Majorana modes in double Rashba Nanowires

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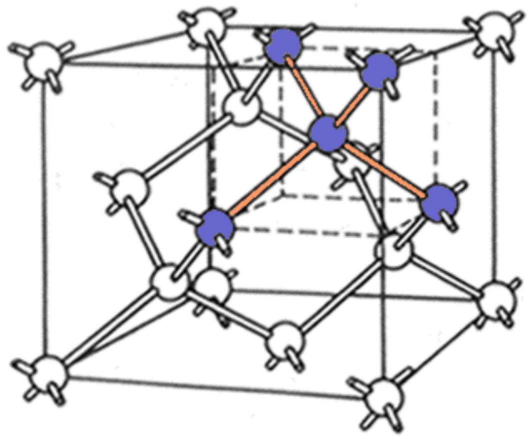
Les Houches 08/09/19

Outline

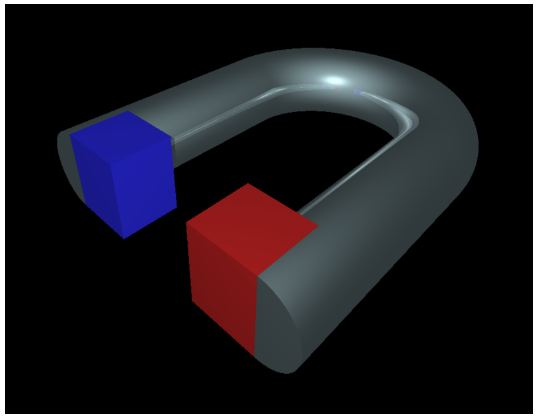
- Topological superconductors in one-dimension
- Majorana Modes in Rashba Nanowire: Review
- Rashba Double-nanowire setup

Topological order – A New Type of Classification

Landau theory: Distinct phases can be characterized by an order parameter corresponding to a spontaneously broken symmetry



Crystal: Broken translational and rotational symmetries



Magnet: Broken rotational and time-reversal symmetries

Quantum Hall systems!!

Moore, Nature 464, 194 (2010), Hasan and Kane, Rev. Mod. Phys. 82, 3045 (2010)

Topological order – A New Type of Classification

- New classification scheme based on topology

- "Topology" : Branch of mathematics where we study those properties of a system which remain the same if small changes are made in the material property or geometry

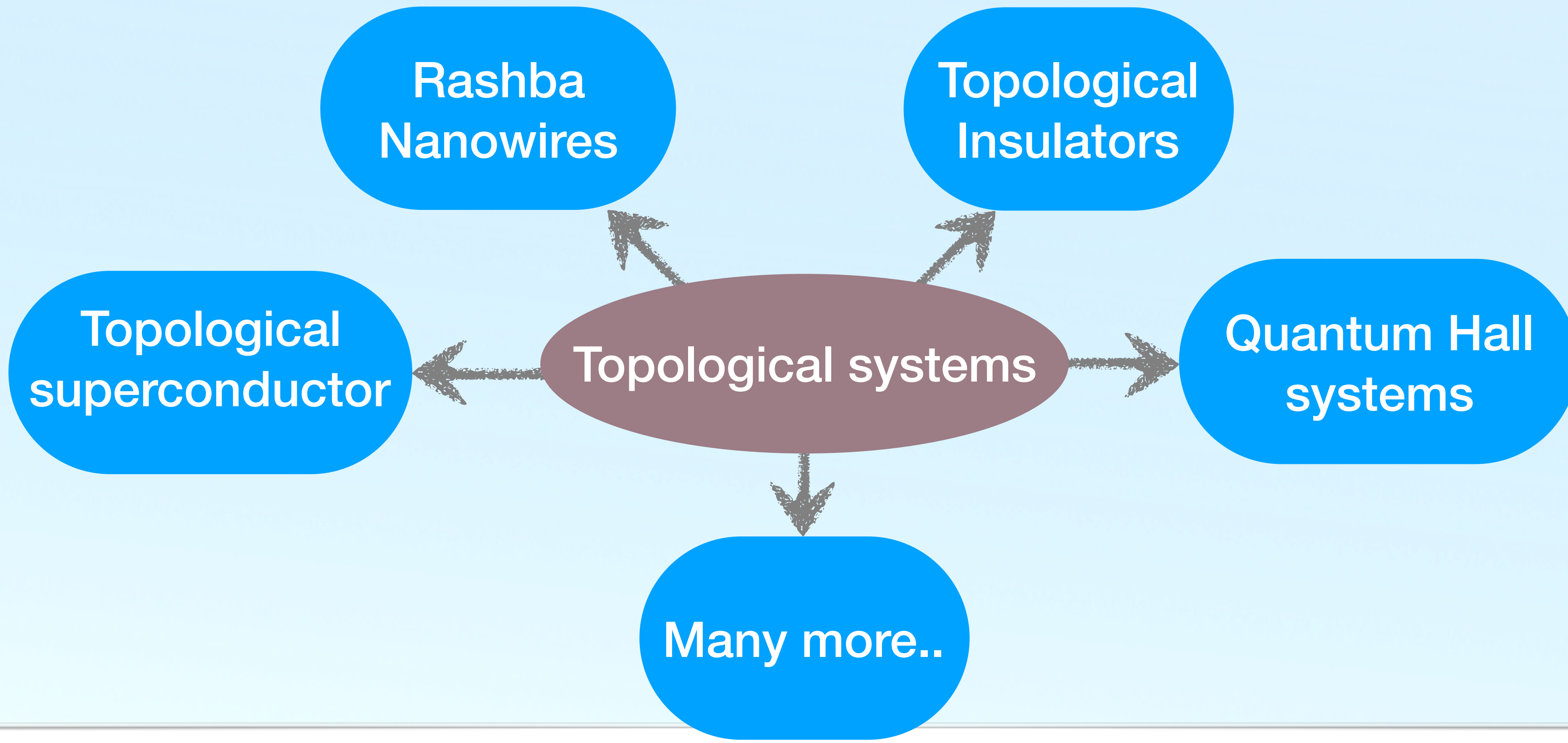
- "Topological Invariant" A number which remains the same under small changes

- "Topological Order" Existence of a topological invariant characterizing some observable

Moore, Nature 464, 194 (2010), Hasan and Kane, Rev. Mod. Phys. 82, 3045 (2010)

Topological condensed matter systems

- Gapped system such as superconductors and insulator
- Existence of gapless state on the boundary: Topological invariant
- Topological invariant can not change unless the bulk gap closes
- Consequence: Bulk-boundary correspondence



Topological superconductors

	Bulk	Edge
Topological Superconductors	Gapped (Pairing Gap)	Majorana Bound States

Majorana Bound States (MBSs)

Ettore Majorana

High-energy definition (Majorana fermion): a particle that is its own anti-particle (*real* solution to Dirac eq.)

$$(i\gamma^{\mu}\partial_{\mu} - m)\psi = 0 \qquad (\psi = \psi^{*})$$

Condensed matter definition: quasiparticle with real operator

$$\boxed{\gamma = \gamma^{\dagger}}$$

- Majorana quasiparticles are equal superposition of electron $(c^{\dagger})$ and hole (c)

$$\gamma_1 = c + c^{\dagger} \qquad c^{\dagger} = \frac{\gamma_1 + i\gamma_2}{2}$$

$$\gamma_2 = i(c - c^{\dagger}) \qquad c = \frac{\gamma_1 - i\gamma_2}{2}$$

$c^{\dagger}$ ● $\quad c$ ○ $\quad \gamma$ ◓

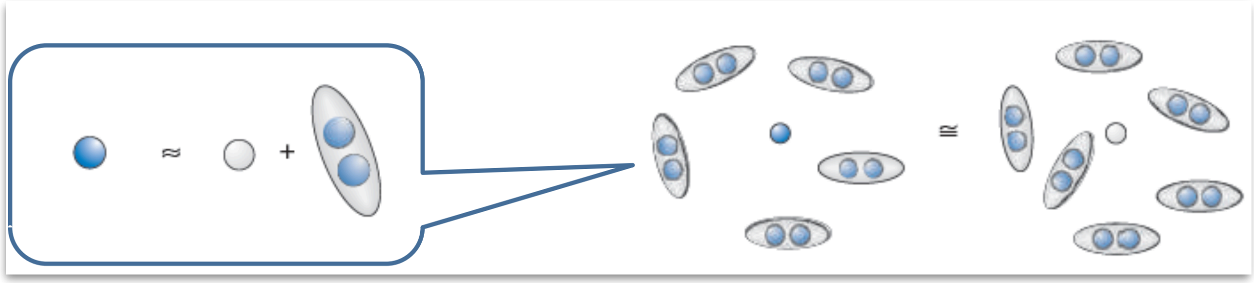
E. Majorana, Nuovo Cimento 14, 171 (1937)

Superconductors are natural hosts!

Bogoliubov meets Majorana

Nikolay Bogoliubov

In superconductors the absolute distinction between electrons and holes is blurred



Wilczek Nature (2009)

Ettore Majorana

Bogoliubov quasiparticle: $\gamma = u\,c + v\,c^\dagger$, if $u = v^* \Rightarrow \gamma = \gamma^\dagger$

Superconductor imposes electron-hole symmetry

$$\gamma(\epsilon) \text{ at } \epsilon \text{ and } \gamma^\dagger(-\epsilon) \text{ at } -\epsilon \Rightarrow \gamma(\epsilon = 0) = \gamma^\dagger(\epsilon = 0)$$

Is spin degeneracy a problem?

s-wave superconductivity: $\gamma_n^\dagger \approx u_n c_\uparrow^\dagger + v_n c_\downarrow$

Zero energy mode ➡ $\gamma_0^\dagger \neq \gamma_0$

p-wave superconductivity: $\gamma_n^\dagger \approx u_n c_\uparrow^\dagger + v_n c_\uparrow$

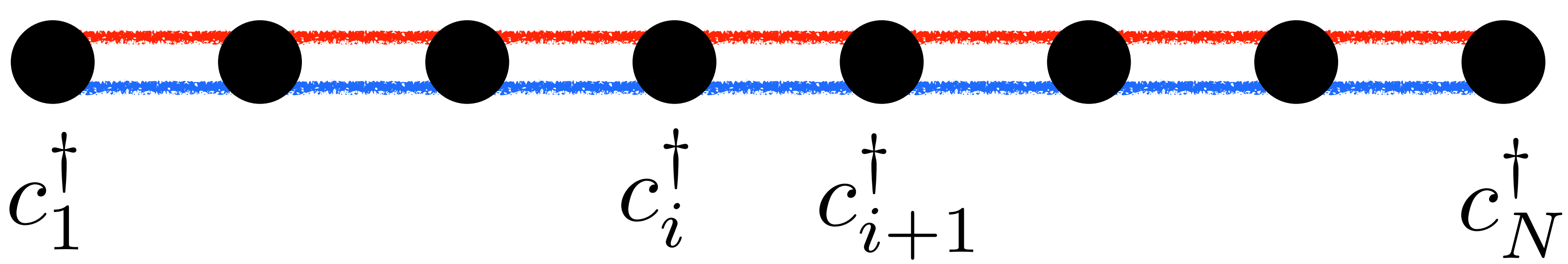
Zero energy mode ➡ $\gamma_0^\dagger = \gamma_0 \quad (\text{for } u_n^* = v_n)$

Kitaev, Physics-Uspekhi 44, 131 (2001)

Introduction to MBS in 1D spinless superconductors

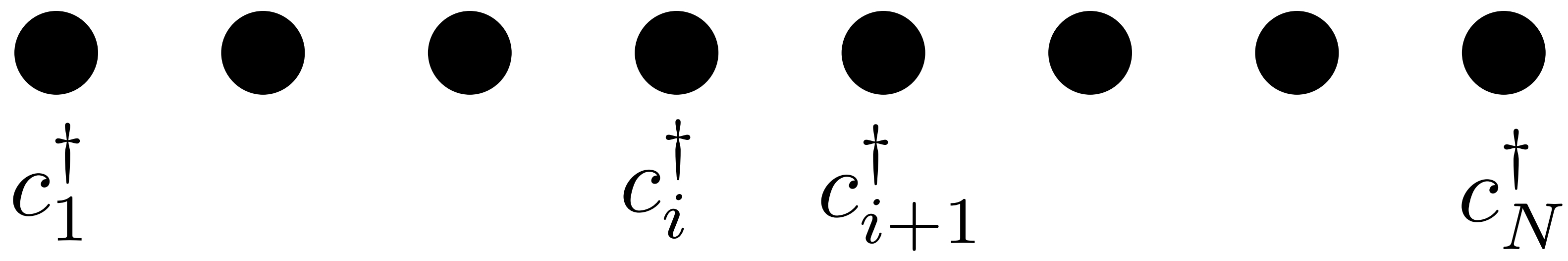
Kitaev model

$$H = -\sum_{n=1}^{N}\left[t c_n^\dagger c_{n+1} + \Delta c_n^\dagger c_{n+1}^\dagger + H.c.\right] - \mu \sum_{n=1}^{N} c_n^\dagger c_n$$



Kitaev, Physics-Uspekhi 44, 131 (2001)

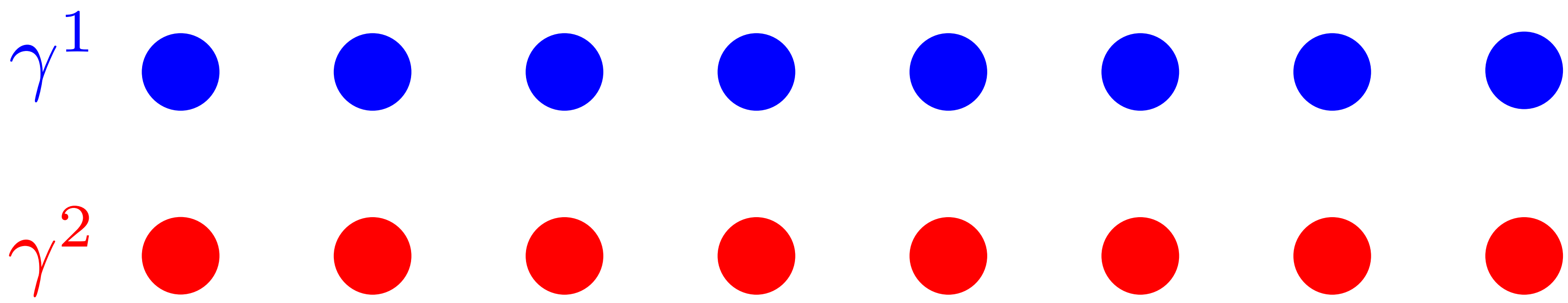
Introduction to MBS in 1D spinless superconductors



Majorana decomposition

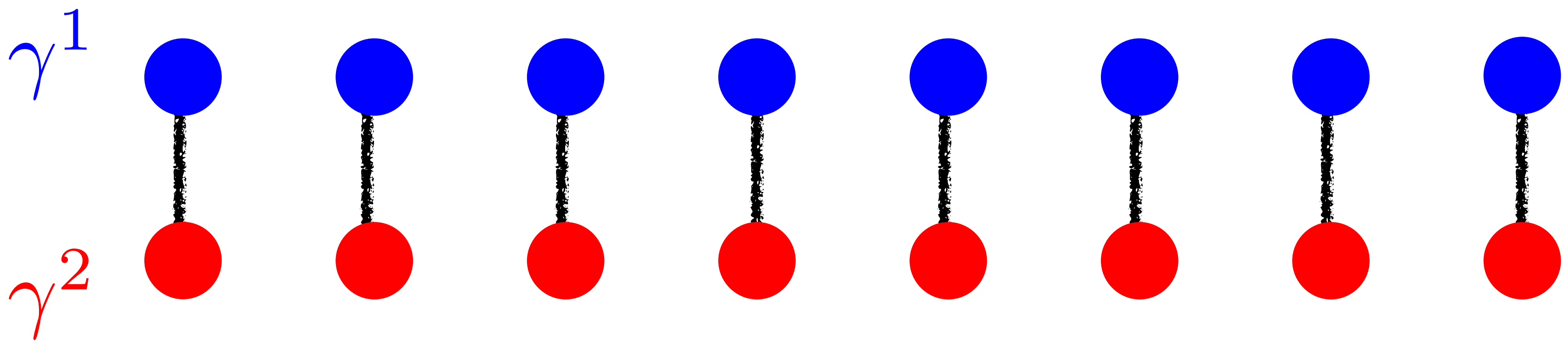
$$\gamma_n^1 = c_n + c_n^\dagger$$

$$\gamma_n^2 = i(c_n - c_n^\dagger)$$



Kitaev, Physics-Uspekhi 44, 131 (2001)

Introduction to Majorana bound state in 1D spinless superconductors



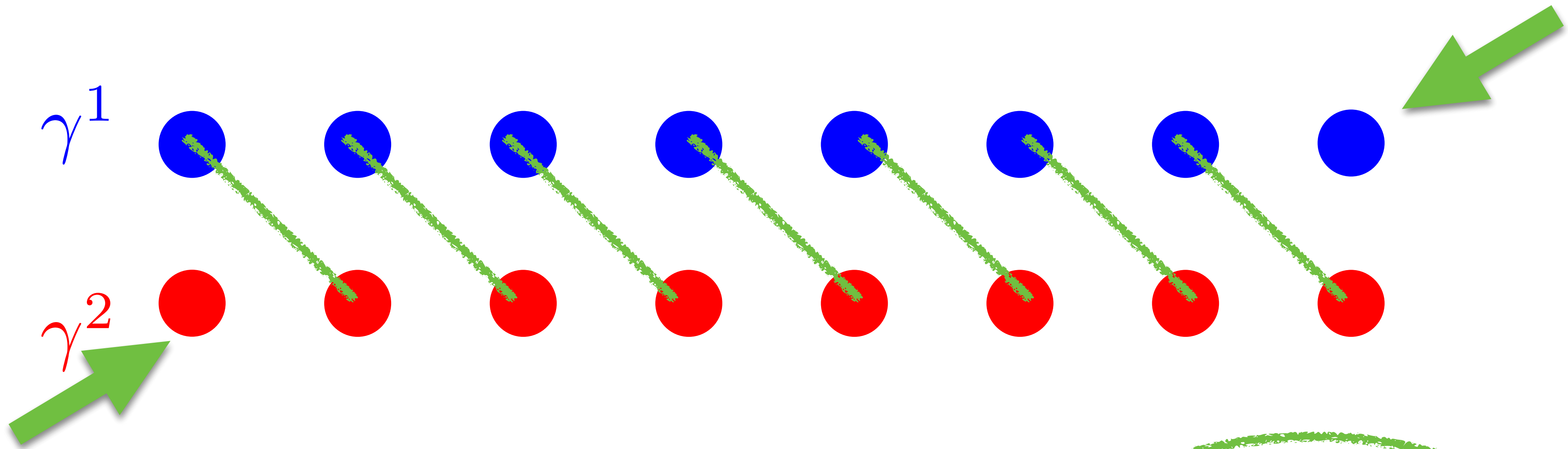
$$\gamma_n^1 = c_n + c_n^\dagger$$

$$\gamma_n^2 = i(c_n - c_n^\dagger)$$

$$H = -\frac{i}{2}\sum_{n=1}^{N-1}\left[(t+\Delta)\gamma_n^2\gamma_{n+1}^1 - (t-\Delta)\gamma_n^1\gamma_{n+1}^2\right] - i\frac{\mu}{2}\sum_{n=1}^{N}\gamma_n^2\gamma_n^1$$

Kitaev, Physics-Uspekhi 44, 131 (2001)

Introduction to Majorana bound state in 1D spinless superconductors



$$\gamma_n^1 = c_n + c_n^\dagger$$
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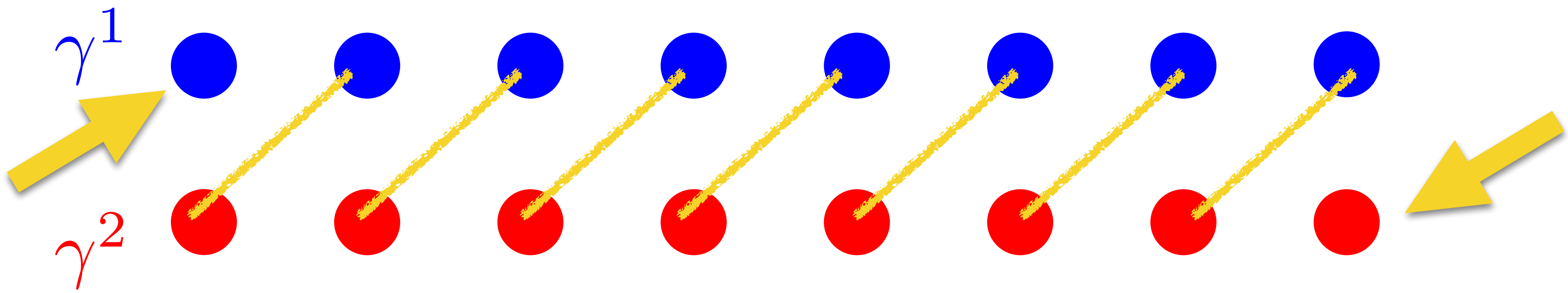
Majorana bound state at the ends of the chain

Kitaev, Physics-Uspekhi 44, 131 (2001)

Introduction to Majorana bound state in 1D spinless superconductors



$$\gamma_n^1 = c_n + c_n^\dagger$$

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Majorana bound state at the ends of the chain

Kitaev, Physics-Uspekhi 44, 131 (2001)

How to implement this in a real system?

