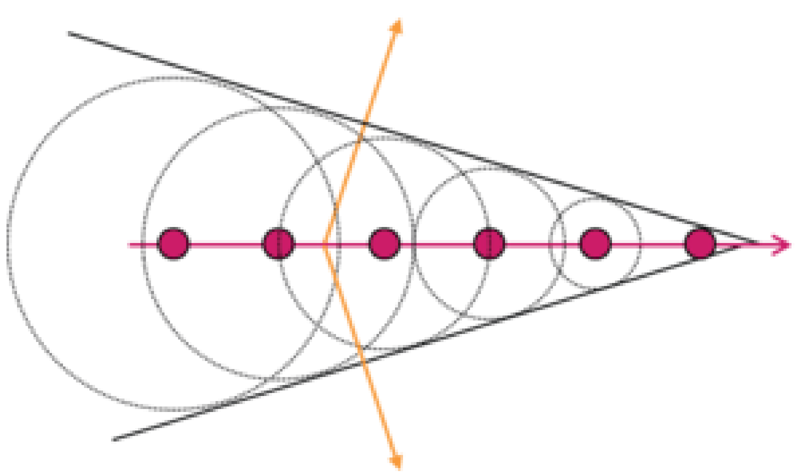
Jump discontinuity

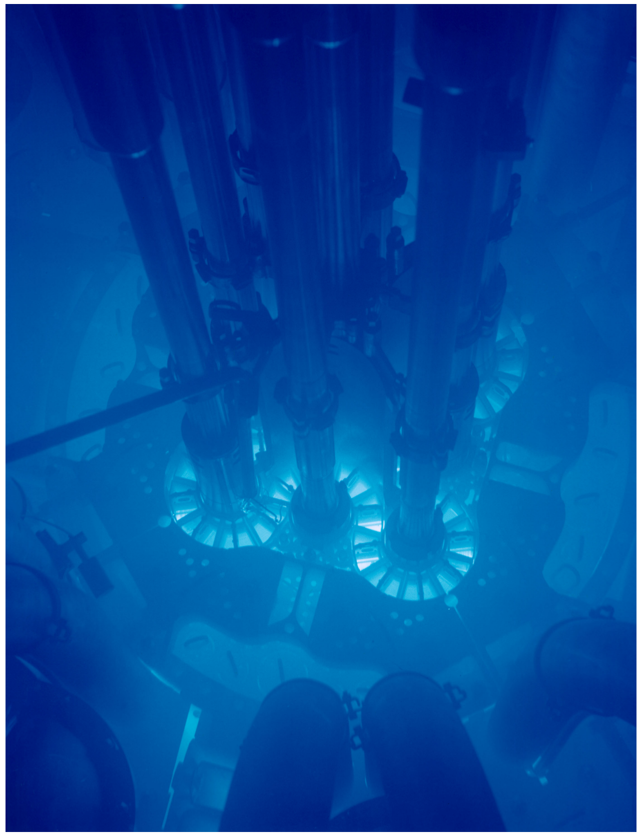
Suppression at finite wavevector

Spinons emit Cerenkov radiation due to slow photon

