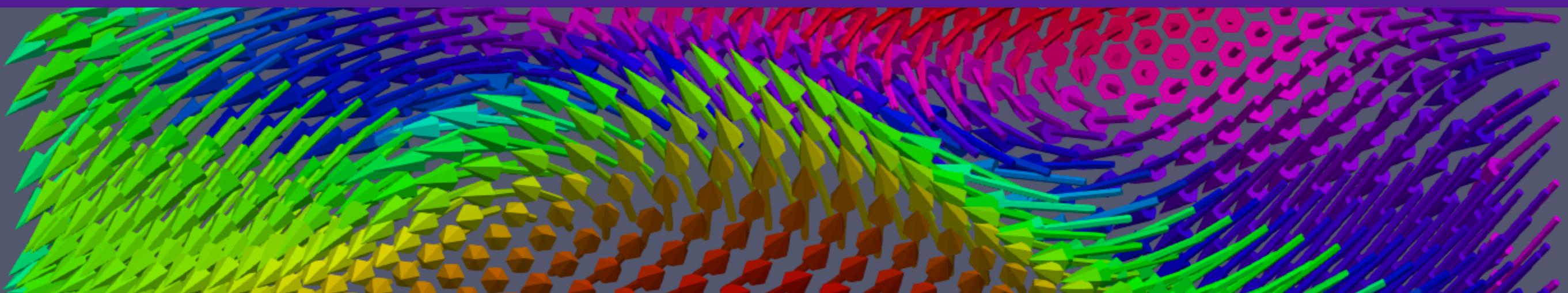


Cavity Optomagnonics with magnetic textures



Silvia Viola Kusminskiy



FRIEDRICH-ALEXANDER
UNIVERSITÄT
ERLANGEN-NÜRNBERG



MAX PLANCK INSTITUTE
for the science of light

Max Planck Research Group



MAX PLANCK INSTITUTE
for the science of light



Erlangen, Germany

Theory of hybrid quantum systems



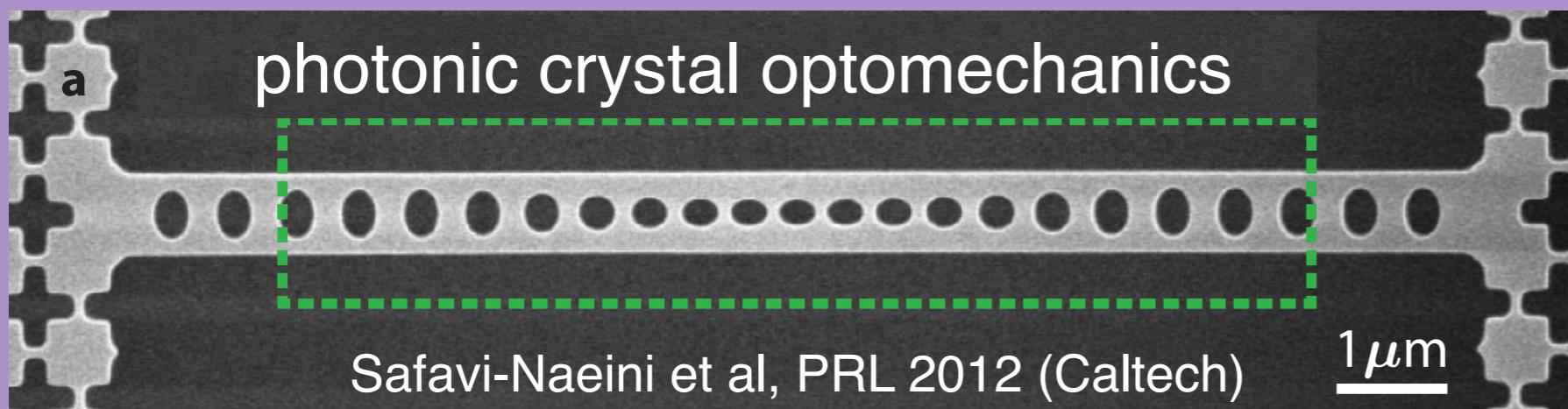
MAX PLANCK INSTITUTE
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Theory of hybrid quantum systems