Need a p-wave superconductor but difficult!!

Sr_2RuO_4

Rare in nature and difficult to manipulate experimentally

How to mimic p-wave superconductivity with available ingredients in the lab?

Topological Superconductivity in single nanowire

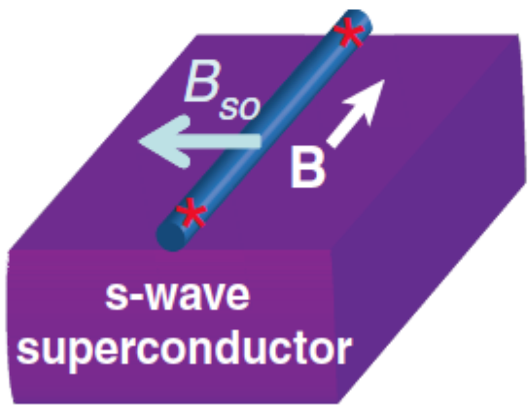
[p-wave vortex: Volovik, JETP Lett. 70, 609 (1999); 2D with SOI: Sato and Fujimoto, PRB 79, 094504 (2009)]

Theory: nanowires: Lutchyn et al., PRL 105, 077001 (2010), Oreg et al., PRL 105, 177002 (2010)

(E_{so}) (Δ_Z) (Δ)

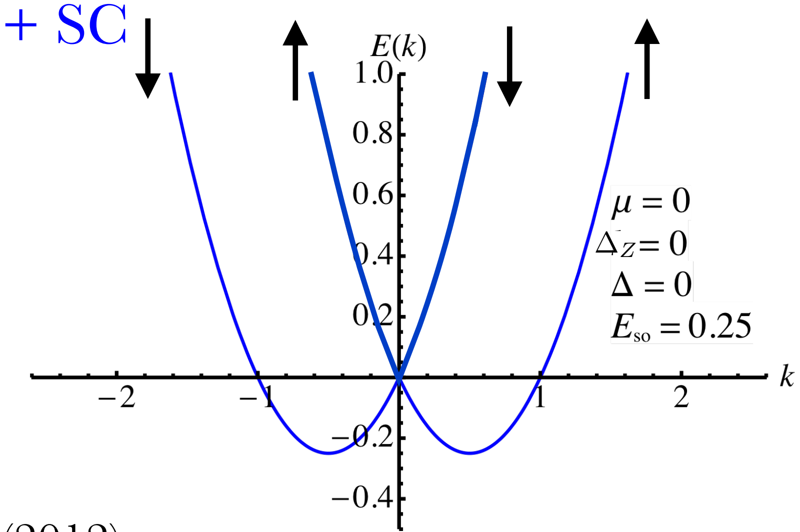
Rashba SOI + Magnetic field + SC

$$E_{\pm}^2 = \Delta_Z^2 + \Delta^2 + \xi_k^2 + \alpha^2 k^2 \pm 2\sqrt{\Delta_Z^2\Delta^2 + \Delta_Z^2\xi_k^2 + \alpha^2k^2\xi_k^2}$$

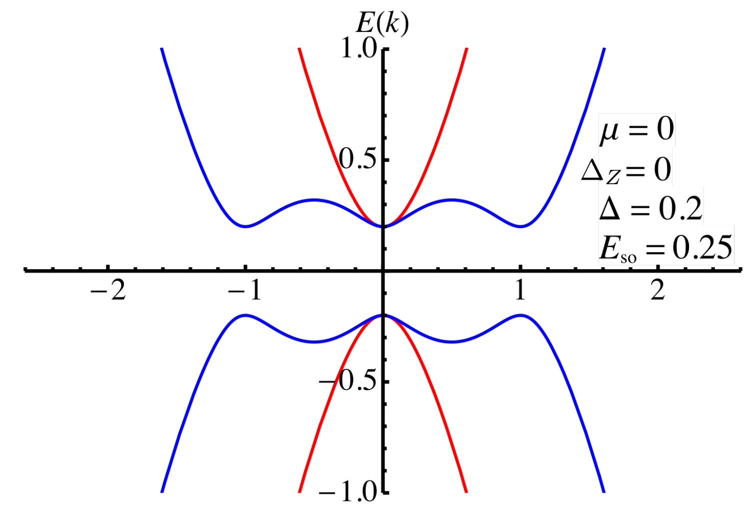


Exp. Mourik *et al.*, Science 336, 1003 (2012)

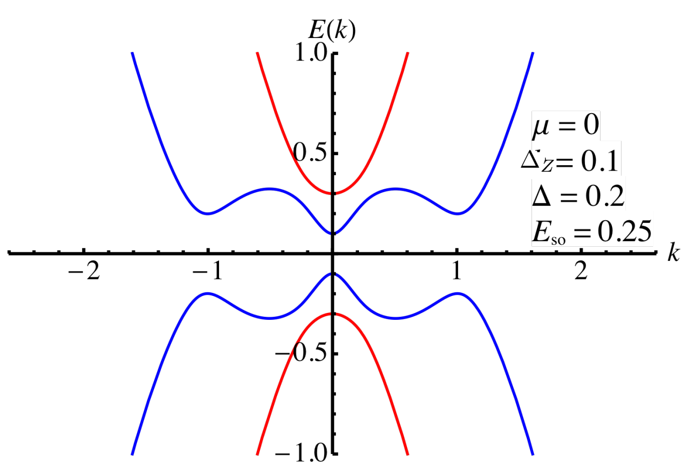
Exp. Deng *et al.*, Science 354, 1557 (2017)



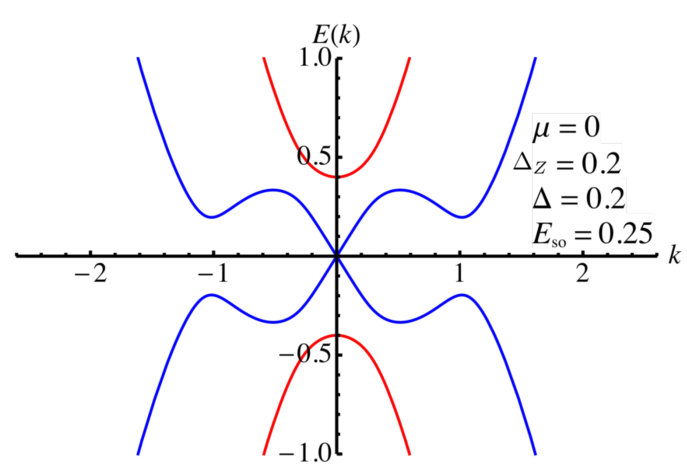
1. SOI lifts spin degeneracy



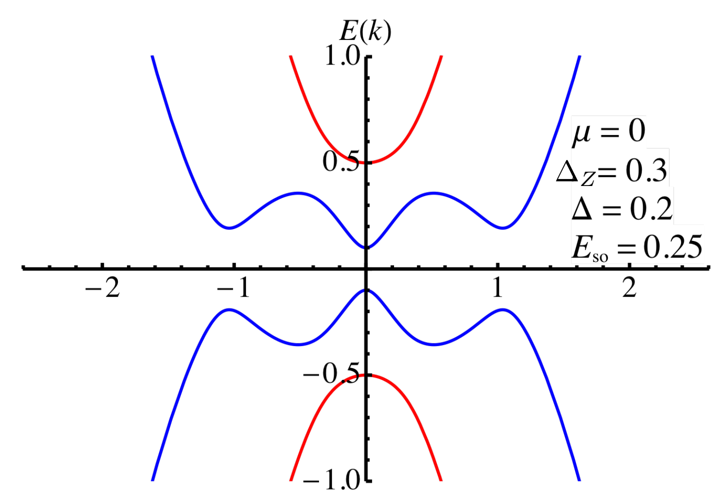
2. SC opens a gap



3. Magnetic Field breaks TR, decreases gap size



4. Gap closes at $\Delta_Z = \Delta$

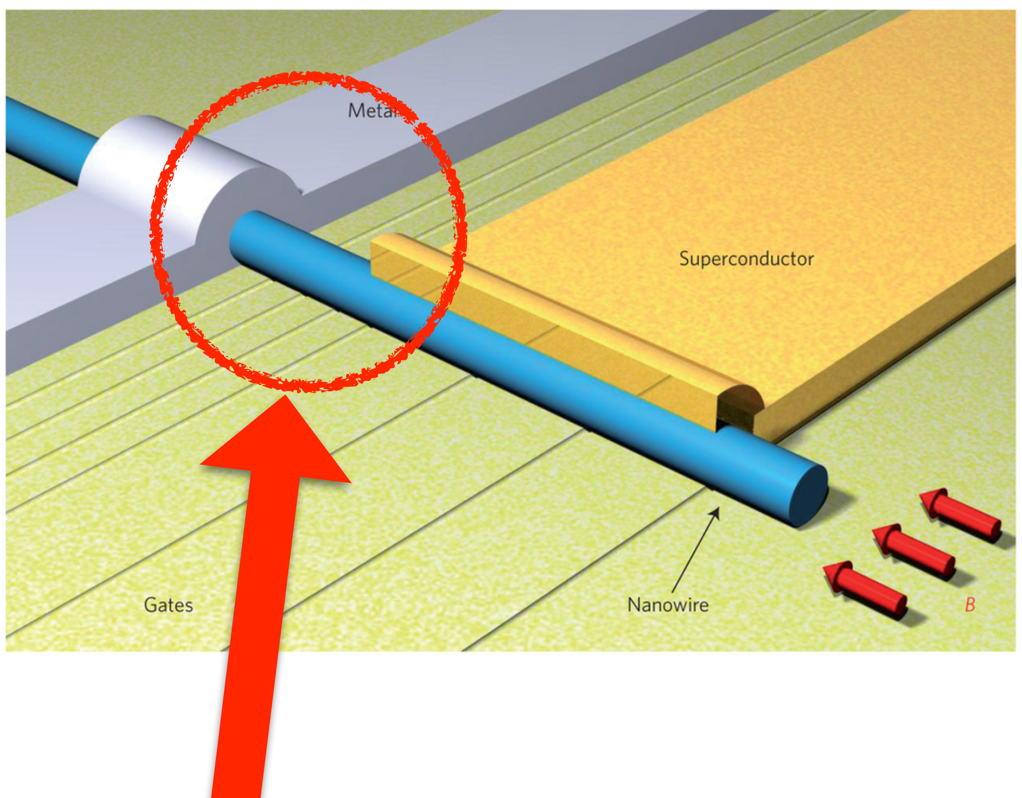


5. Increasing B-field reopens gap in topological phase

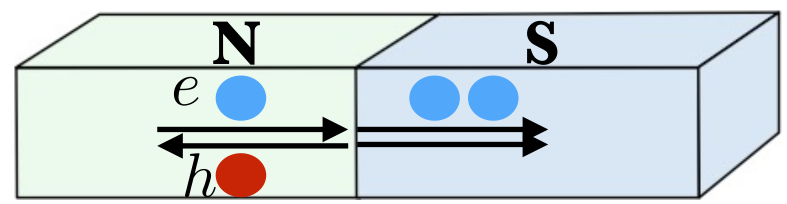
How to detect Majorana bound state?

➔ Transport experiment: zero bias peak

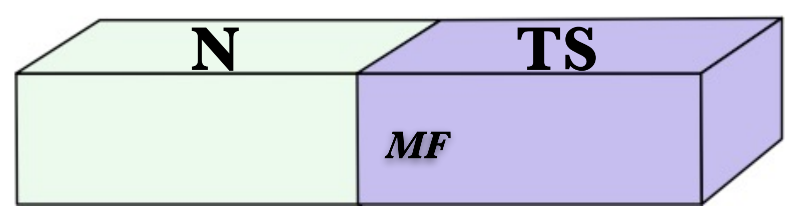
M. Franz, Nature Nanotechnology 8, 149 (2013)



Tunnel barrier:
NS interface with the wire



$$G = \frac{2e^2}{h}T$$

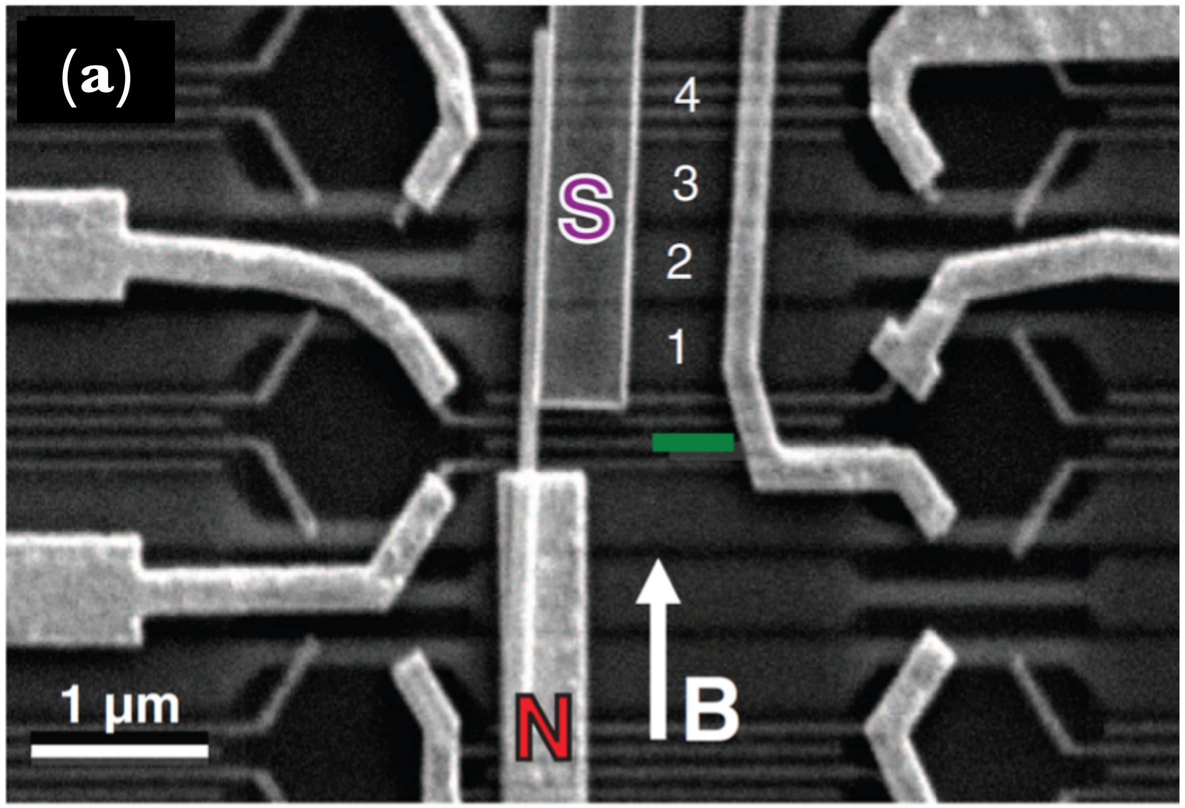


$$G = \frac{2e^2}{h}$$

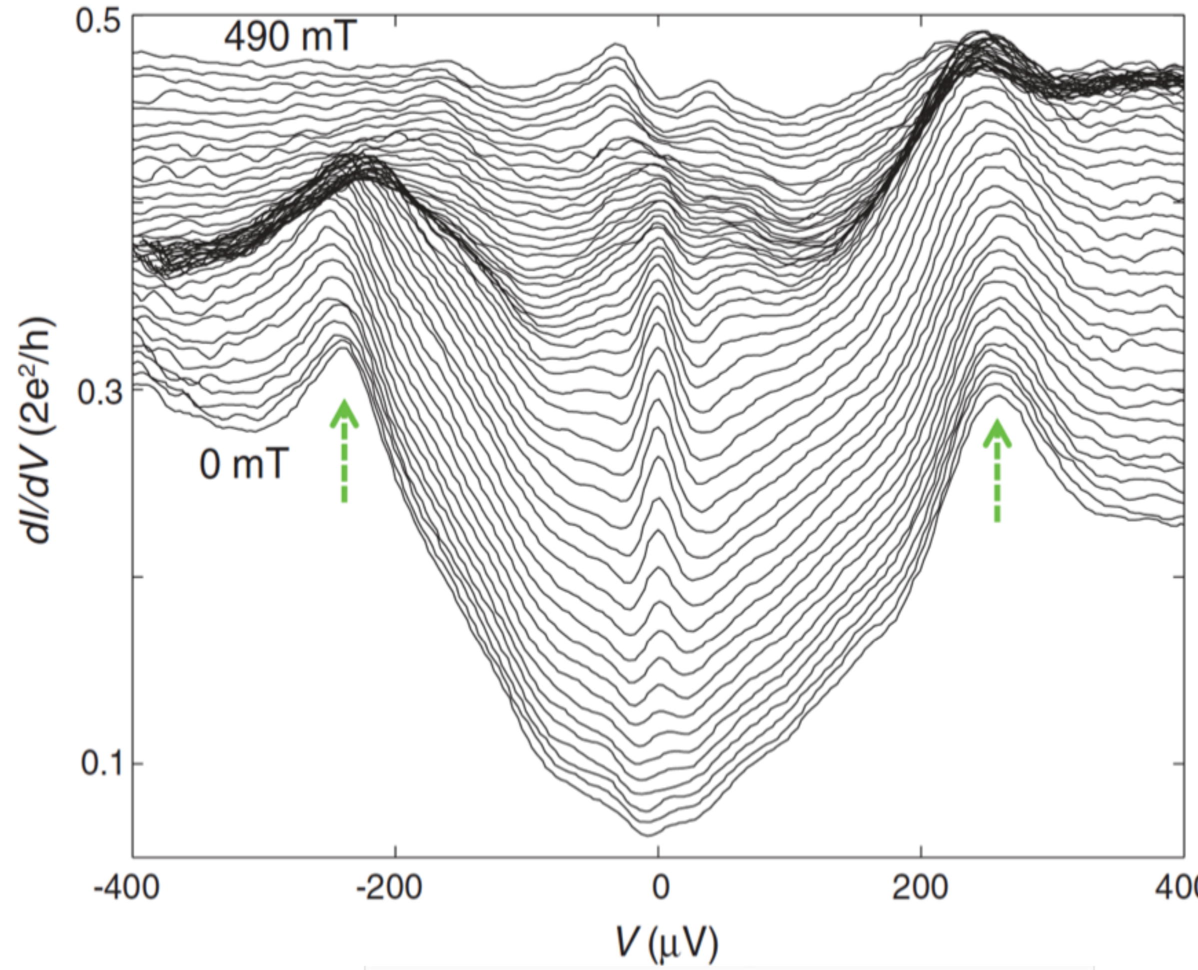
Andreev Resonant Tunneling

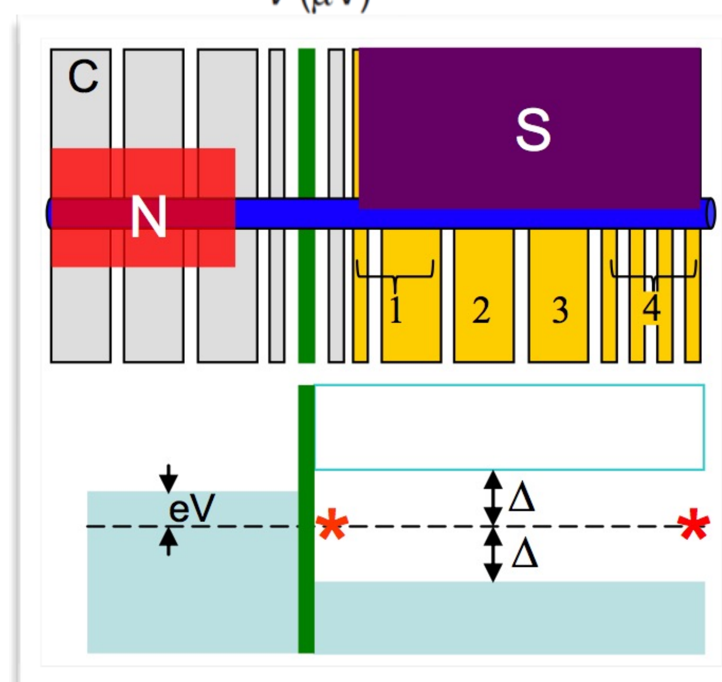
$$T = 1$$

First Experiment



Mourik *et al*, Science **336**, 1003 (2012)