$$k > mc$$

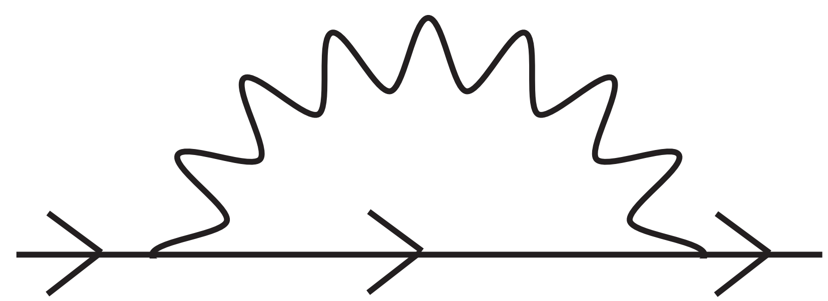
$$\cos(\theta) = \frac{mc}{k}$$





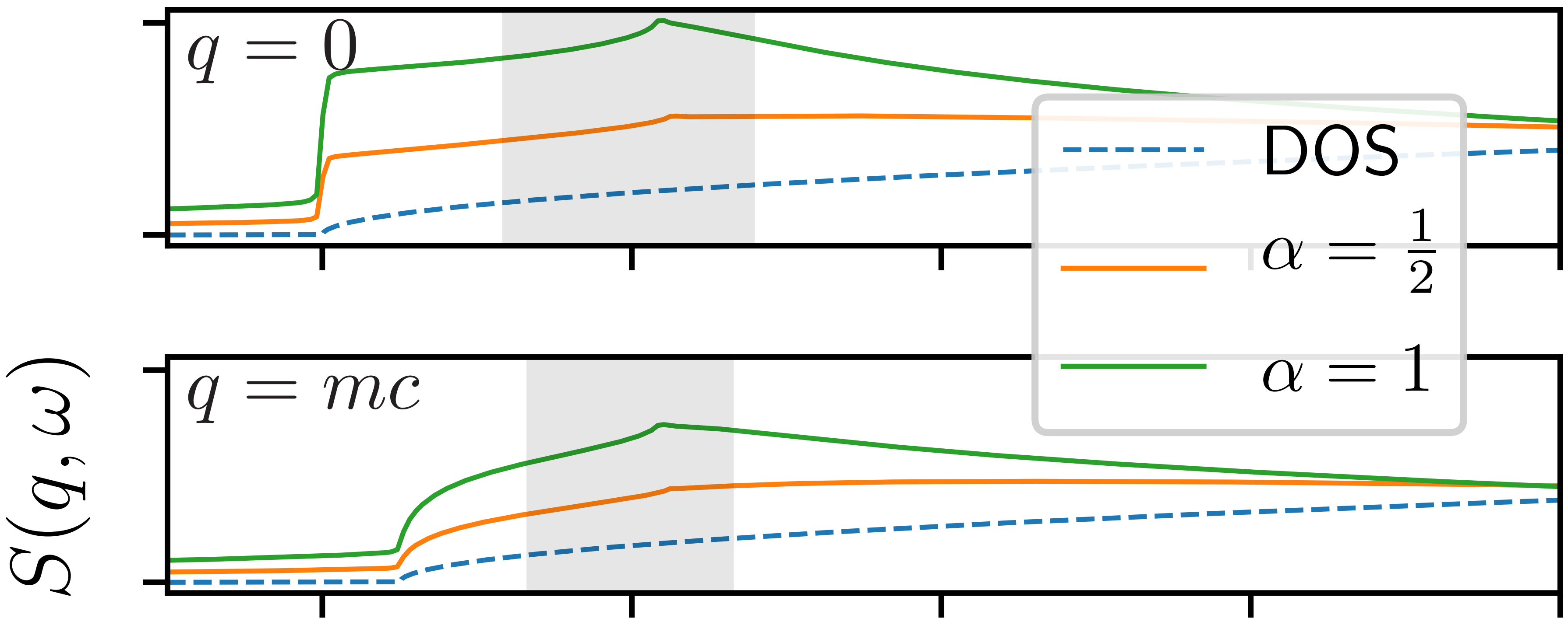
Cerenkov radiation gives a lifetime to the spinons