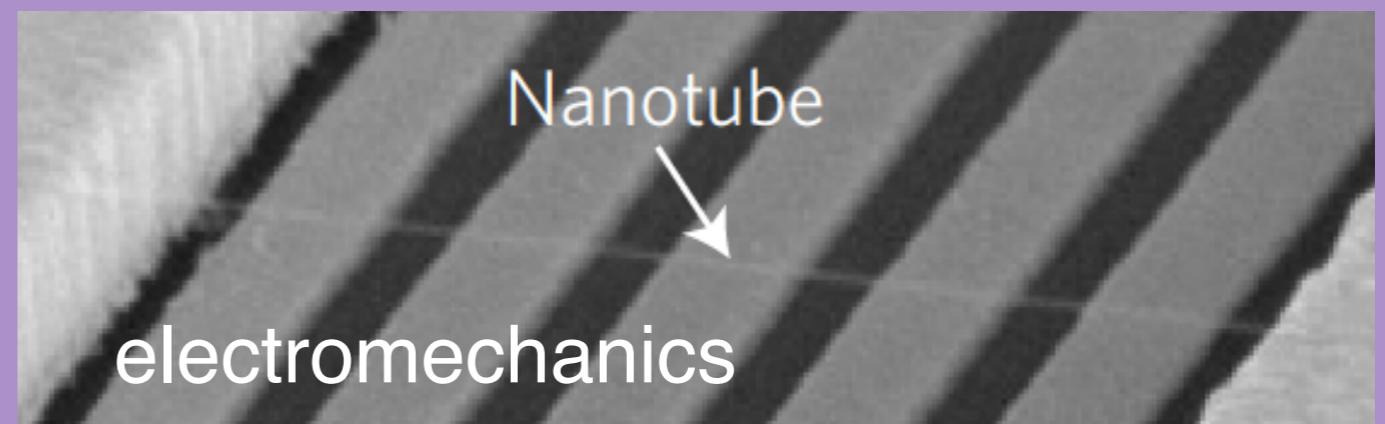
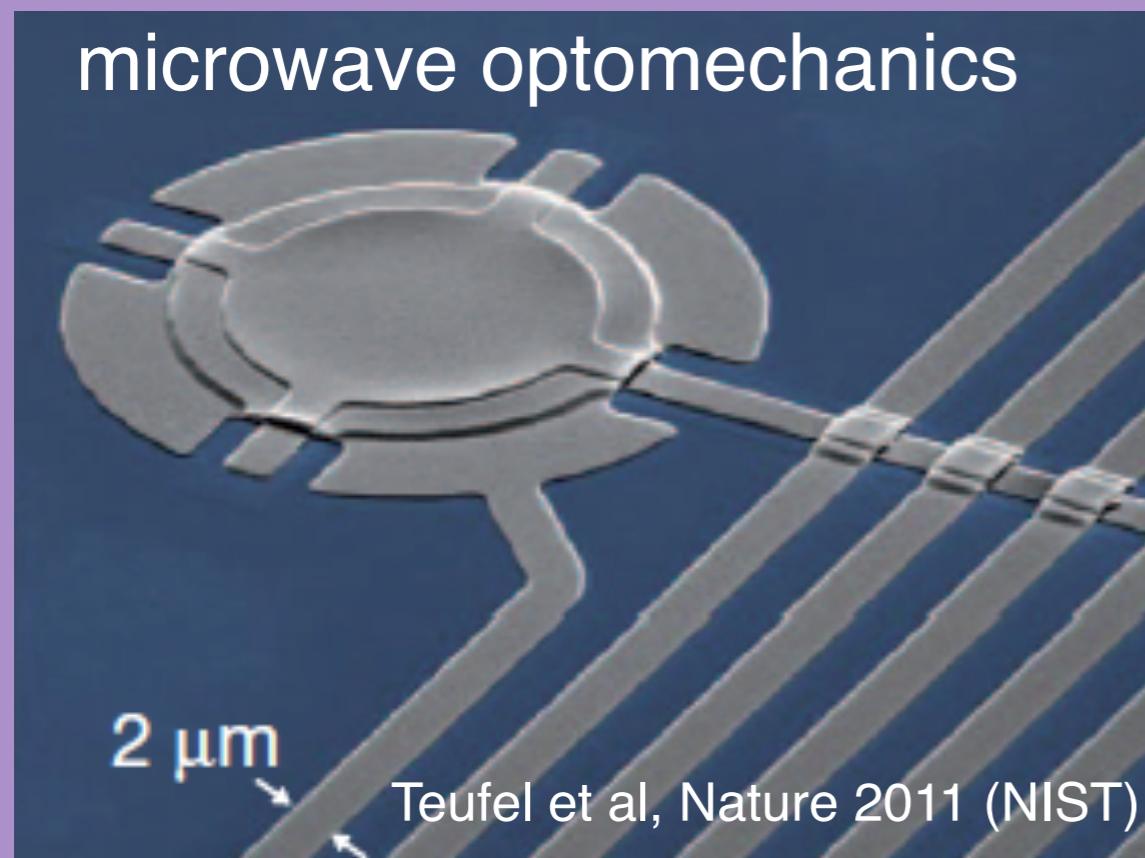