- Zero-bias conductance peak above ~ 100 mT (Majorana) $\Delta_Z > \Delta$
- Small gap (~ 250 μeV much smaller than NbTiN gap ~ 2 meV)
  - Due to weak coupling with superconductor

Signatures of Majorana Fermions in Hybrid Superconductor-Semiconducto Nanowire Devices

V. Mourik,[1*] K. Zuo,[1*] S. M. Frolov,[1] S. R. Plissard,[2] E. P. A. M. Bakkers,[1,2] L. P. Kouwenhoven[1*†]

nature physics ARTICLES
PUBLISHED ONLINE: 11 NOVEMBER 2012 | DOI: 10.1038/NPHYS2479

Zero-bias peaks and splitting in an Al-InAs nanowire topological superconductor as a signature of Majorana fermions

Anindya Das[†], Yuval Ronen[†], Yonatan Most, Yuval Oreg, Moty Heiblum* and Hadas Shtrikman

RESEARCH ARTICLES

TOPOLOGICAL MATTER

Observation of Majo ferromagnetic atomi a supercond

Stevan Nadj-Perge,[1*] Ilya K.
Jungpil Seo,[1] Allan H. MacI

LETTER
doi:10.1038/nature17162

Exponential protection of zero modes in Majorana islands

S. M. Albrecht[1*], A. P. Higginbotham[1,2*], M. Madsen[1], F. Kuemmeth[1], T. S. Jespersen[1], J. Nygård[1], P. Krogstrup[1] & C. M. Marcus[1]

npj Qu

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nctions
Pb surface

oss[1] and Ernst Meyer[1]

NA

Anor
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M. T. I

LETTER
doi:10.1038/nature26142

Quantized Majorana conductance

Hao Zhang[1*], Chun-Xiao Liu[2*], Sasa Gazibegovic[3*], Di Xu[1], John A. Logan[4], Guanzhong Wang[1], Nick van Loo[1], Jouri D. S. Bommer[1], Michiel W. A. de Moor[1], Diana Car[3], Roy L. M. Op het Veld[3], Petrus J. van Veldhoven[3], Sebastian Koelling[3], Marcel A. Verheijen[3,5], Mihir Pendharkar[6], Daniel J. Pennachio[4], Borzoyeh Shojaei[4,7], Joon Sue Lee[7], Chris J. Palmstrøm[4,6,7], Erik P. A. M. Bakkers[3], S. Das Sarma[2] & Leo P. Kouwenhoven[1,8]

Challenges with single nanowire setups

- Magnetic field and superconductivity have detrimental effect on each other : High magnetic field kills superconductivity
- Small and soft Gap: Majorana bound states are less protected
- Height of Zero bias peak is not quantized to $2e^2/h$
- Clear distinction between the zero bias peak coming from Andreev bound state and Majorana bound state is missing

Outline

Is it possible to achieve topological phases

(a) without magnetic field?

Interaction

(b) with weak magnetic field?

Magnetic Field

Supercurrent

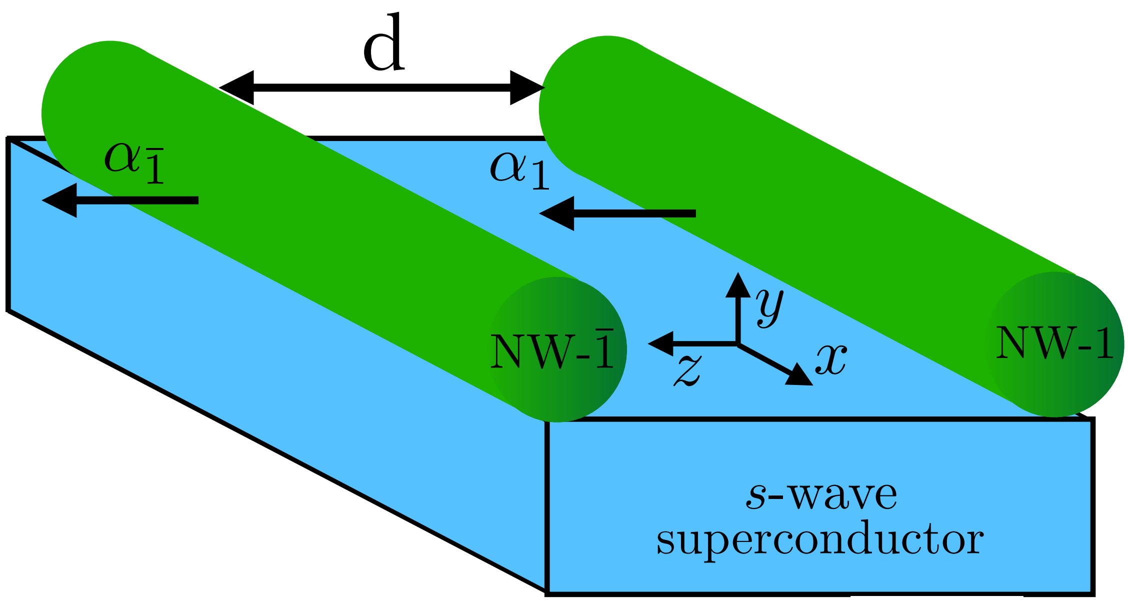
(c) with bulk features showing the existence of MBS?

Transport

Majorana Bound states

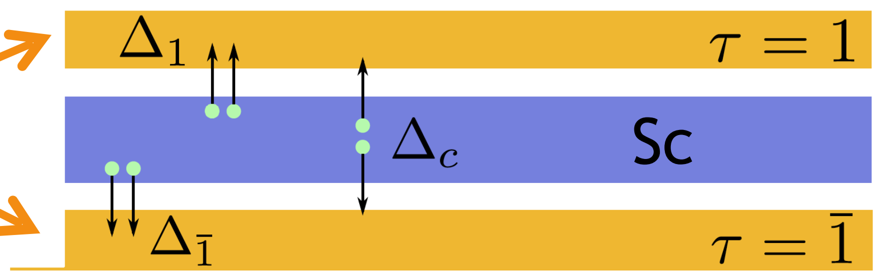
(a) Without magnetic field

Topological phase without magnetic field ?
Double Rashba wire + Superconductor



unequal SOI

No B-field! **'DIII class'**



Consider inter- and intra-wire pairing

'crossed Andreev' term Δ_c
(non-local Cooper pairs)

usual Δ_τ
(local Cooper pairs)

➔ Two competing gap mechanisms

Klinovaja and Loss, Phys. Rev. B 90, 045118 (2014)

Topological phase without magnetic field ?
→ Double Rashba nanowire + Superconductor

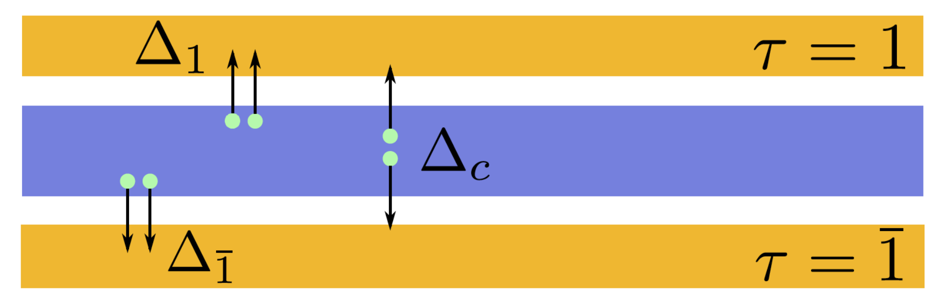
Double-NW setup in proximity to an s-wave superconductor underneath and with Rashba spin orbit interaction

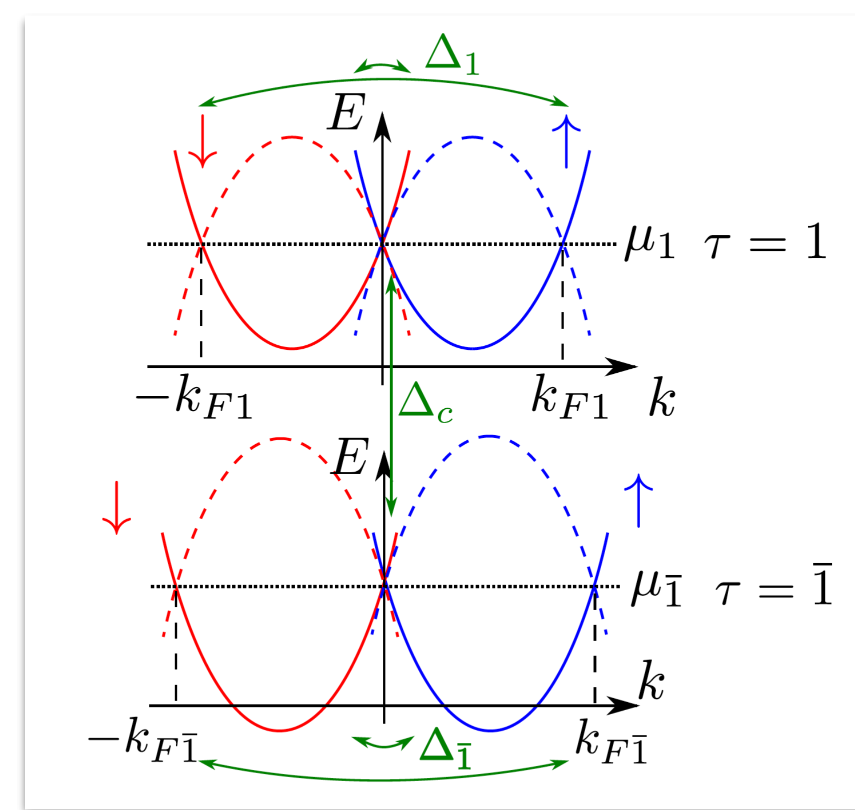
$$H_0 = \sum_{\tau,\sigma} \int \mathrm{d}x \; \Psi^\dagger_{\tau\sigma} \left(-\frac{\hbar^2 \partial_x^2}{2m} - \mu_\tau \right) \Psi_{\tau\sigma'}$$

$$H_{so} = i \sum_{\tau,\sigma,\sigma'} \alpha_\tau \int \mathrm{d}x \; \Psi^\dagger_{\tau\sigma} \left(\sigma_z\right)_{\sigma\sigma'} \partial_x \Psi_{\tau\sigma'} .$$

$$H_d = \sum_{\tau,\sigma,\sigma'} \frac{\Delta_\tau}{2} \int \mathrm{d}x \left[\Psi_{\tau\sigma} (i\sigma_y)_{\sigma\sigma'} \Psi_{\tau\sigma'} + \mathrm{H.c.} \right]$$

$$H_c = \frac{\Delta_c}{2} \sum_{\tau,\sigma,\sigma'} \int \mathrm{d}x \left[\Psi_{\tau\sigma} (i\sigma_y)_{\sigma\sigma'} \Psi_{\bar{\tau}\sigma'} + \mathrm{H.c.} \right]$$

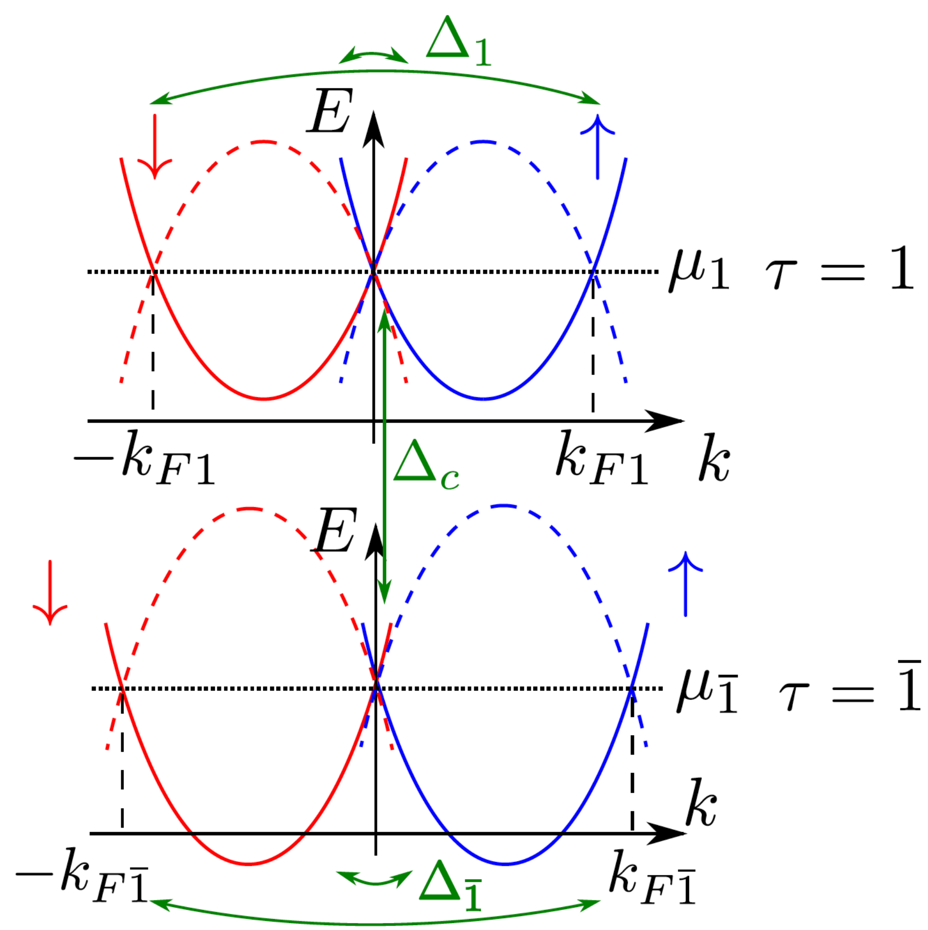




Unequal strength of spin-orbit interaction

Klinovaja and Loss, Phys. Rev. B 90, 045118 (2014)
Baba, et al., NJP 20 (2018) 063021

Double Rashba nanowire + Superconductor



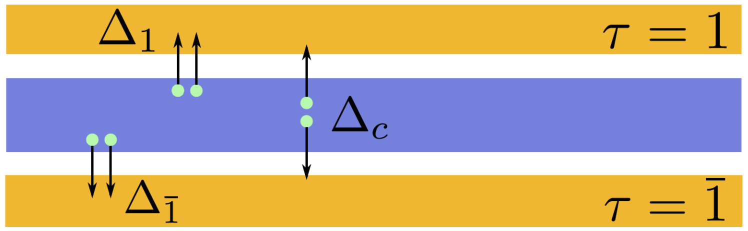
- Gap at $k = 0$ in the spectrum is given by $\Delta_g = \sqrt{|\Delta_c^2 - \Delta_1 \Delta_{\bar{1}}|}$

- Topological criterion: For $\Delta_c^2 > \Delta_1 \Delta_{\bar{1}}$, setup hosts Kramers pairs of MBSs [J. Klinovaja and D. Loss, Phys. Rev. B 90, 045118 (2014)] $\Delta_c > \Delta_\tau$

- For non-interacting systems, $\Delta_\tau > \Delta_c$ [Reeg et al., Phys. Rev. B 96, 081301(R) (2017)]

*See also Gaidamauskas et al., Phys. Rev. Lett. 112, 126402 (2014)

How to obtain $\Delta_c > \Delta_\tau$?

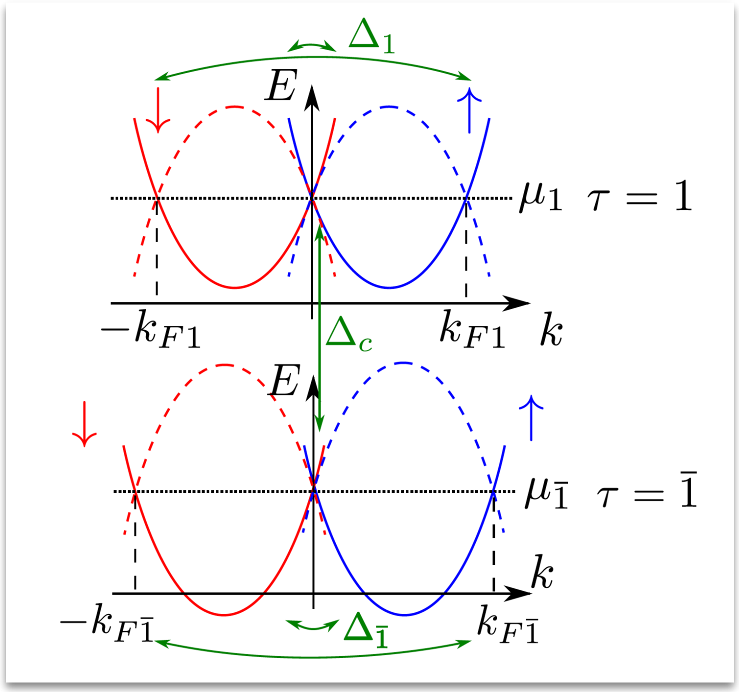


Include electron-electron interaction

- Electron-electron interaction $\rightarrow$ Bosonization
- Disorder averaging $\rightarrow$ Replica Method
- Which term in the Hamiltonian dominates over the other terms (to identify topological phase) $\rightarrow$ Renormalization Group analysis

Thakurathi et al., Phys. Rev. B 97, 045415 (2018)
Yosuke Sato, et al., arXiv:1810.06259 [Exp: Strong electron-electron interaction in TLL]

Bosonized effective Hamiltonian



- $H_{eff} = H_0 + H_s + H_c + H_g +$ Disorder

$$H_0 = \sum_i u_i \int \frac{dx}{2\pi}\left[\frac{(\partial_x \phi_i)^2}{K_i} + K_i\,(\partial_x \theta_i)^2\right]$$

- $$H_s = \frac{\tilde{\Delta}_1^{ext} u}{\pi\,\alpha^2}\int dx\,\cos(2\,\phi_1) + \frac{\tilde{\Delta}_{\bar{1}}^{ext} u}{\pi\,\alpha^2}\int dx\,\cos(2\,\phi_2) + \frac{\tilde{\Delta}_1^{int} u}{\pi\,\alpha^2}\int dx\,\cos(2\,\phi_3) + \frac{\Delta_{\bar{1}}^{int}}{\pi\,\alpha}\int dx\,\cos(2\,\phi_4)$$

$$H_c = \frac{2\tilde{\Delta}_c u}{\pi\,\alpha^2}\int dx\,\cos(\phi_3 + \phi_4)\cos(\theta_3 - \theta_4)$$

- $$H_g = \frac{u}{\pi\,\alpha^2}\int dx\Big[\,y_1\cos\{2\,(\phi_1 - \phi_3)\} + y_{\bar{1}}\cos\{2\,(\phi_2 - \phi_4)\}\Big]$$

$$S_{dis,1/\bar{1}} = -\frac{u^2\,\tilde{D}_{1/\bar{1}}}{\pi\alpha^3}\Big[\sum_{m,n}\int dx\,dt\,dt'\,e^{i\left\{\theta^m_{1/2}(x,t)+\theta^m_{3/4}(x,t)\right\}} \times e^{-i\left\{\theta^n_{1/2}(x,t')+\theta^n_{3/4}(x,t')\right\}}\cos\left\{\phi^m_{1/2}(x,t) - \phi^m_{3/4}(x,t)\right\} \times \cos\left\{\phi^n_{1/2}(x,t') - \phi^n_{3/4}(x,t')\right\} + \text{H.c.}\Big]$$

- Dimensionless coupling constants defined as $\tilde{\Delta}_{\tau/c} = \Delta_{\tau/c}\,\alpha/u$, $\tilde{D}_\tau = \alpha\,D_\tau/(2\pi u^2)$, and $y_\tau = g_\tau/(2\pi u)$

Thakurathi et al., Phys. Rev. B 97, 045415 (2018)

RG equations for effective Hamiltonian

changing the cut off from $\alpha \to \alpha(1+dl)$ and asking how the coupling constant should change to preserve the partition function.