$$k > mc$$



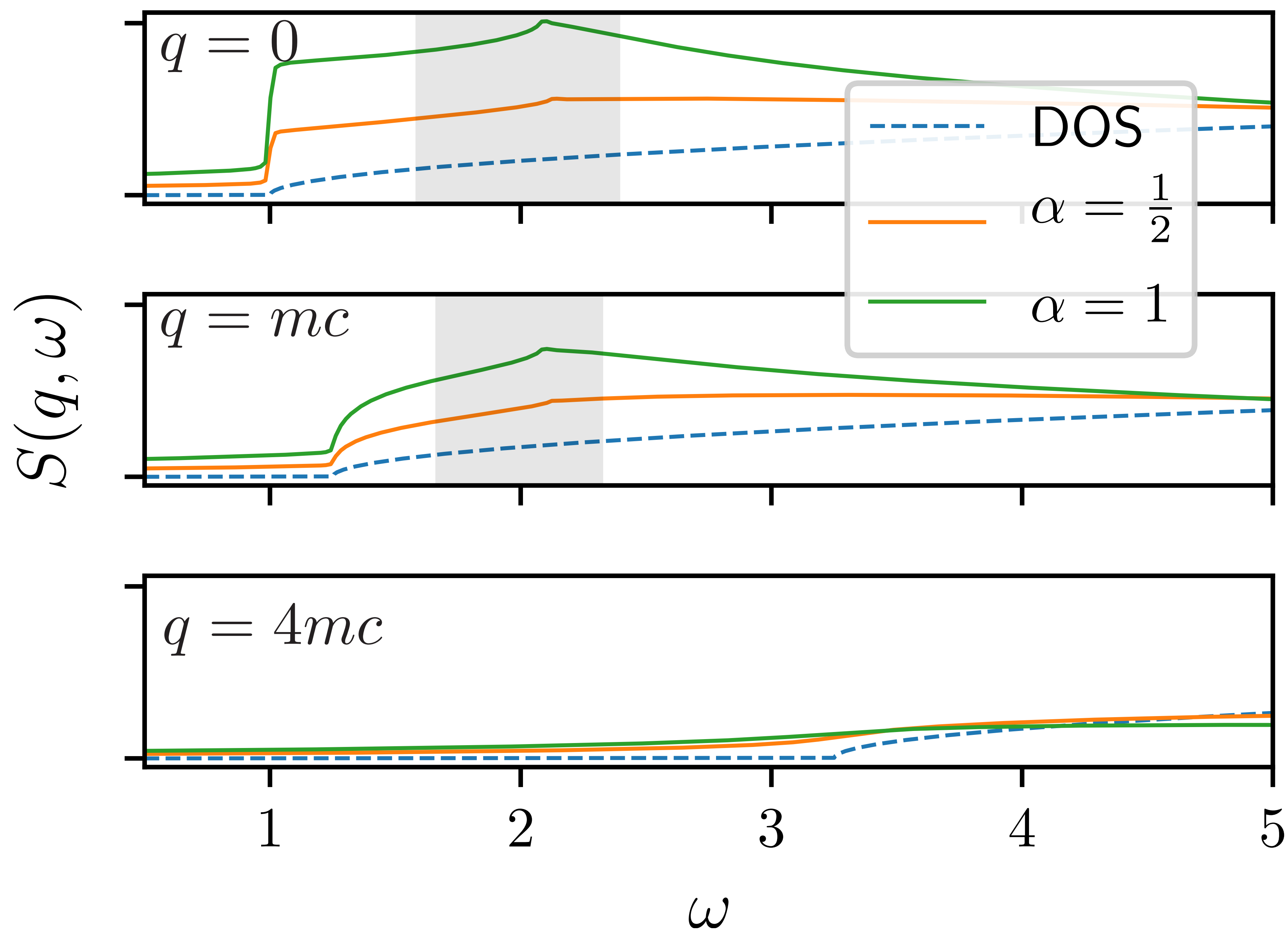
$$\frac{1}{\tau} \sim \frac{\alpha k^2}{m}(\frac{mc}{k} - \frac{1}{3}(\frac{mc}{k})^3 - \frac{2}{3})$$