MAX PLANCK INSTITUTE
for the science of light

Hybrid Quantum Systems



mesoscopic:
nano/micro scale
systems



Benyamini et al, Nature Physics 10, 151 (2014)

optomagnonics



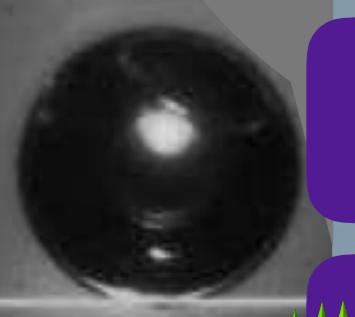
Osada et. al PRL 116, 223601 (2016)

use collective excitations

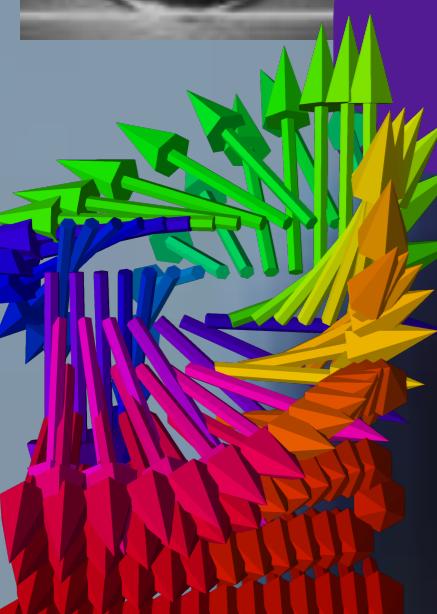
Optomagnonics



Picture from Tabuchi et al, PRL 113, 083603 (2014)



Introduction: cavity optomagnonics

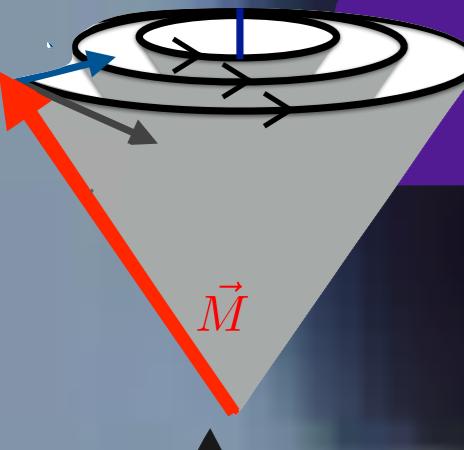


Magnetic textures

Why do they form?

Equilibrium condition

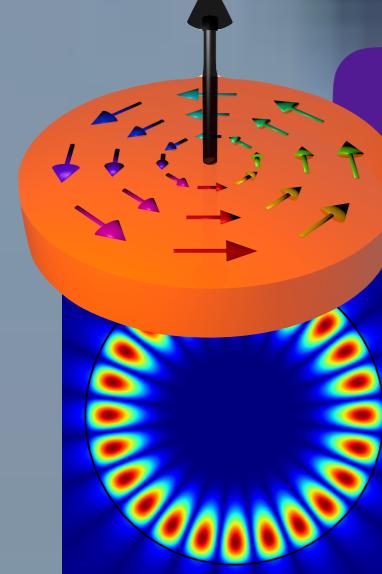
Topology of smooth textures



Dynamics of the magnetization

Landau Lifschitz Gilbert equation

Thiele equation for topological defects



Cavity optomagnonics with magnetic textures

Hamiltonian: Holstein-Primakoff

Optomagnetic coupling: gyrotropic mode

Introduction: cavity optomagnonics

Magnetic textures

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Dynamics of the magnetization