- For $K_1 = K_2$ and $K_3 = K_4$

$$\frac{d\tilde{\Delta}_\tau^{ext}}{dl} = [2 - K_1]\tilde{\Delta}_\tau^{ext}, \quad \frac{d\tilde{\Delta}_\tau^{int}}{dl} = \Big[2 - K_3\Big]\tilde{\Delta}_\tau^{int},$$

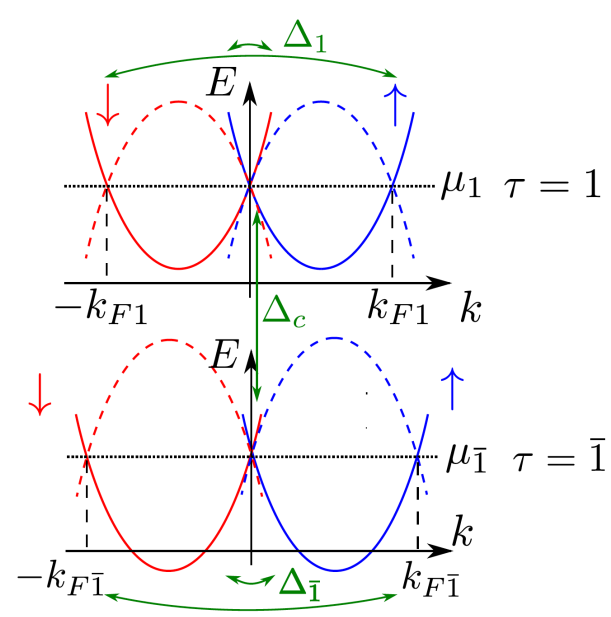
$$\frac{d\tilde{\Delta}_c}{dl} = \Big[2 - \frac{1}{2}\Big(K_3 + \frac{1}{K_3}\Big)\Big]\tilde{\Delta}_c,$$

$$\frac{dK_1}{dl} = -\Big[\Big(\tilde{\Delta}_\tau^{ext}\Big)^2 + y_1^2\Big]K_1^2 + \frac{\tilde{D}_\tau(1-K_1^2)}{2},$$

$$\frac{dK_3}{dl} = -\Big[\Big(\tilde{\Delta}_\tau^{int}\Big)^2 + y_1^2\Big]K_3^2 + \frac{(\tilde{\Delta}_c^2 + \tilde{D}_\tau)\,(1-K_3^2)}{2},$$

$$\frac{d\tilde{D}_\tau}{dl} = \Big[3 - \frac{1}{2}\Big(K_1 + K_3 + \frac{1}{K_1} + \frac{1}{K_3}\Big) - y_\tau\Big]\tilde{D}_\tau,$$

$$\frac{dy_\tau}{dl} = (2 - K_1 - K_3)\,y_\tau - \tilde{D}_\tau,$$



where $l = \ln(\alpha/\alpha_0)$, $\tilde{\Delta}_{\tau/c} = \Delta_{\tau/c}\,\alpha/u$, $\tilde{D}_\tau = \alpha\,D_\tau/(2\pi u^2)$, and $y_\tau = g_\tau/(2\pi u)$

Thakurathi et al., Phys. Rev. B 97, 045415 (2018)

Renormalization group flow for interacting Hamiltonian

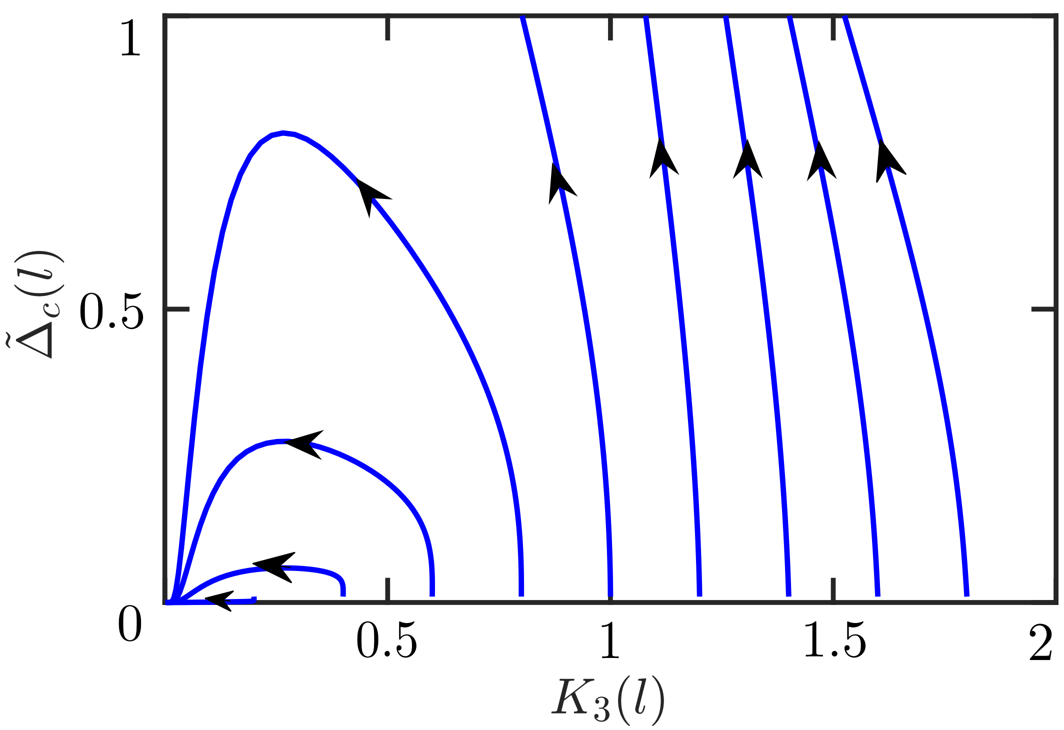
$$\frac{d\tilde{\Delta}_\tau^{int}}{dl} = (2 - K_3)\tilde{\Delta}_\tau^{int}, \quad \frac{d\tilde{\Delta}_c}{dl} = (2 - \frac{K_3}{2} - \frac{1}{2K_3})\tilde{\Delta}_c$$

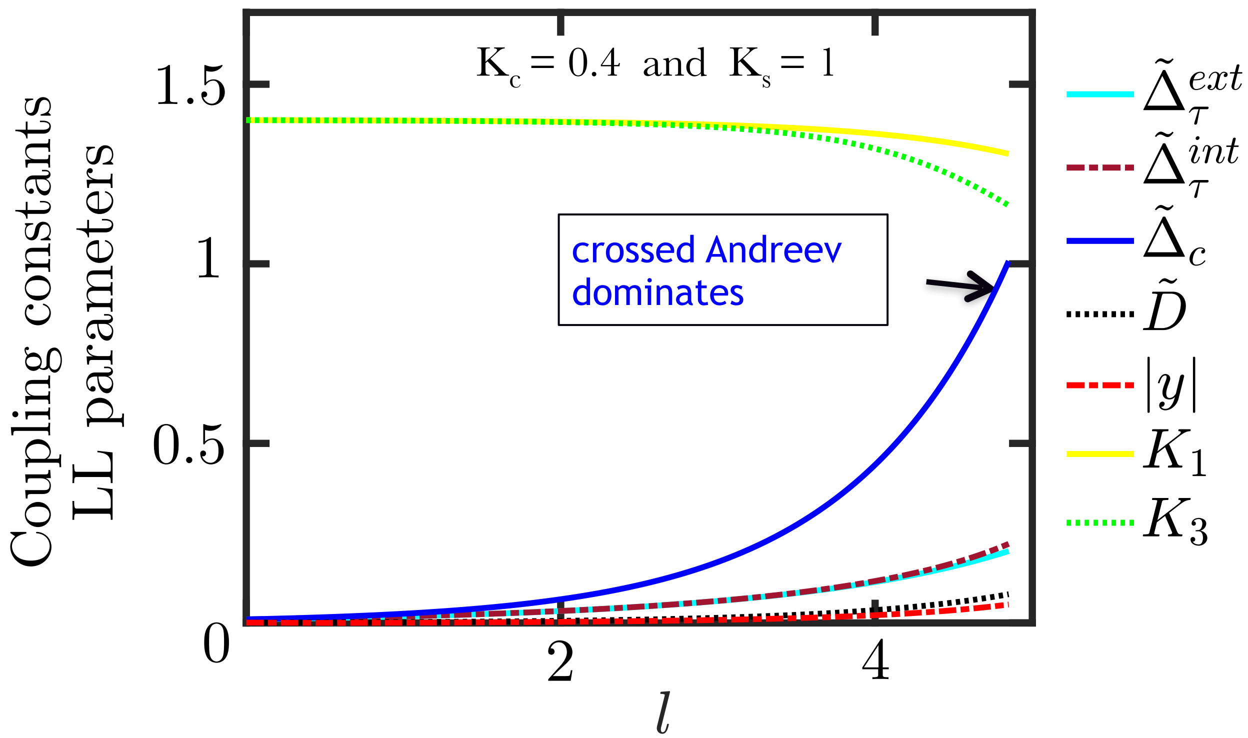
$K_c = 1$: non-interacting

$K_3 \propto \frac{K_s}{K_c}$

K_s Spin Luttinger parameter

K_c Charge Luttinger parameter





Topological Phase $K_c < K_s$

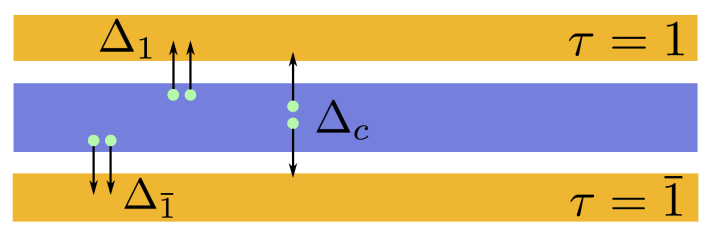
For $K_s \geq 1$, repulsive interaction with $K_c < 1$

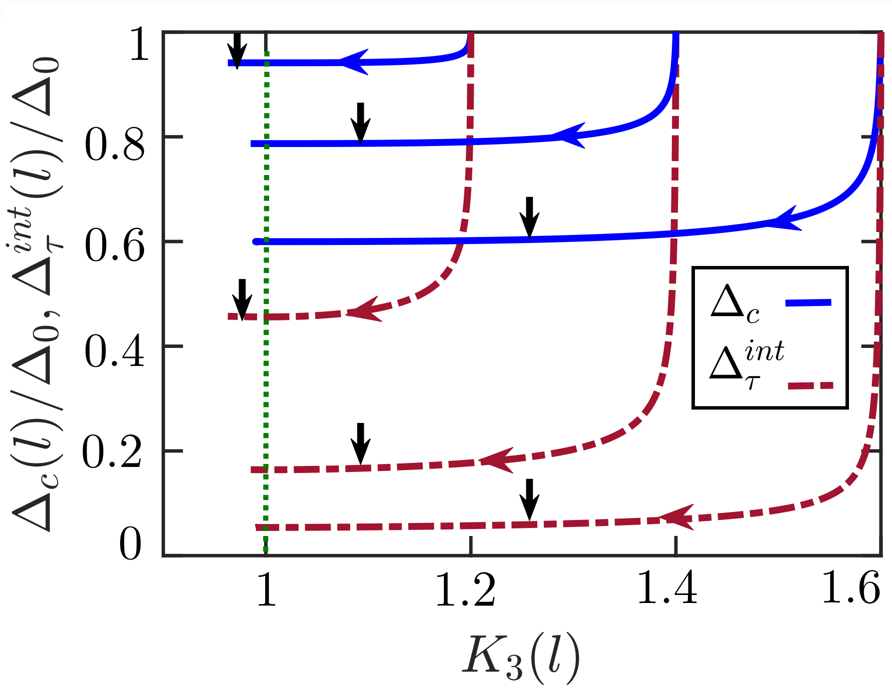
$K_c < 1$: repulsive

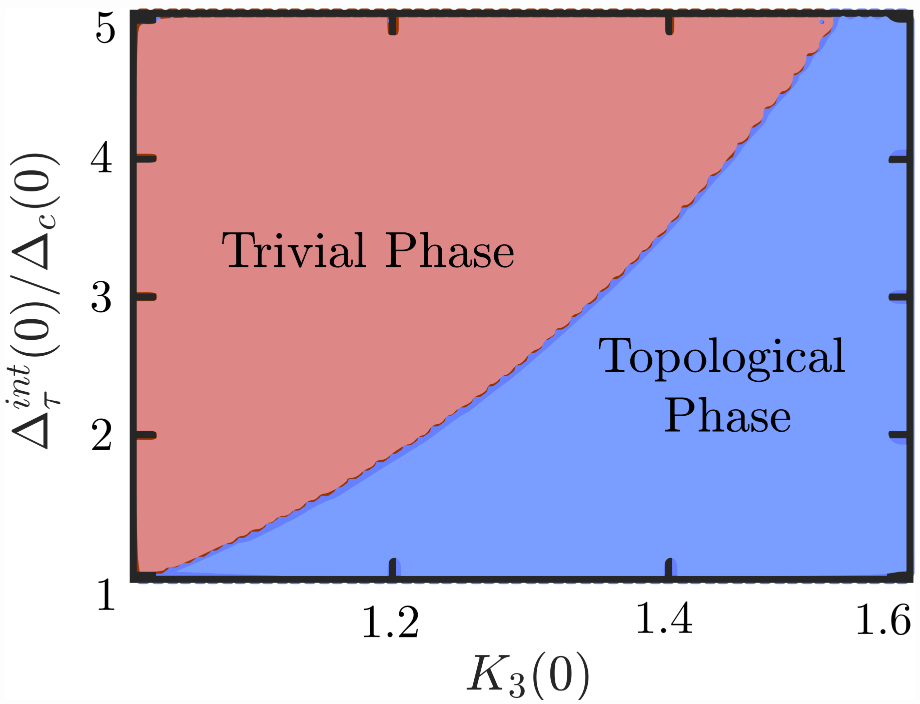
$K_c > 1$: attractive

Thakurathi et al., Phys. Rev. B 97, 045415 (2018)

The crossed Andreev pairing gets less suppressed by electron-electron interactions than the direct pairing terms





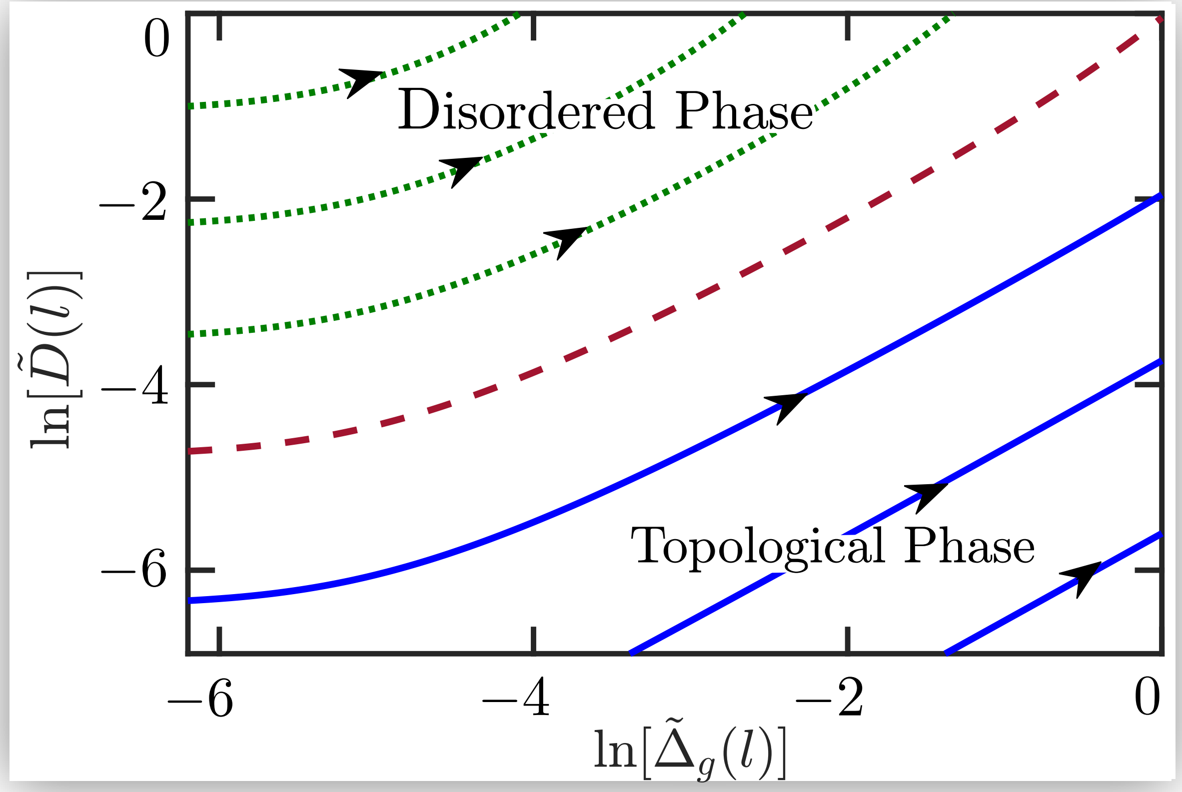


Physical gaps $\Delta_{\tau/c} = \tilde{\Delta}_{\tau/c} u/\alpha$ $\quad$ $\Delta_0 = 0.01u/a_0$

Phase diagram for different initial values of pairing amplitudes and LL parameters $\tilde{D}(0) = y(0) = 0.001$

Thakurathi et al., Phys. Rev. B 97, 045415 (2018)

Stability against disorder



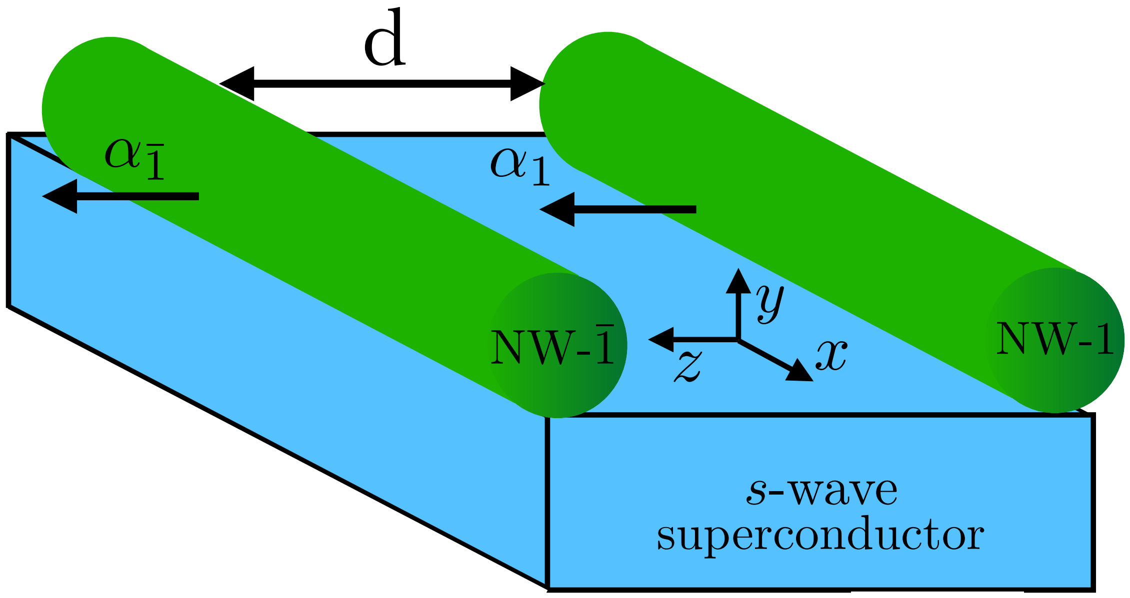
$$\tilde{\Delta}_g = \sqrt{\tilde{\Delta}_c^2 - \tilde{\Delta}_1^{int}\tilde{\Delta}_{\bar{1}}^{int}}$$

$$\tilde{\Delta}_\tau^{int}(0) = 0.01, y(0) = 0.001, \text{ and } K_3(0) = 1.4$$

Thakurathi et al., Phys. Rev. B 97, 045415 (2018)