Interplay of enhancement and Cerenkov effects

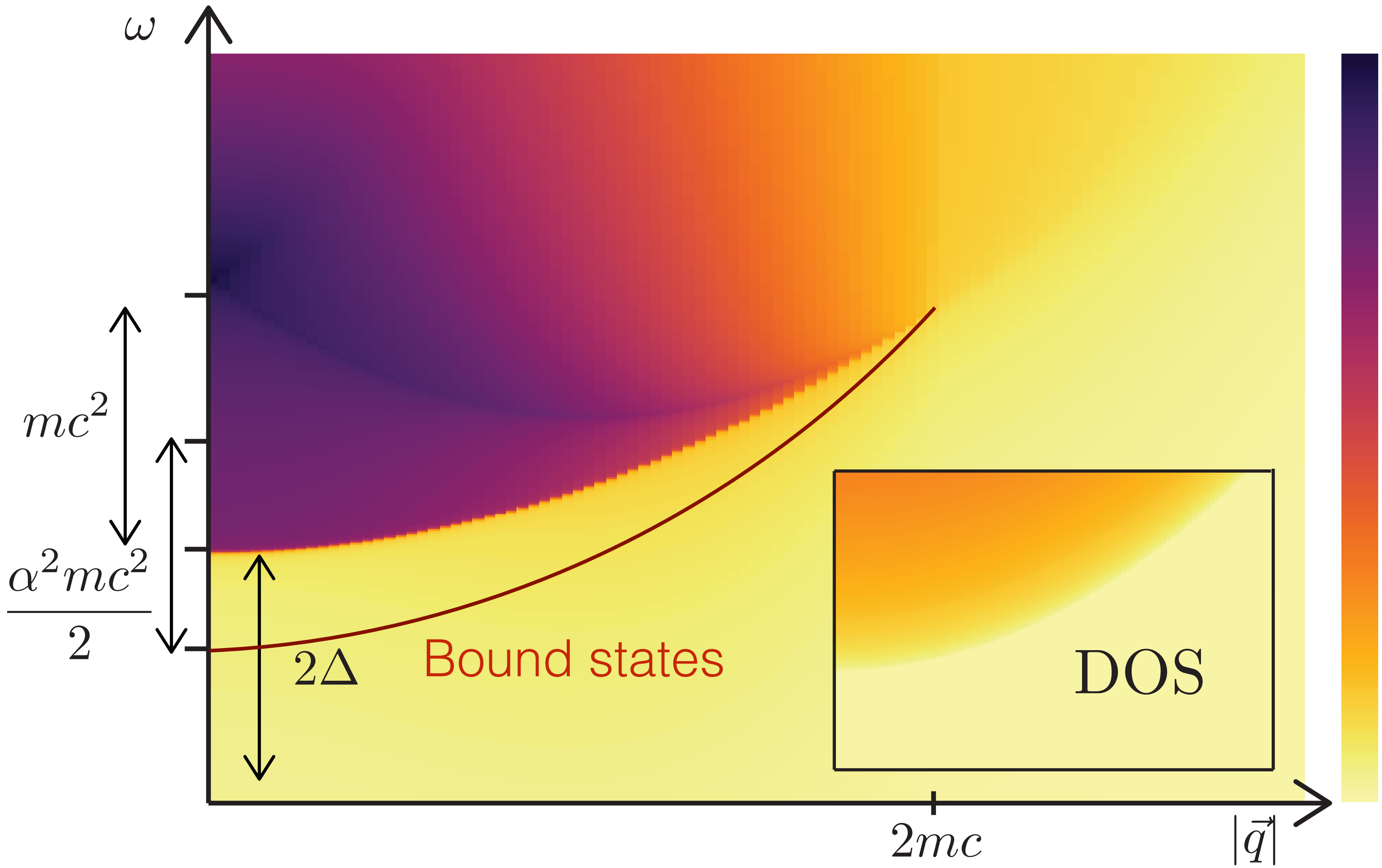


$$q_{\text{critical}} = 2mc$$