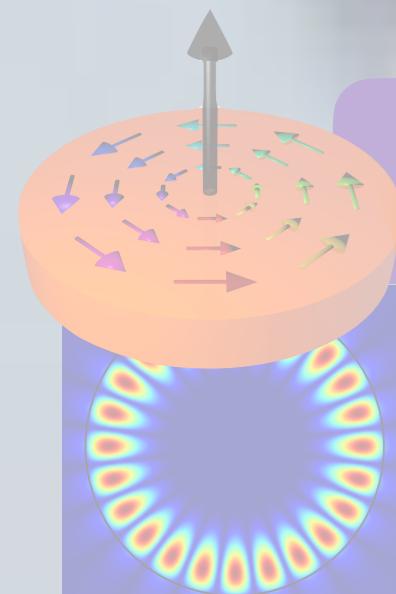
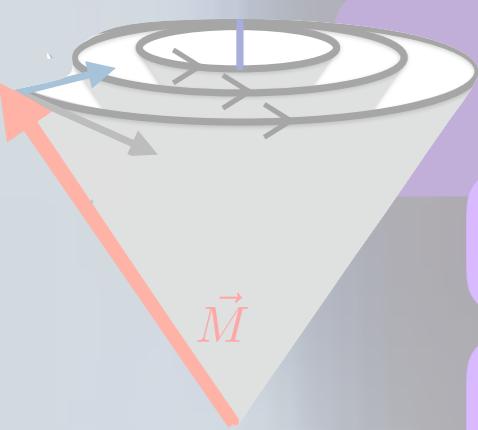
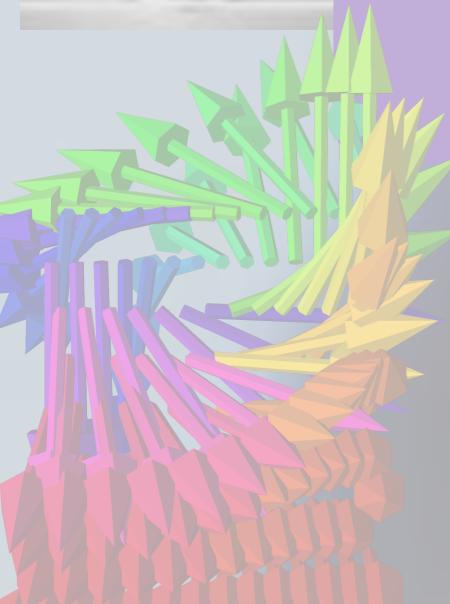
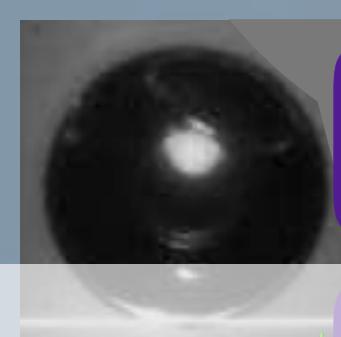
Landau Lifschitz Gilbert equation

Thiele equation for topological defects

Cavity optomagnonics with magnetic textures

Hamiltonian: Holstein-Primakoff

Optomagnetic coupling: gyrotropic mode



Magnonics



elementary magnetic
excitation
(quantum of spin wave)

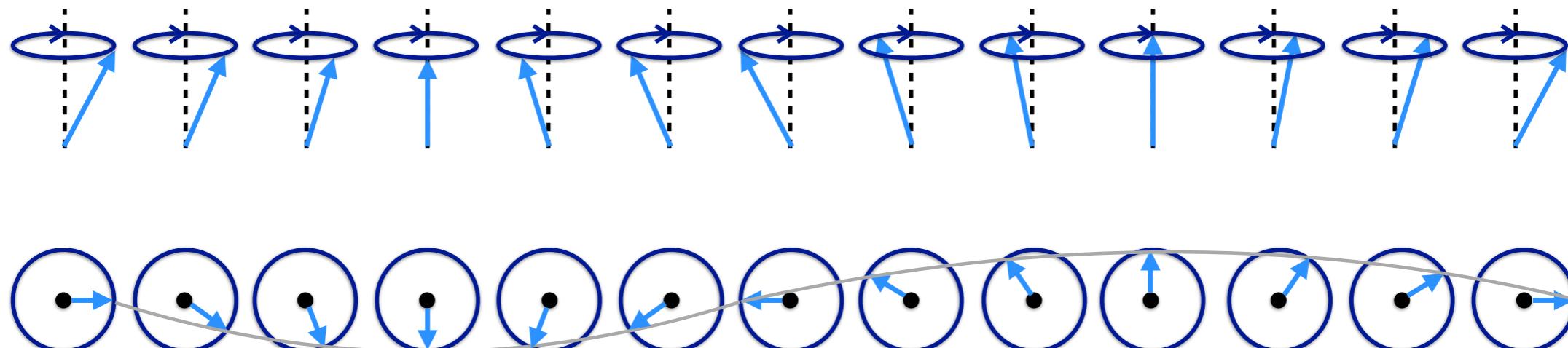
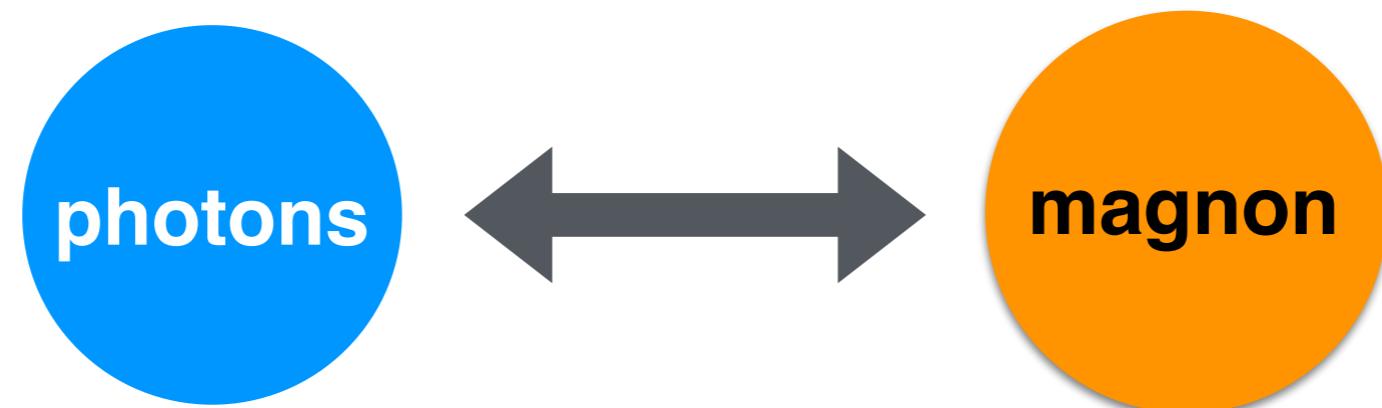