RG equations for tunneling Hamiltonian

In the previous part, we included intra- and interwire superconducting pairing terms in the Hamiltonian as model parameters



- More microscopic approach: Begin with the tunneling Hamiltonian between the superconductor and NWs

- Model the coupling between the three-dimensional bulk s-wave SC and the NWs by the following tunneling Hamiltonian,

$$H_T = \sum_\tau \int dx\, d\mathbf{r}\, \{[t'_{ext,\tau}(x,\mathbf{r})\, e^{-i\,k_{F\tau}\,x}\, R^\dagger_{\tau 1}(x) + t'_{int,\tau}(x,\mathbf{r})\, L^\dagger_{\tau 1}(x)]\, \Psi_\uparrow(\mathbf{r})$$
$$+[t'_{int,\tau}(x,\mathbf{r})\, R^\dagger_{\tau \bar{1}}(x) + t'_{ext,\tau}(x,\mathbf{r}) e^{i\,k_{F\tau}\,x}\, L^\dagger_{\tau\bar{1}}(x)]\, \Psi_\downarrow(\mathbf{r}) + \text{H.c.}\}$$

- $t'_{int/ext,\tau}(x,\mathbf{r}) = t_{int/ext}\,\delta(r_x - x)\,\delta(r_y - d_\tau)\,\delta(r_z)$ and $\Psi_\sigma(\mathbf{r})$ is an annihilation operator acting on electrons with spin σ located at point $\mathbf{r}$ of the SC

RG equations for tunneling Hamiltonian

- $\tilde{t}_{int/ext} = t_{int/ext}\sqrt{\frac{\alpha}{\xi^2\,L}} \times \frac{\alpha}{u}$

$$\frac{d\tilde{t}_{int}}{dl} = \left[2 - \frac{1}{4}\left(K_3 + \frac{1}{K_3}\right)\right]\tilde{t}_{int}\,, \quad \frac{d\tilde{t}_{ext}}{dl} = \left[2 - \frac{1}{4}\left(K_1 + \frac{1}{K_1}\right)\right]\tilde{t}_{ext}\,,$$

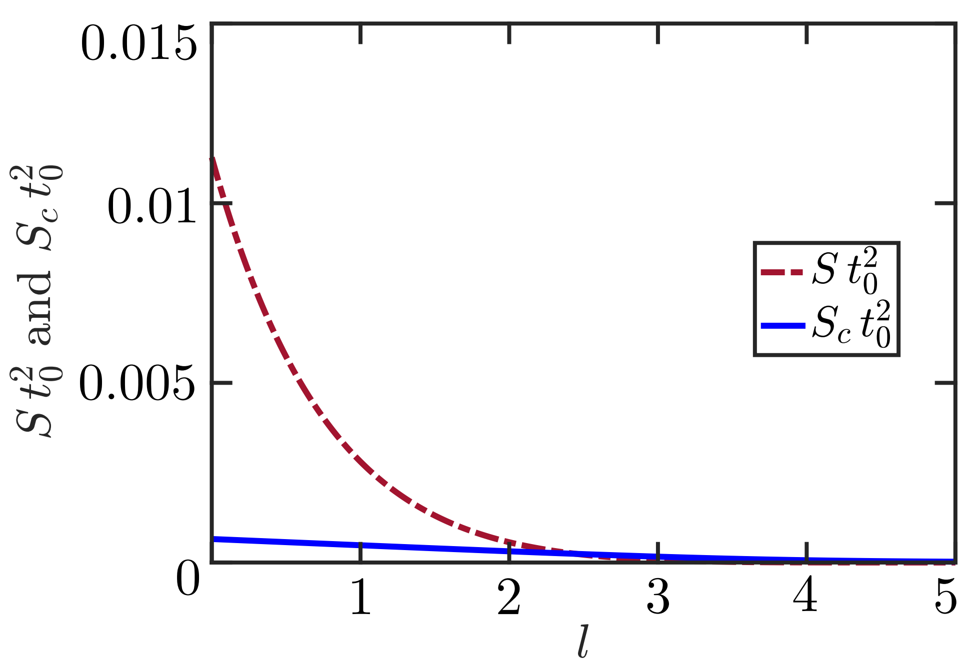
$$\frac{d\tilde{\Delta}_\tau^{ext}}{dl} = \left[2 - K_1\right]\tilde{\Delta}_\tau^{ext} + S\,\tilde{t}_{ext}^2\,, \quad \frac{d\tilde{\Delta}_\tau^{int}}{dl} = \left[2 - K_3\right]\tilde{\Delta}_\tau^{int} + S\,\tilde{t}_{int}^2\,,$$

$$\frac{d\tilde{\Delta}_c}{dl} = \left[2 - \frac{1}{2}\left(K_3 + \frac{1}{K_3}\right)\right]\tilde{\Delta}_c + S_c\,\tilde{t}_{int}^2\,,$$

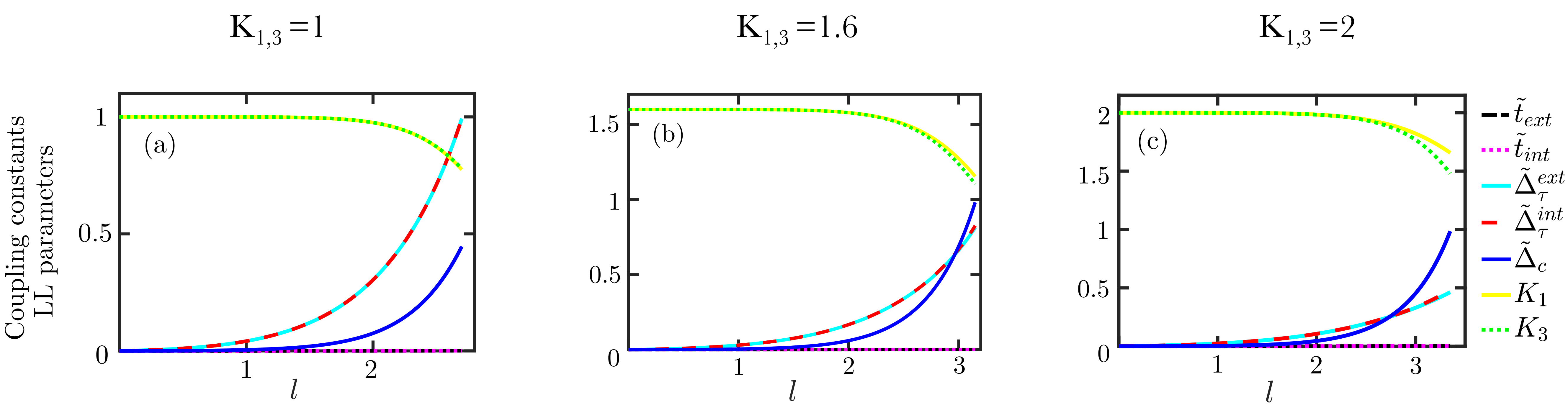
$$\frac{dK_1}{dl} = -\left(\tilde{\Delta}_\tau^{ext}\right)^2 K_1^2\,, \quad \frac{dK_3}{dl} = -\left(\tilde{\Delta}_\tau^{int}\right)^2 K_3^2 + \frac{\tilde{\Delta}_c^2\,(1-K_3^2)}{2}$$

- Source terms $S = \frac{m_e\,v_{F,sc}^2\,L}{2\,\pi\,\Delta\,\alpha} K_0\left(\frac{\alpha\Delta}{\hbar\,u}\right)$,

$$S_c = \frac{m_e\,v_{F,sc}^2\,L\,|\sin(k_{F,sc}\,d)|\,e^{-\frac{d}{\xi}}}{\pi^2\,d\,\Delta} \times \int_0^{\pi/2} d\theta'\,K_0\left(\frac{|\alpha\,\sin(\theta')|\Delta}{\hbar\,u}\right)$$



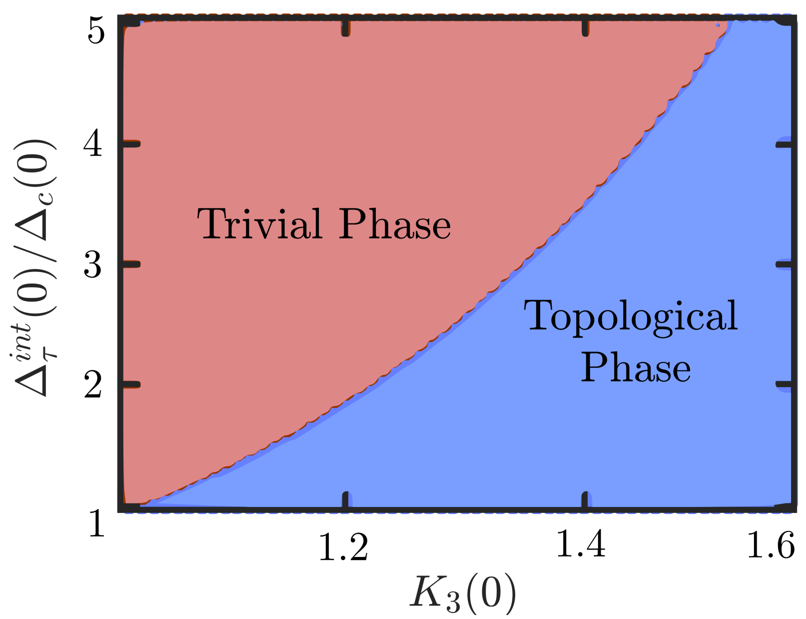
RG flow for tunneling Hamiltonian



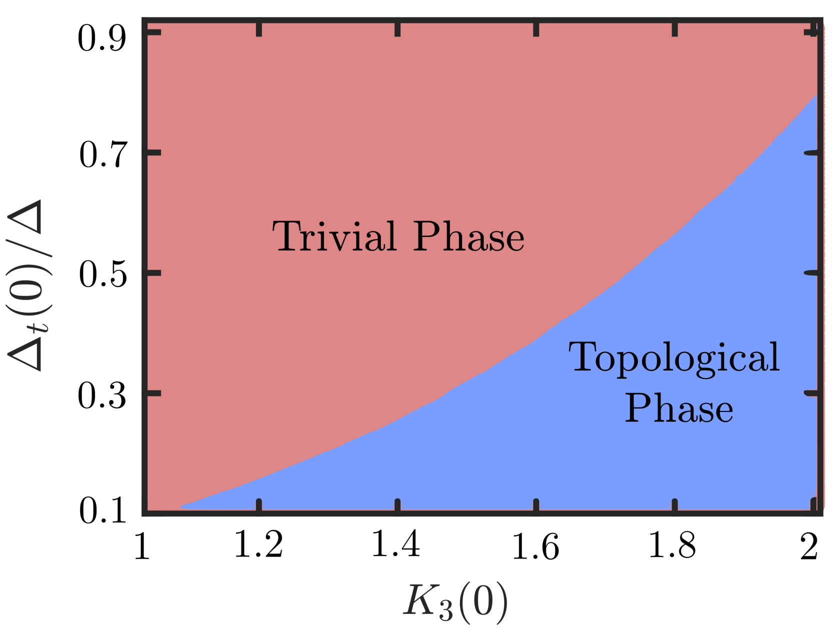
$\Delta = 0.35$ meV, $u = 10^4$ m/s, $v_{F,sc} = 10^6$ m/s, $\alpha_0 = 1\,\text{nm}$, $d = 15\,\alpha_0$, $L = 1\,\mu\text{m}$, $\alpha_{\text{sc}} = 1/k_{F,sc} = 1$ Å. We use the initial conditions: $\tilde{t}_{int,ext}(0) = 3.8 \times 10^{-5}$ and $\tilde{\Delta}_\tau^{int,ext}(0) = \tilde{\Delta}_c(0) = 0$.

Comparison between two methods

From Effective Hamiltonian



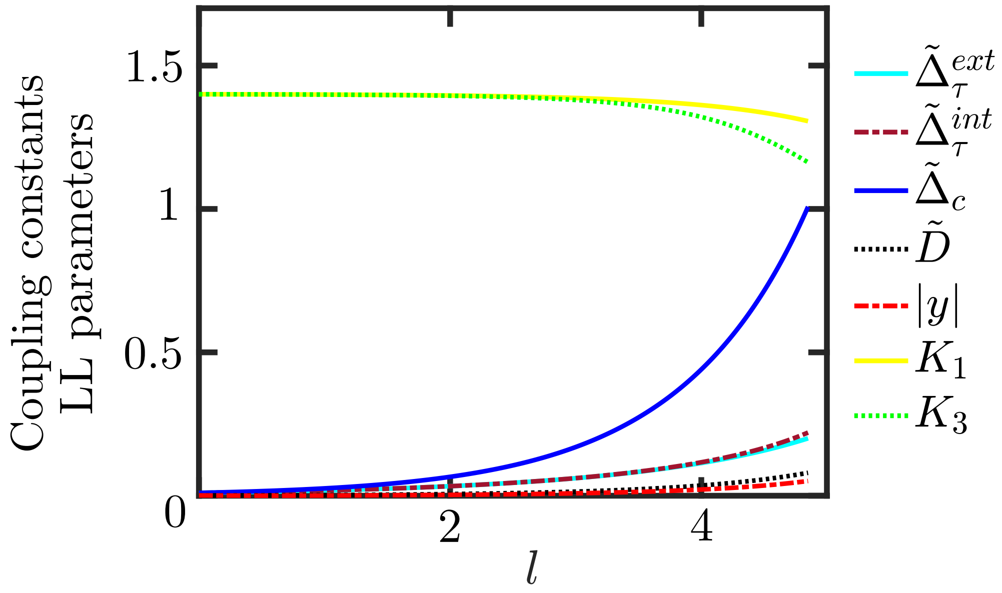
From Tunneling Hamiltonian



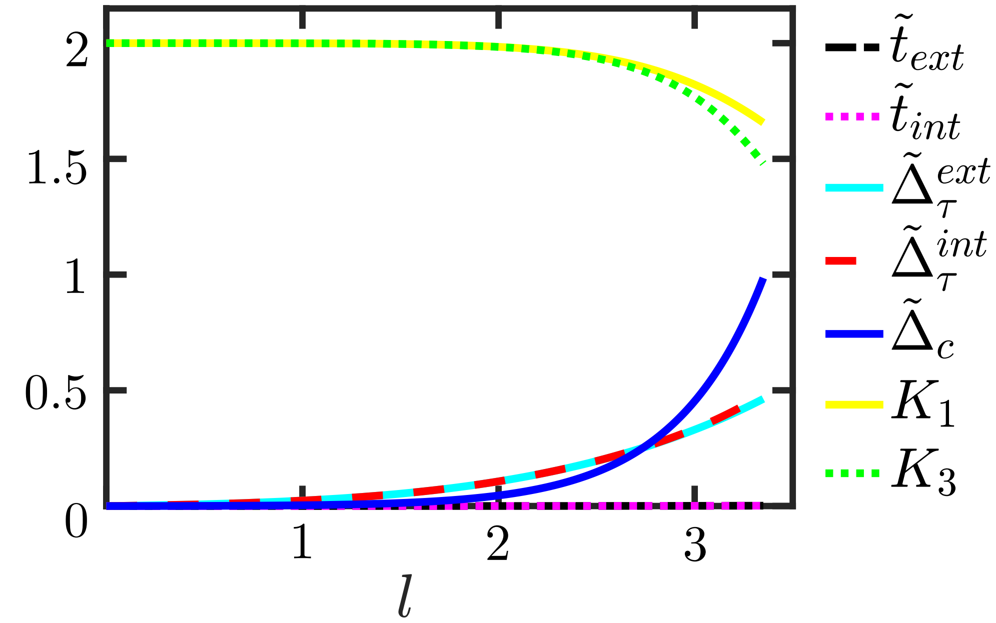
$$\Delta_t = S(0)\,\tilde{t}^2_{int/ext}(0)\,\frac{\hbar u}{\alpha_0}$$

Comparison between two methods

From Effective Hamiltonian



From Tunneling Hamiltonian



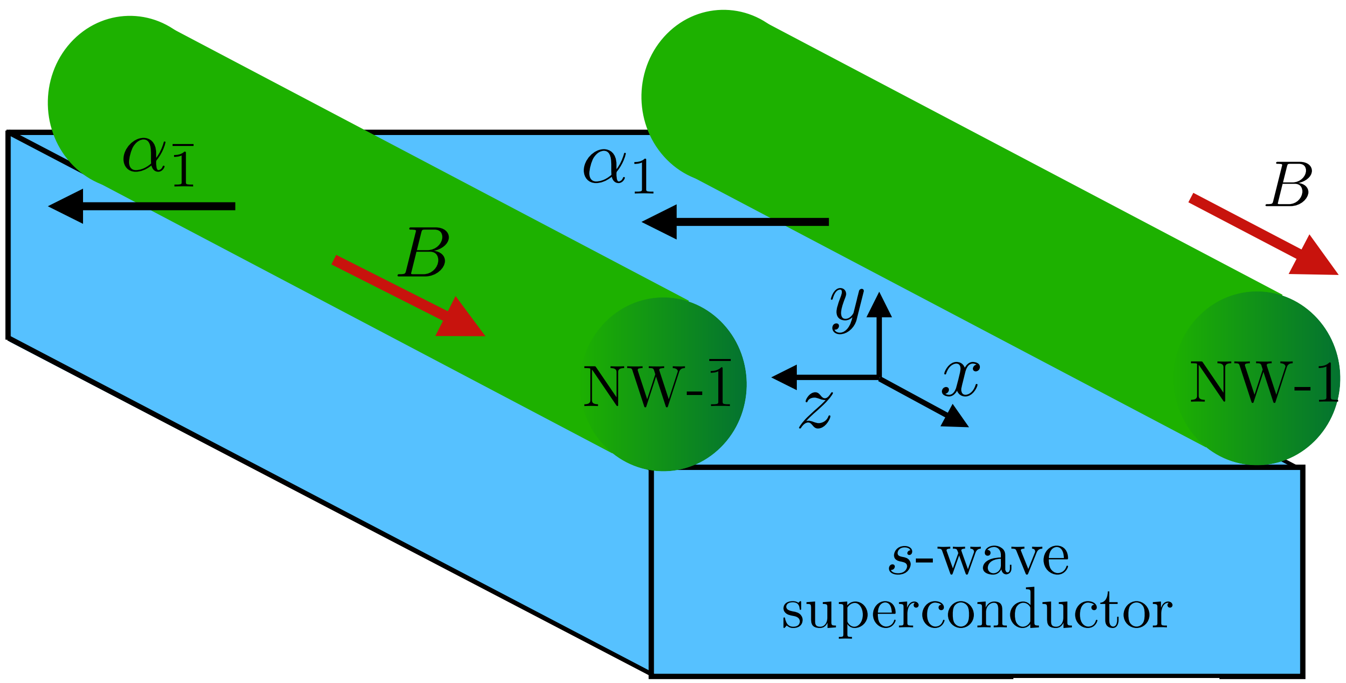
Majorana Bound states

(b) with weak magnetic field?