Interplay of enhancement and Cerenkov effects



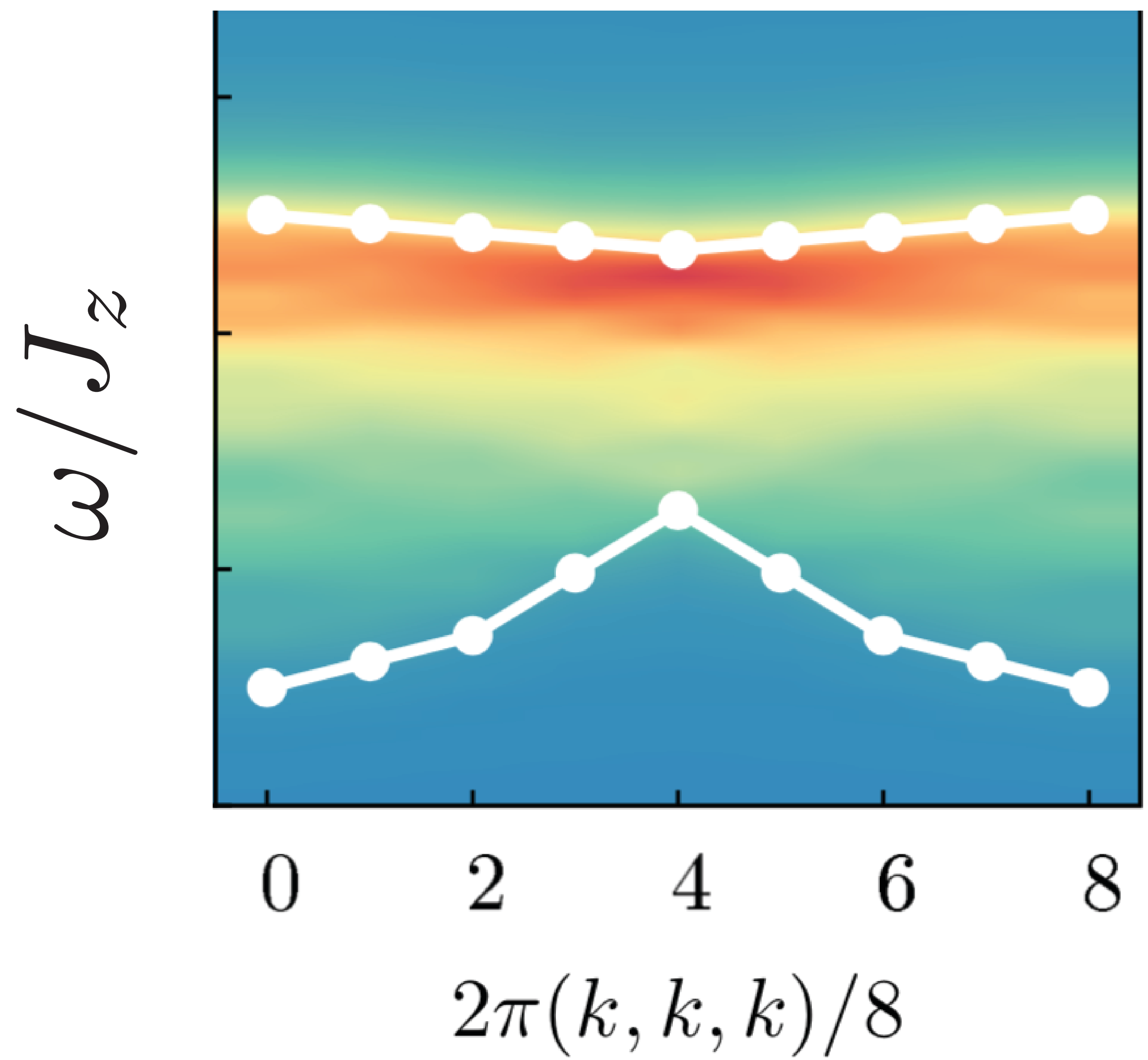
Interplay of enhancement and Cerenkov effects



Consistent with recent numerics and experiments

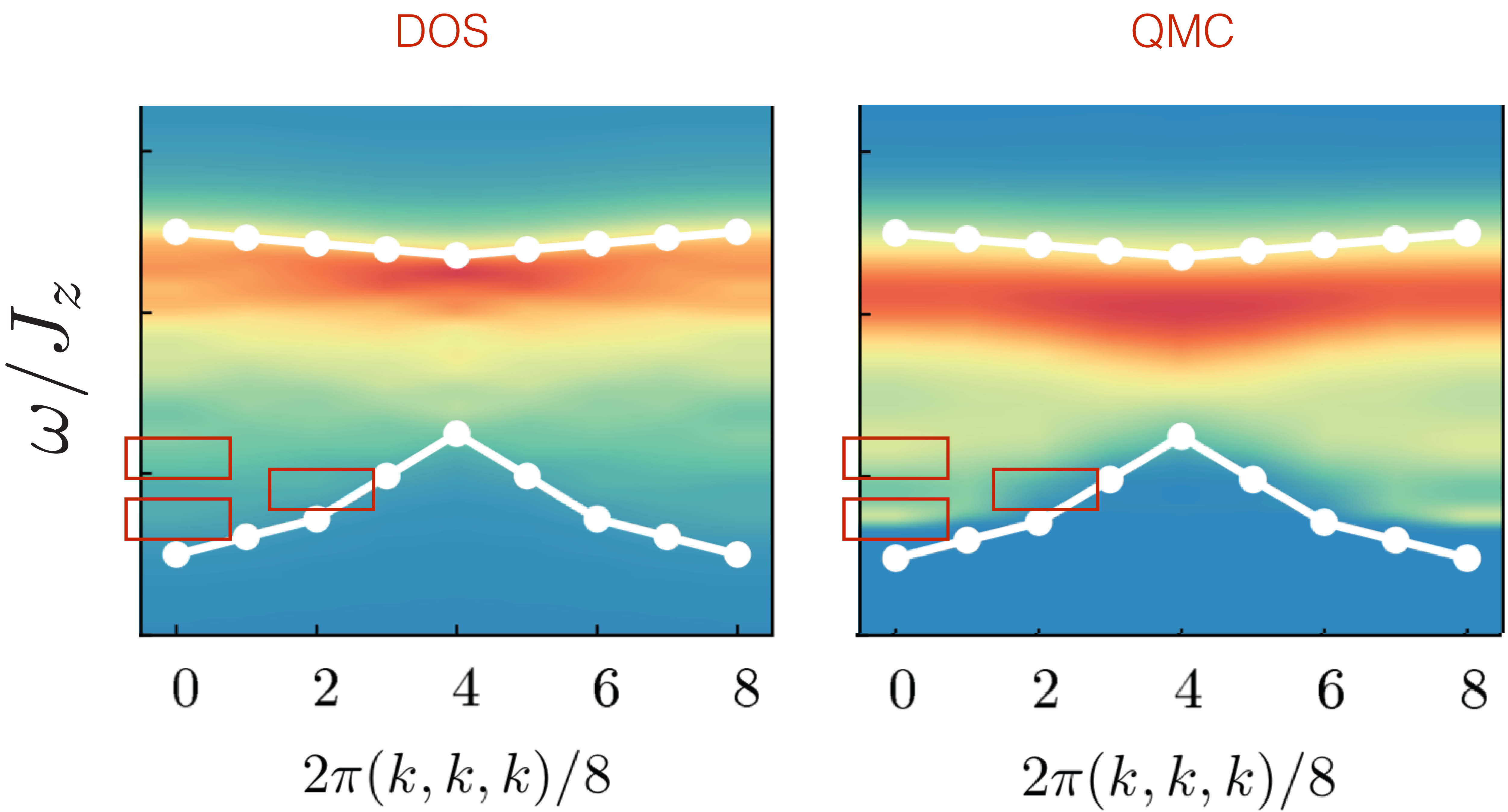
Pyrochlore XXZ

DOS



Huang et al - Phys. Rev. Lett. (2018)