Magnetic Field

Supercurrent

Low-field topological threshold in Majorana double nanowires



Hamiltonian:

$$H_0 = \sum_{\tau,\sigma} \int \mathrm{d}x \; \Psi^\dagger_{\tau\sigma} \left(-\frac{\hbar^2 \partial_x^2}{2m} - \mu_\tau \right) \Psi_{\tau\sigma'}$$

$$H_{so} = i \sum_{\tau,\sigma,\sigma'} \alpha_\tau \int \mathrm{d}x \; \Psi^\dagger_{\tau\sigma} \, (\sigma_z)_{\sigma\sigma'} \, \partial_x \Psi_{\tau\sigma'}$$

$$H_Z = \sum_{\tau,\sigma,\sigma'} \Delta_{Z\tau} \int \mathrm{d}x \; \Psi^\dagger_{\tau\sigma} (\sigma_x)_{\sigma\sigma'} \Psi_{\tau\sigma}$$

$$H_d = \sum_{\tau,\sigma,\sigma'} \frac{\Delta_\tau}{2} \int \mathrm{d}x \, [\Psi_{\tau\sigma} (i\sigma_y)_{\sigma\sigma'} \Psi_{\tau\sigma'} + \mathrm{H.c.}]$$

$$H_c = \frac{\Delta_c}{2} \sum_{\tau,\sigma,\sigma'} \int \mathrm{d}x \, [\Psi_{\tau\sigma} (i\sigma_y)_{\sigma\sigma'} \Psi_{\bar{\tau}\sigma'} + \mathrm{H.c.}]$$

Energy scales: $E_{so,\tau} \gg |E_{so,1} - E_{so,\bar{1}}| \gg \Delta_{Z\tau}, \Delta_\tau, \Delta_c \gg |\Delta_{Z1} - \Delta_{Z\bar{1}}|, |\Delta_1 - \Delta_{\bar{1}}|$

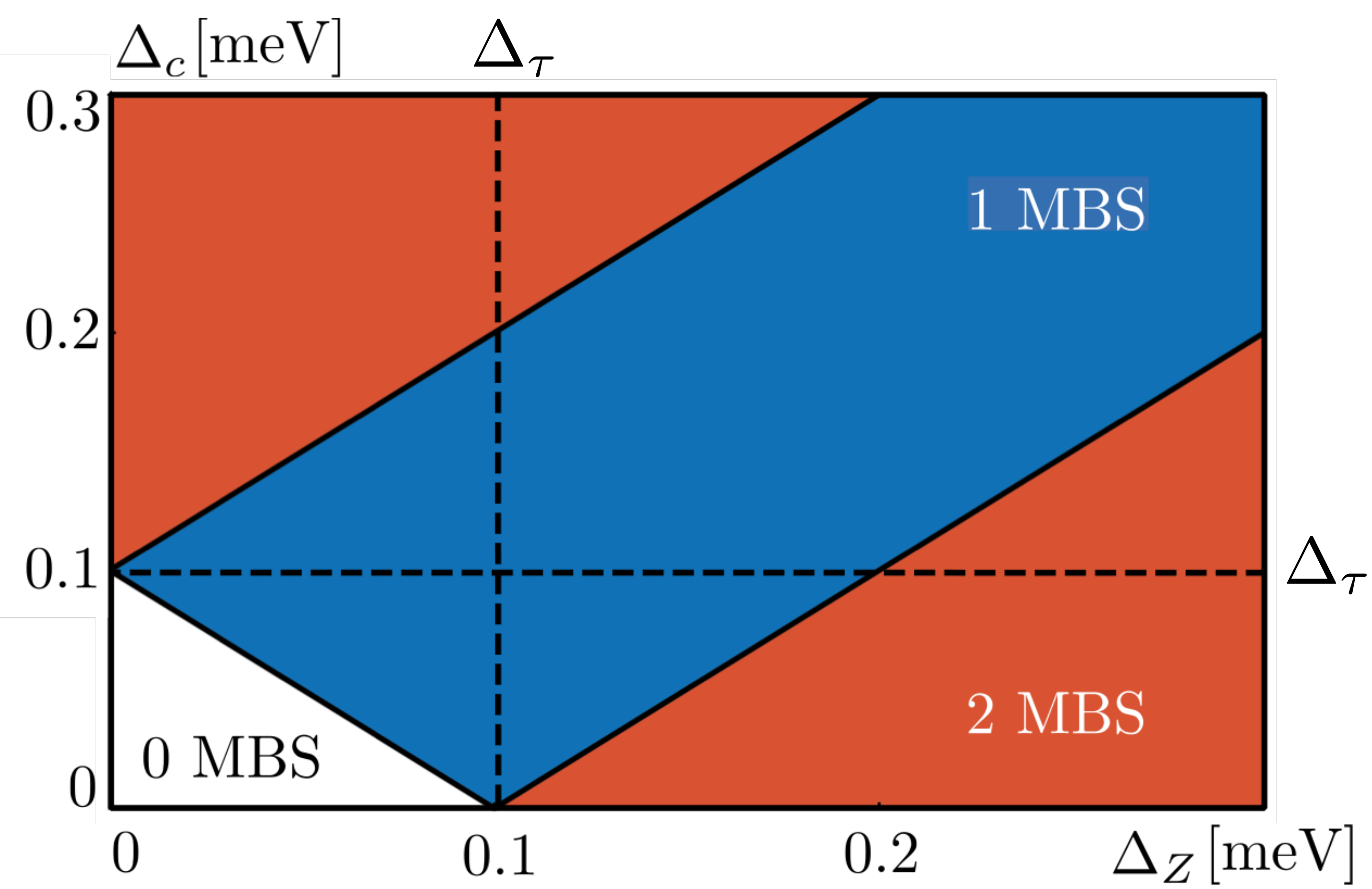
The effects of interwire tunneling can always be compensated by adjusting the chemical potentials!

Schrade, Thakurathi et al., PRB 96, 035306 (2017)

Topological phase diagram

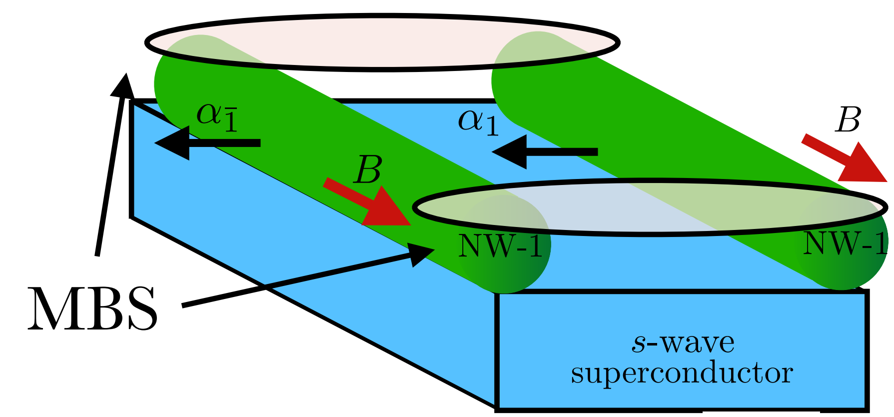
Single NW, topological phase for $\Delta_Z > \Delta_\tau$

Topological Phase Transitions: $$\Delta_c^2 = (\Delta_\tau \pm \Delta_Z)^2$$



Low-Field Topological Threshold

1 MBS phase due to crossed-Andreev pairing!



double NW, topological phase for

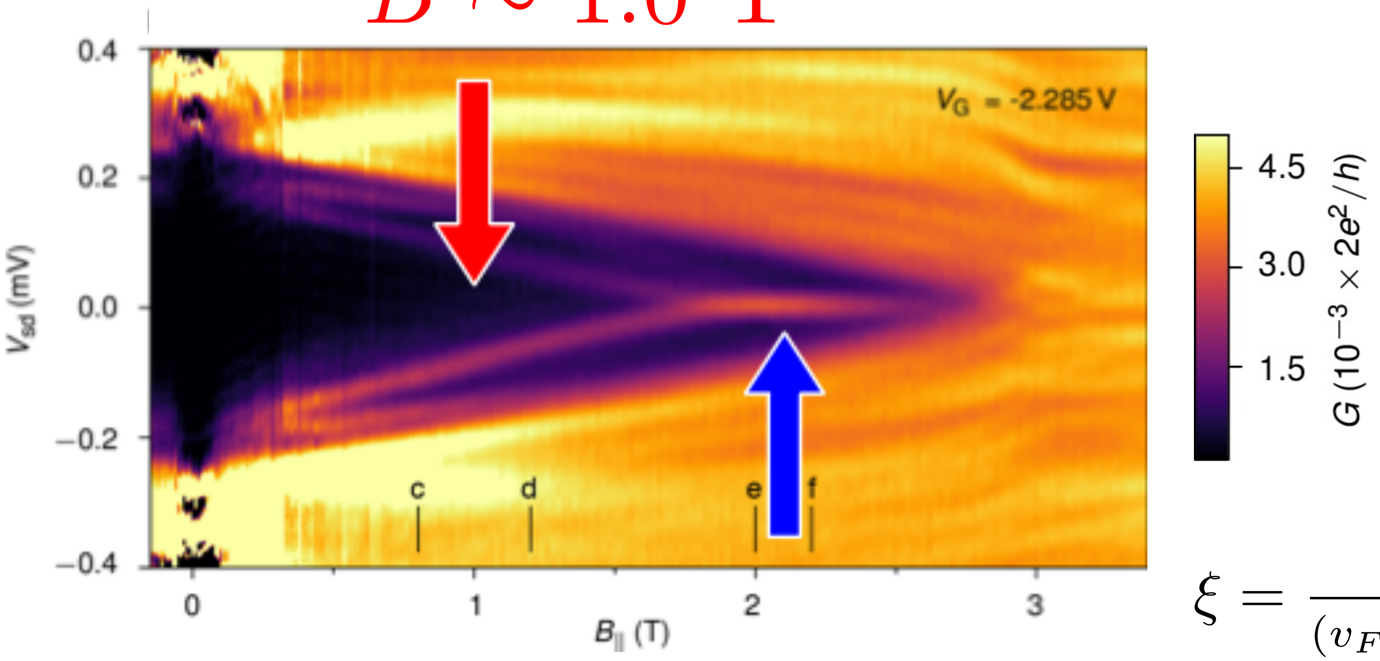
$\Delta_Z < \Delta_\tau$

Schrade, Thakurathi et al., PRB 96, 035306 (2017)

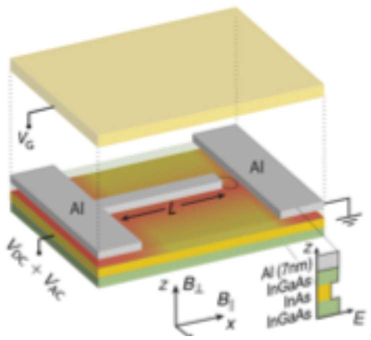
Localization length of the Majorana bound states

Suominen et al., Phys. Rev. Lett. 119, 176805 (2017)

$B \sim 1.0$ T



$B \sim 2.1$ T



Single Nanowire:

$$\xi = \frac{\hbar v_F}{\Delta_Z - \Delta_\tau}$$

Double Nanowire:

$$\xi = \frac{2\hbar v_{F1} v_{F\bar{1}}}{(v_{F1}+v_{F\bar{1}})(\Delta_Z-\Delta_\tau)+\sqrt{[(v_{F1}-v_{F\bar{1}})(\Delta_Z-\Delta_\tau)]^2+4v_{F1}v_{F\bar{1}}\Delta_c^2}}$$

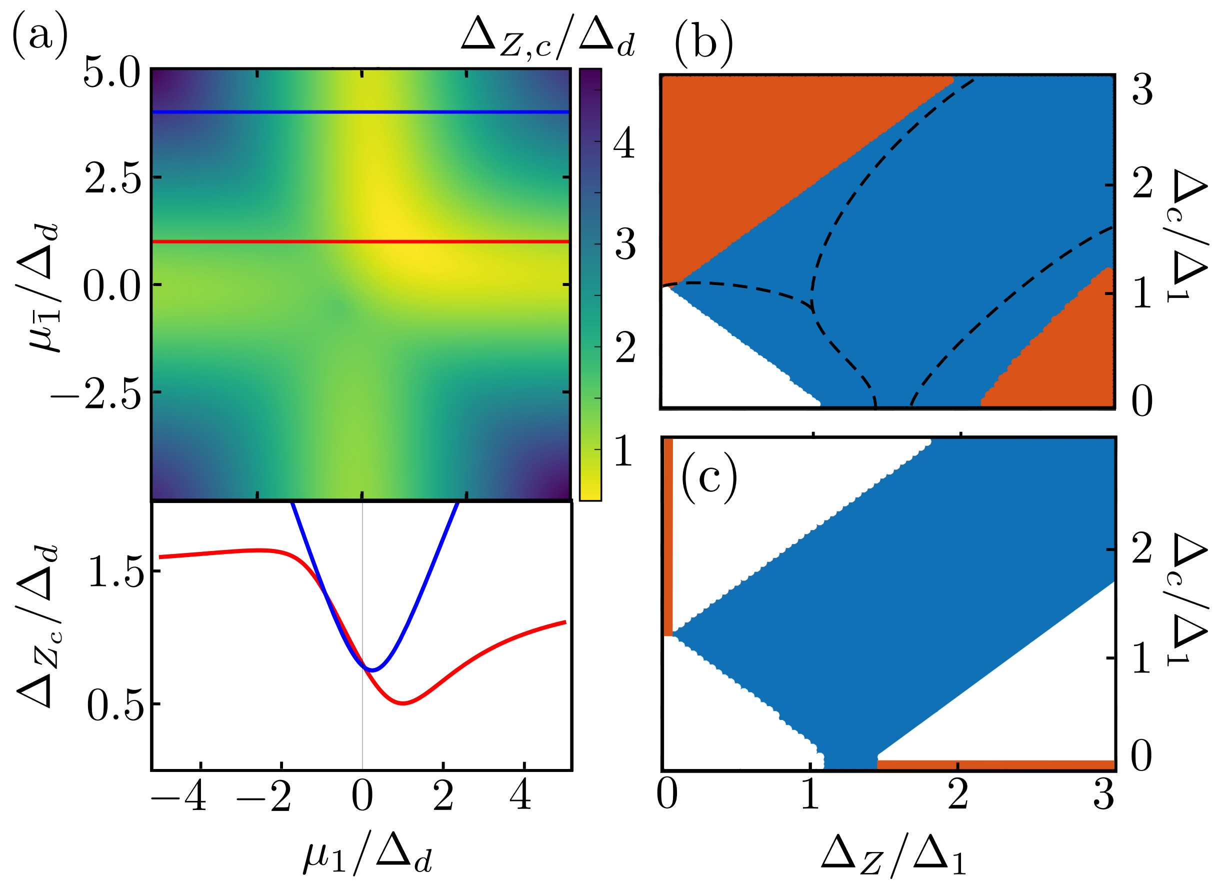
$\Delta_\tau \sim 0.2$ meV $\quad \Delta_c \sim 0.14$ meV $\quad v_{F1} = 1.5 \times 10^4$ m/s and $v_{F\bar{1}} = 2.5 \times 10^4$ m/s

Localization Length $\quad \xi \sim 170$ nm

strongly reduced magnetic field
(with $\xi \approx$ const.!)
due to crossed Andreev pairing Δ_c

Schrade, Thakurathi et al., PRB 96, 035306 (2017)

Stability analysis of one-MBS phase



(a) Topological threshold $\Delta_{Z,c}/\Delta_d$ for the one-MBS phase as a function of μ_τ/Δ_d for $\Delta_c/\Delta_d = 0.5, \Gamma/\Delta_d = 1$

(b) Topological phase diagram as a function of Δ_Z/Δ_1 and Δ_c/Δ_1 for finite interwire tunneling

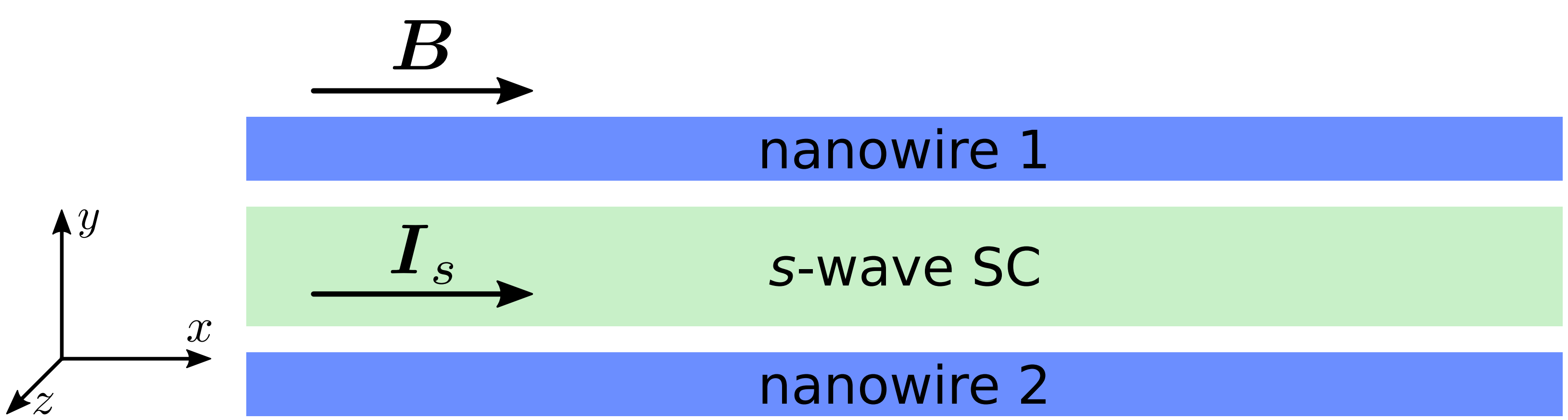
$$E_{so,1}/\Delta_1 = 6.25, E_{so,\bar{1}}/\Delta_1 = 12.25, \Delta_{\bar{1}}/\Delta_1 = 1.3, \Gamma/\Delta_1 = 1, \mu_1 = \mu_{\bar{1}} = \Gamma$$

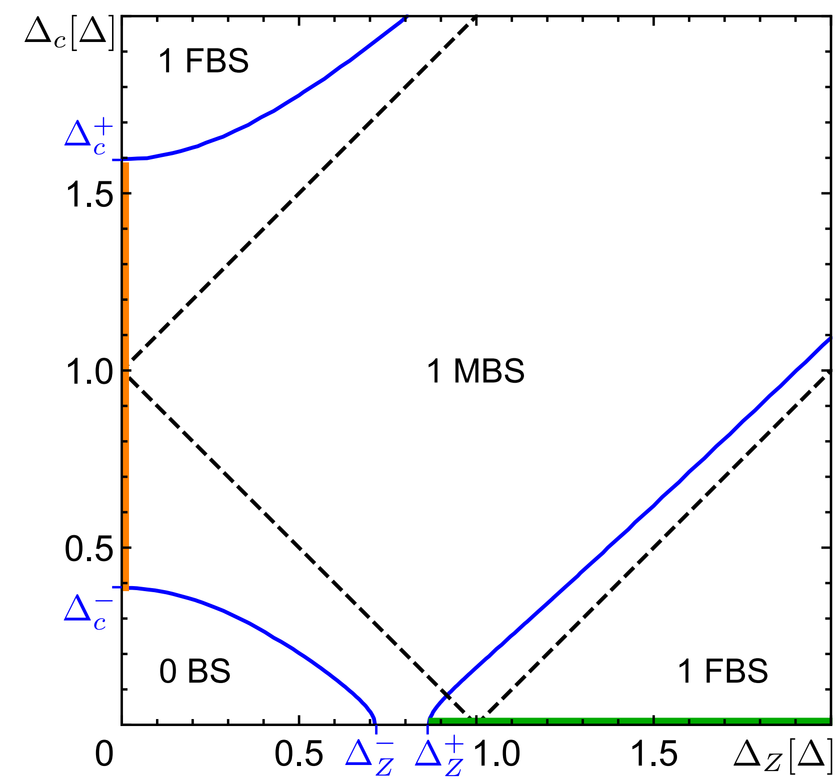
(c) Same phase diagram as in (b) but with the two SOI vectors not being parallel to each other but still orthogonal to the magnetic field

Schrade, Thakurathi et al., PRB 96, 035306 (2017)

Double-NW setup in the presence of supercurrent

For Single nanowire: Romito, Alicea et al., Phys. Rev. B 85, 020502(R) (2012)





$$\Delta_\tau(x) = \Delta_\tau e^{-i\varphi(x)}, \Delta_c(x) = \Delta_c e^{-i\varphi(x)}$$

$\varphi(x) = x/\xi$, where ξ is the characteristic length scale $\quad \delta = \hbar^2/2\,m\,\xi^2$

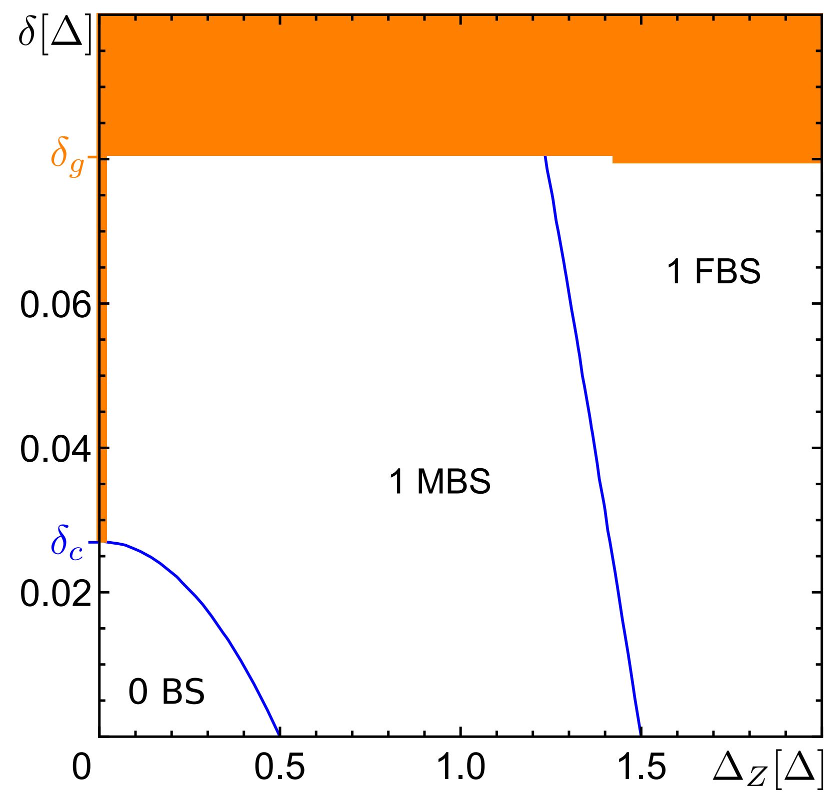
$$\Delta_c^\pm \sim \Delta \pm (\beta+1)\sqrt{E_{so,1}\delta}/2 \qquad \beta = \sqrt{\frac{E_{so,1}}{E_{so,2}}}$$

$$\Delta_Z^- \sim \Delta - \beta^2 E_{so,1}\delta/2\Delta \qquad \Delta_Z^+ \sim \Delta - E_{so,1}\delta/2\Delta$$

Low Field Topological Threshold due to Supercurrent!

Dmytruk, Thakurathi, Loss, Klinovaja, arXiv:1902.11232

Double-NW setup in the presence of supercurrent



$$\Delta_c/\Delta = 0.5$$

- Low Field Topological Threshold due to Supercurrent
- Any finite nonzero magnetic field will result topological phase in the system

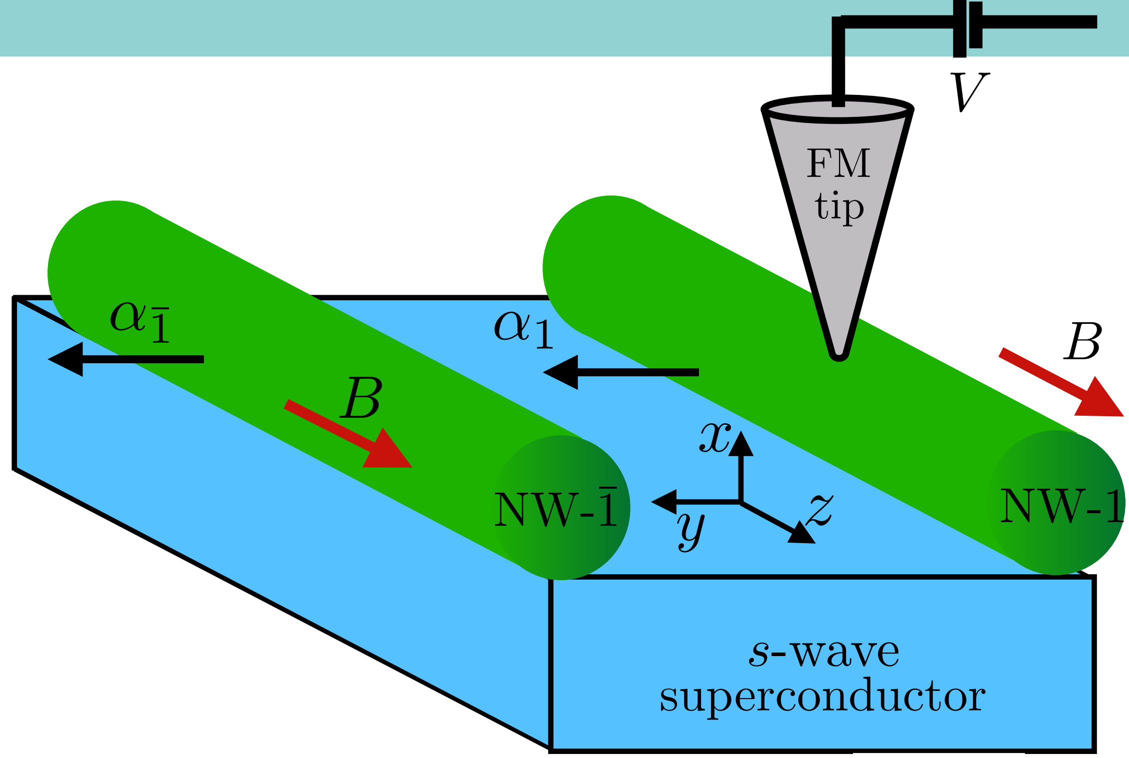
Dmytruk, Thakurathi, Loss, Klinovaja, arXiv:1902.11232

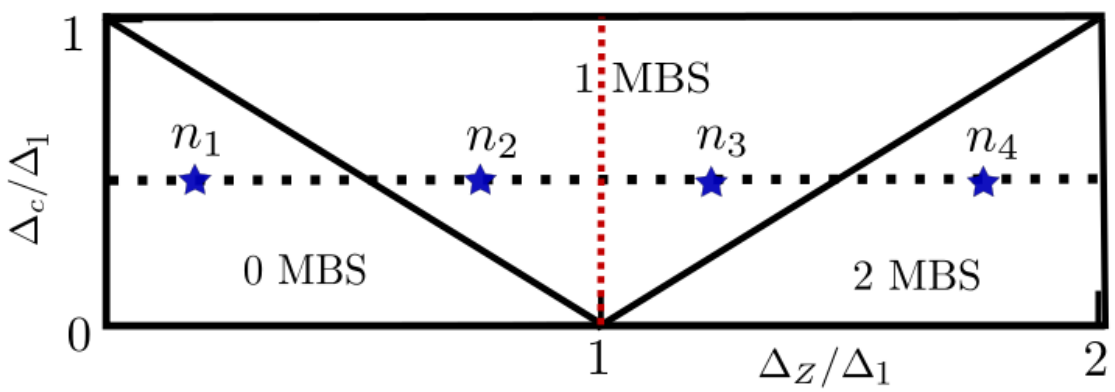
Majorana Bound states

(c) bulk features showing MBS presence

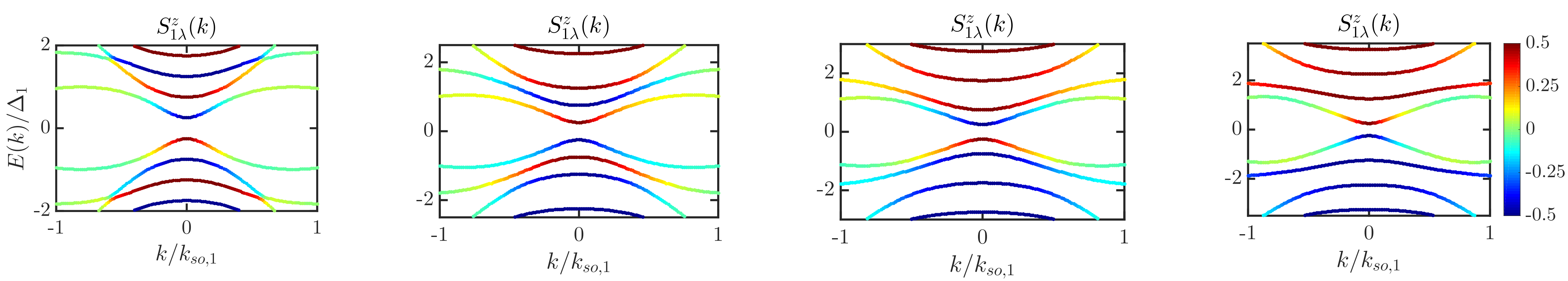
Transport

Transport Signature in Double-NW setup using a ferromagnetic STM tip



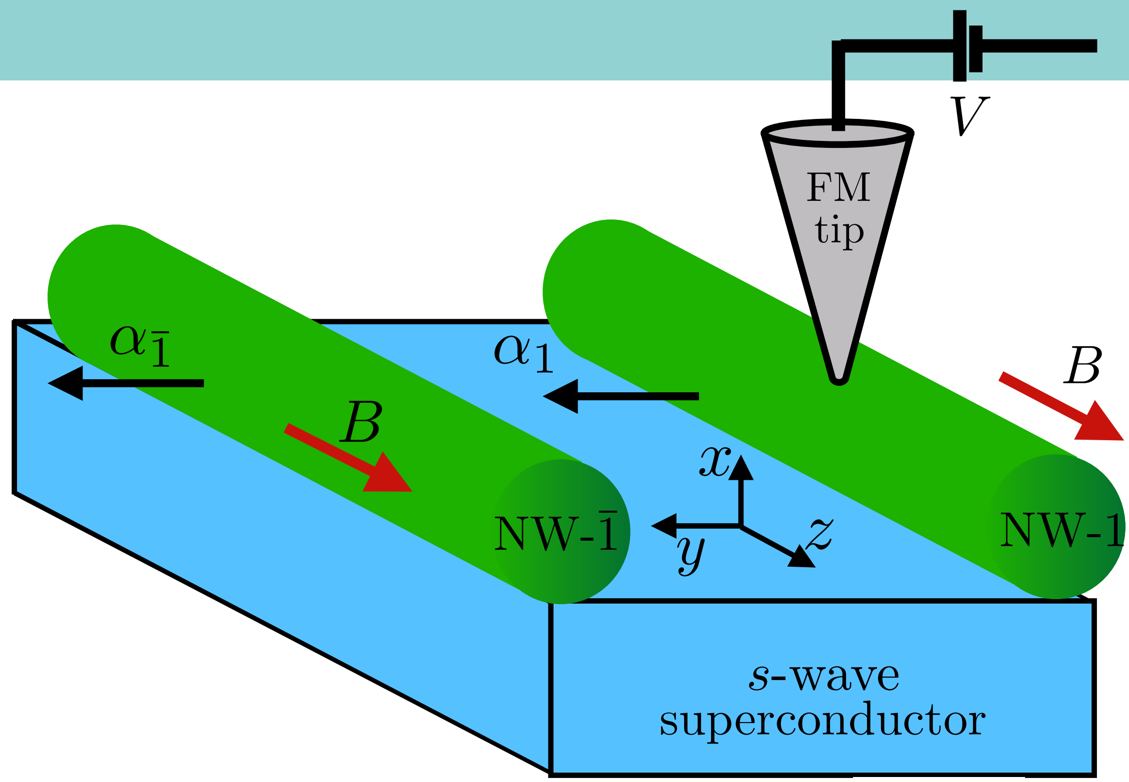


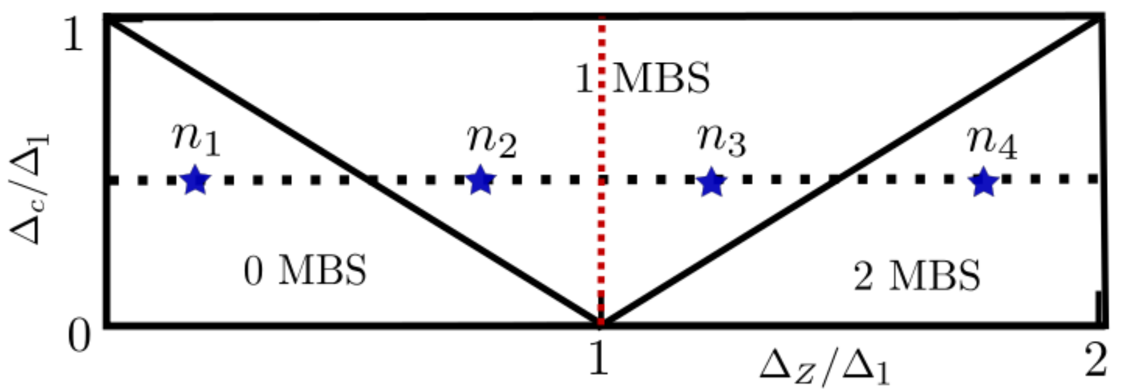
$$\bar{S}_{\eta\lambda}(k) = \Phi^{\dagger}_{\eta\lambda}(k)\,\bar{\sigma}\,\Phi_{\eta\lambda}(k)$$



Thakurathi, Chevallier, Loss, Klinovaja, arXiv soon

Transport Signature in Double-NW setup using a ferromagnetic STM tip

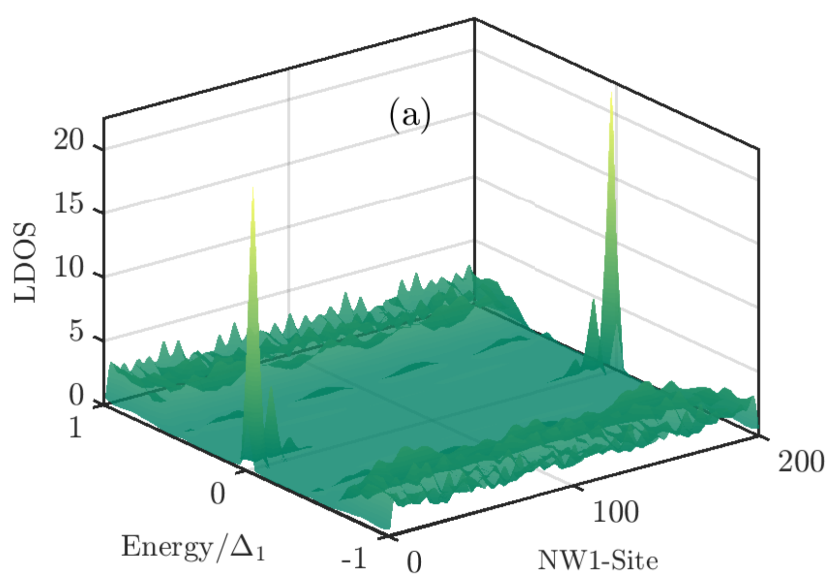


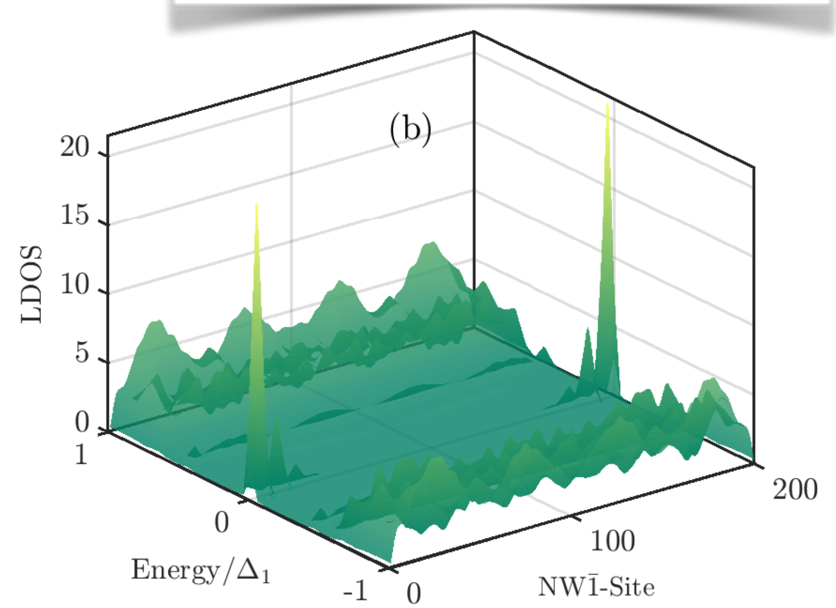


$$H_{0t} = \sum_\eta \left(\sum_{j=1}^{N} \Psi^\dagger_{\eta j}[-(\mu_\eta - 2t)\tau_z + \Delta_\eta \tau_x + \Delta_{Z\eta}\sigma_z]\Psi_{\eta j} + \sum_{j=1}^{N-1} \Psi^\dagger_{\eta j+1}(-t - i\bar{\alpha}_\eta \sigma_y)\tau_z \Psi_{\eta j} + \text{H.c.} \right) + \sum_{j=1}^{N} \Psi_{\bar{1},j}(\Delta_c \tau_x)\Psi_{1,j} + \text{H.c.}$$

$$\rho_j(\omega) = -\frac{1}{\pi}\sum_\sigma \text{Im}[G_{0R}(\omega)]_{jj,\sigma\sigma}$$

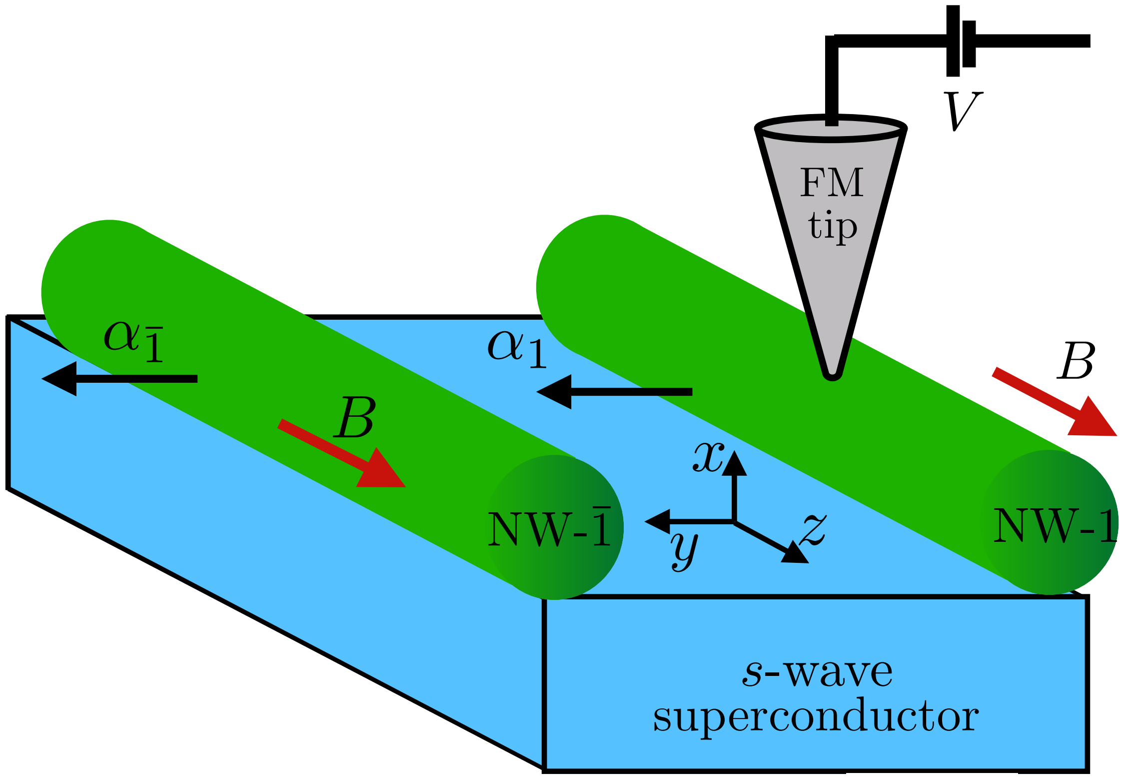
$$G^{-1}_{0R/A} = \omega \pm i\delta - H_{0t}$$