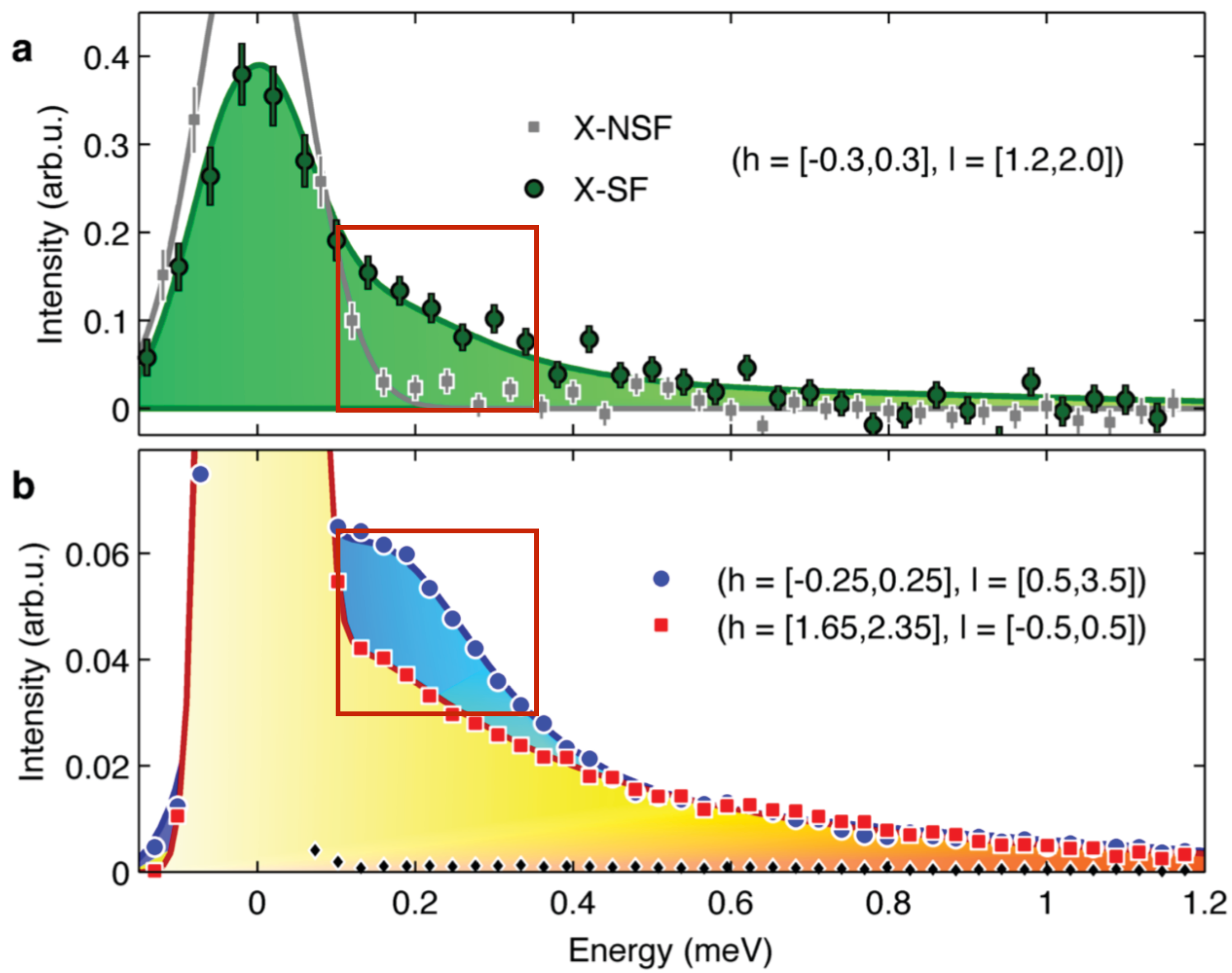
Consistent with recent numerics and experiments



Huang et al - Phys. Rev. Lett. (2018)

Consistent with recent numerics and experiments

$Pr_2Hf_2O_7$



Sibille et al - Nat. Phys. 2018

Summary

- Gapless and slow photon significantly modifies scattering cross-section of spinons in Coulomb spin liquids
- Universal enhancement near the bottom of band of spinons which can be used to determine the fine-structure constant
- Suppression of intensity at finite wavevectors and diffuse cross-section at critical momentum and energy due to Cerenkov radiation
- In line with recent numerical and experimental data

$$S(\vec{q},\omega) \sim m^2 c\alpha \left(1 - \frac{1}{4}\left(\frac{q}{mc}\right)^2\right) \theta(\omega - 2\Delta - \frac{q^2}{4m})$$