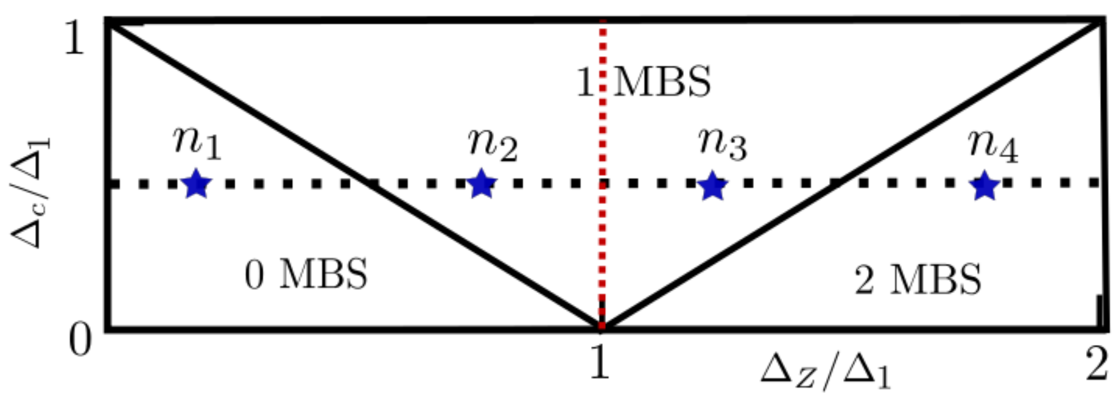




Thakurathi, Chevallier, Loss, Klinovaja, arXiv soon

Transport Signature in Double-NW setup using a ferromagnetic STM tip





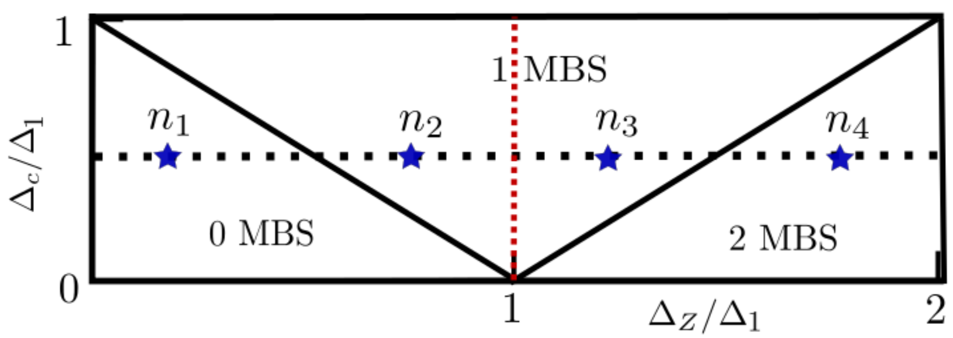
$$(G^s_{R/A})^{-1}(\omega) = G^{-1}_{0R/A} - \Sigma^s_{R/A}(\omega)$$

$$I^s_{DC} = \frac{e}{\hbar} Tr\left(\tau_z \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} Re[G^s_R(\omega)\Sigma^s_K(\omega) + G^s_K(\omega)\Sigma^s_A(\omega)]\right)$$

Spin filtered current $I_{\text{spin}} = I^+_{DC} - I^-_{DC}$

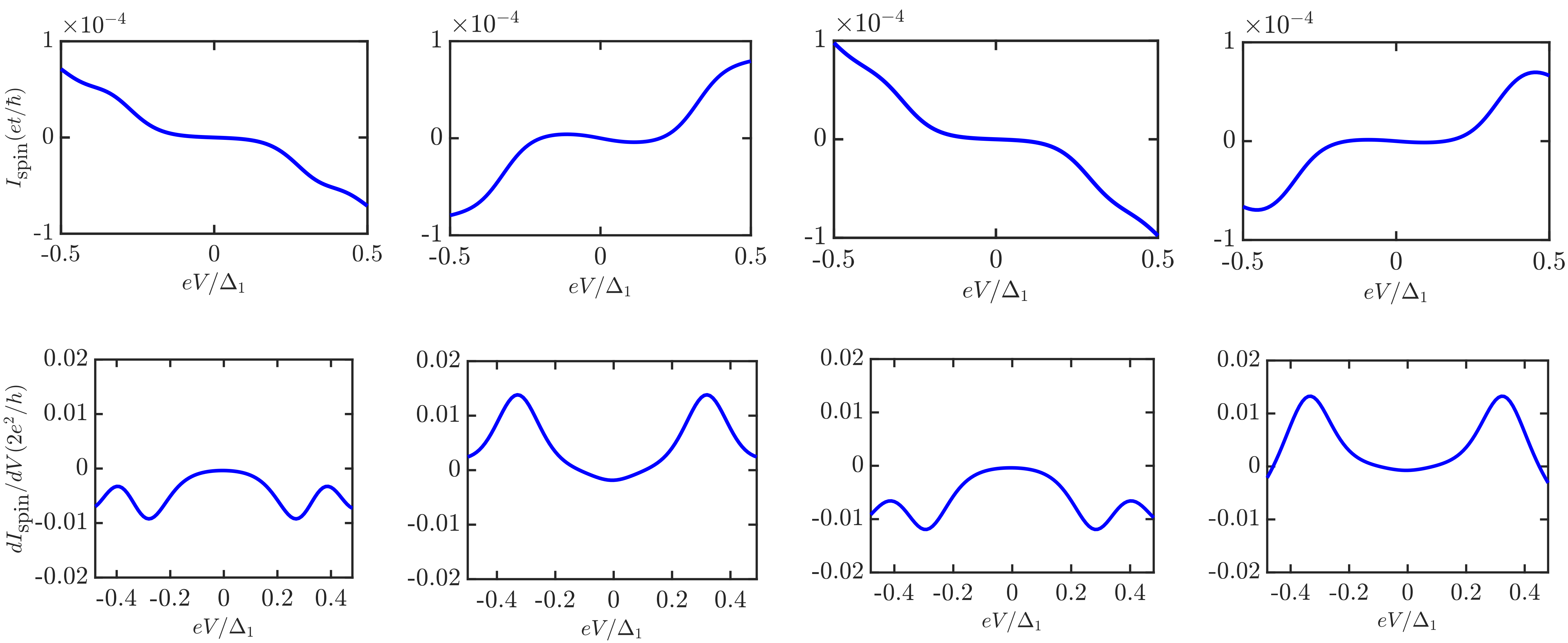
Thakurathi, Chevallier, Loss, Klinovaja, arXiv soon

Transport Signature in Double-NW setup using a ferromagnetic STM tip



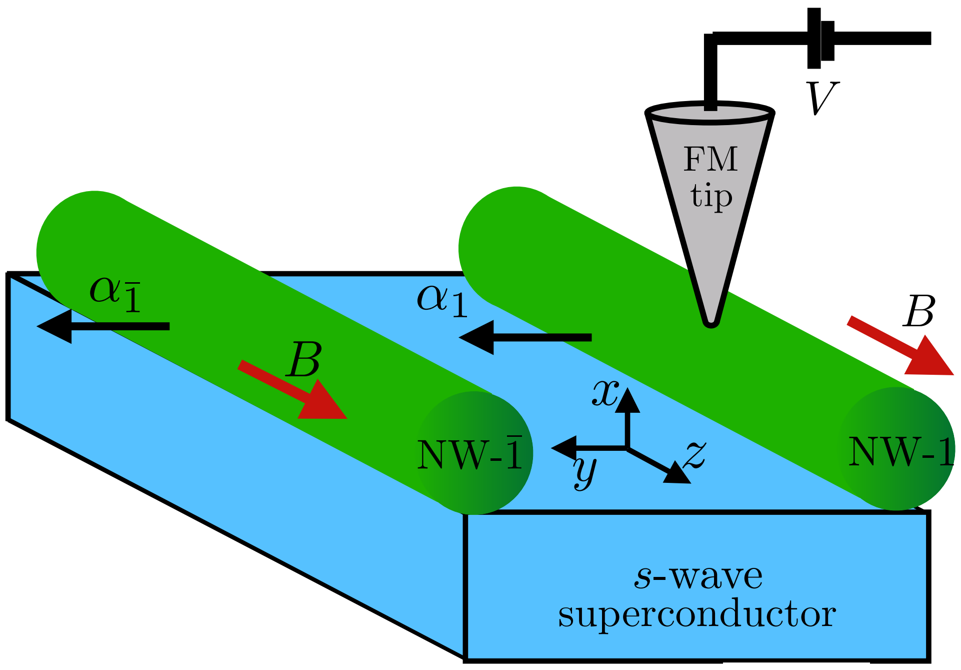
Spin filtered current $I_{\rm spin} = I^{+}_{DC} - I^{-}_{DC}$

$$I^{s}_{DC} = \frac{e}{\hbar} Tr\left(\tau_z \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} Re[G^{s}_{R}(\omega)\Sigma^{s}_{K}(\omega) + G^{s}_{K}(\omega)\Sigma^{s}_{A}(\omega)]\right)$$

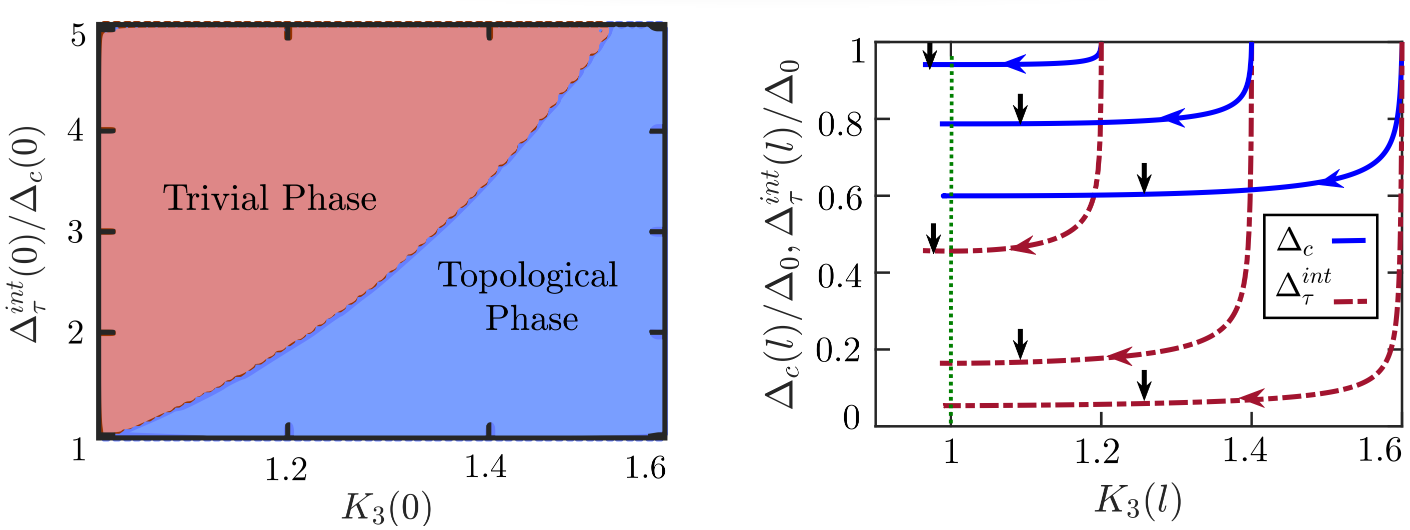


Thakurathi, Chevallier, Loss, Klinovaja, arXiv soon

Conclusions

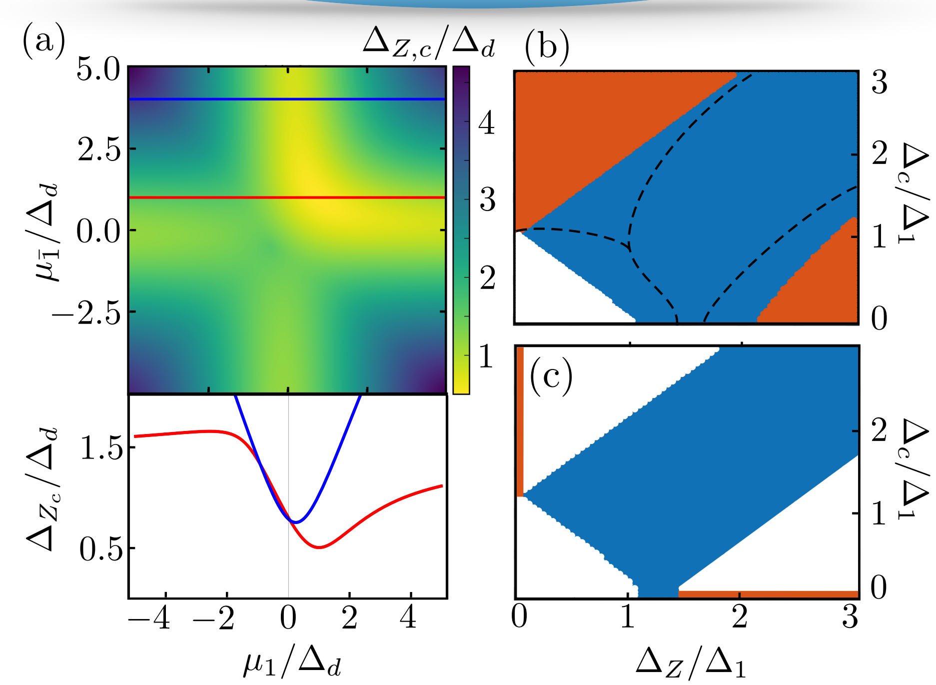


Interaction



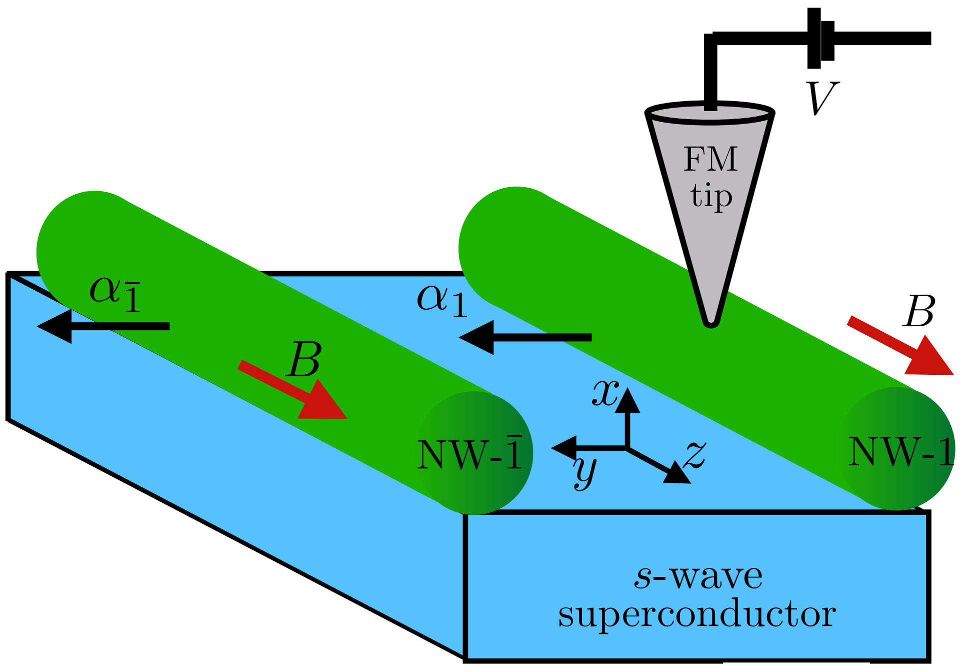
Thakurathi, Simon, Mandal, Klinovaja, Loss, PRB 97, 045415 (2018)

Magnetic Field

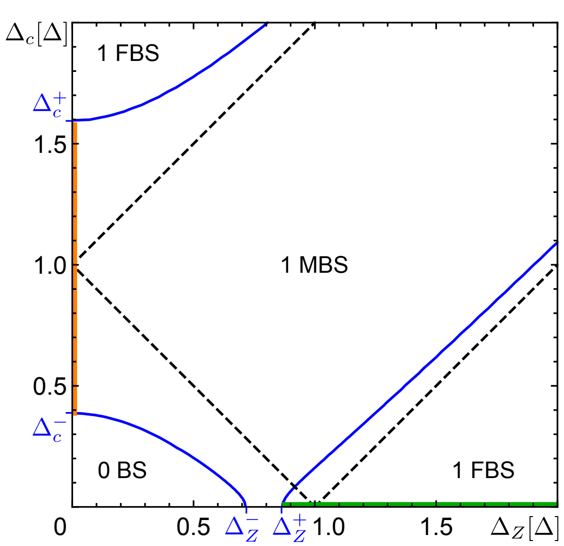


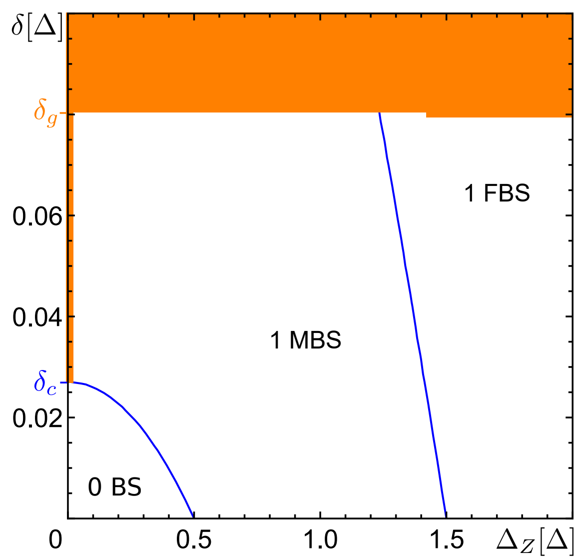
Schrade, Thakurathi, Reeg, Hoffman, Klinovaja, Loss
PRB 96, 035306 (2017)

Conclusions



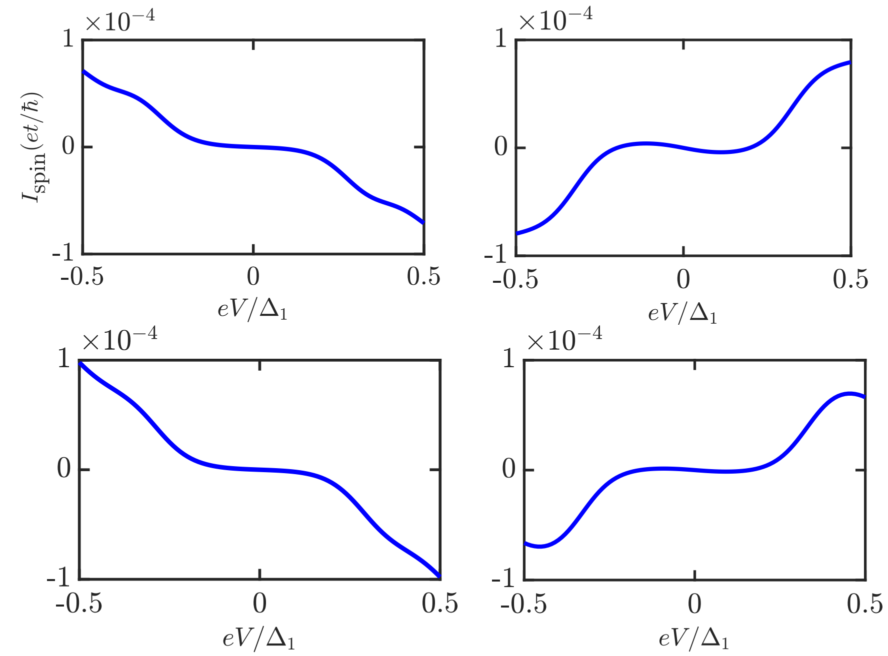
Supercurrent





Dmytruk, Thakurathi, Loss, Klinovaja, PRB 99, 245416 (2019)

Transport



Thakurathi, Chevallier, Loss, Klinovaja, arXiv soon

Outlook

- Floquet Topological systems

External driving gives us a powerful tool to turn initially non-topological materials into topological ones [Nature Physics 7, 490–495 (2011)]…

- Higher order topological systems

Corner and Hinge states [Science 357, 61 (2017).]...

- Planer Josephson junctions

[Phys. Rev. X 7, 021032 (2017)]…

Work done in collaboration with

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