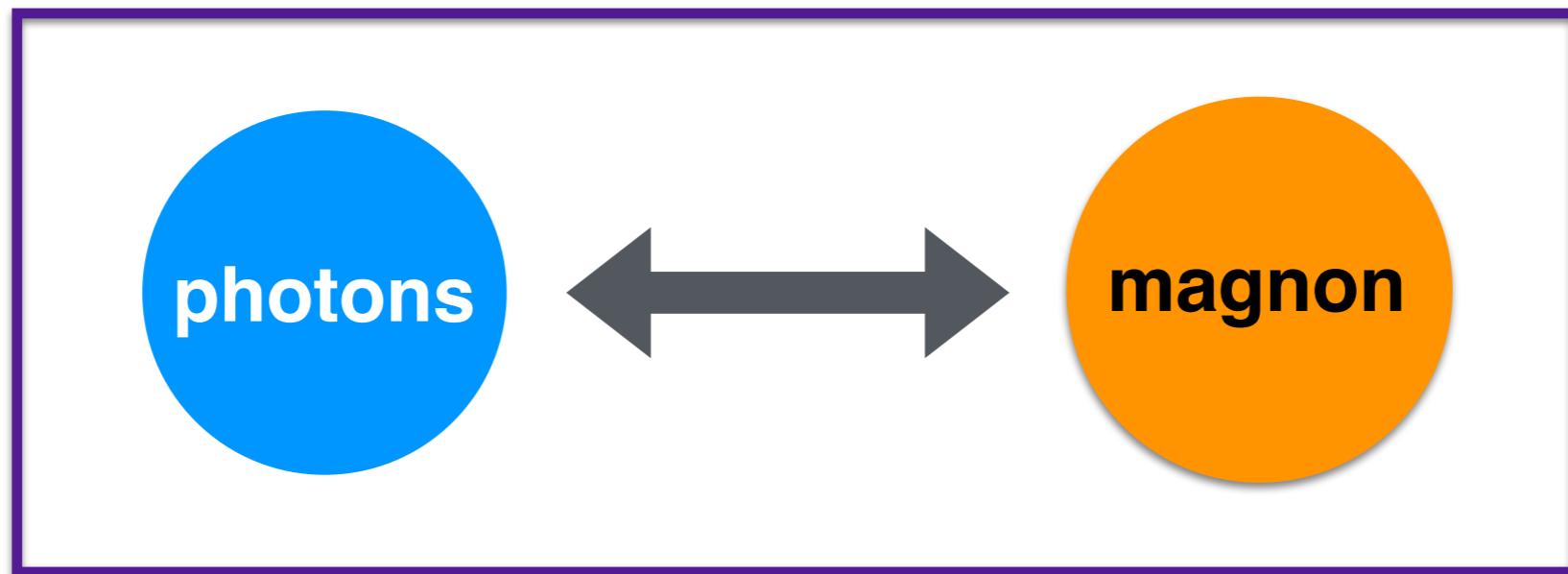
image: Jasmin Graf

Optomagnonics



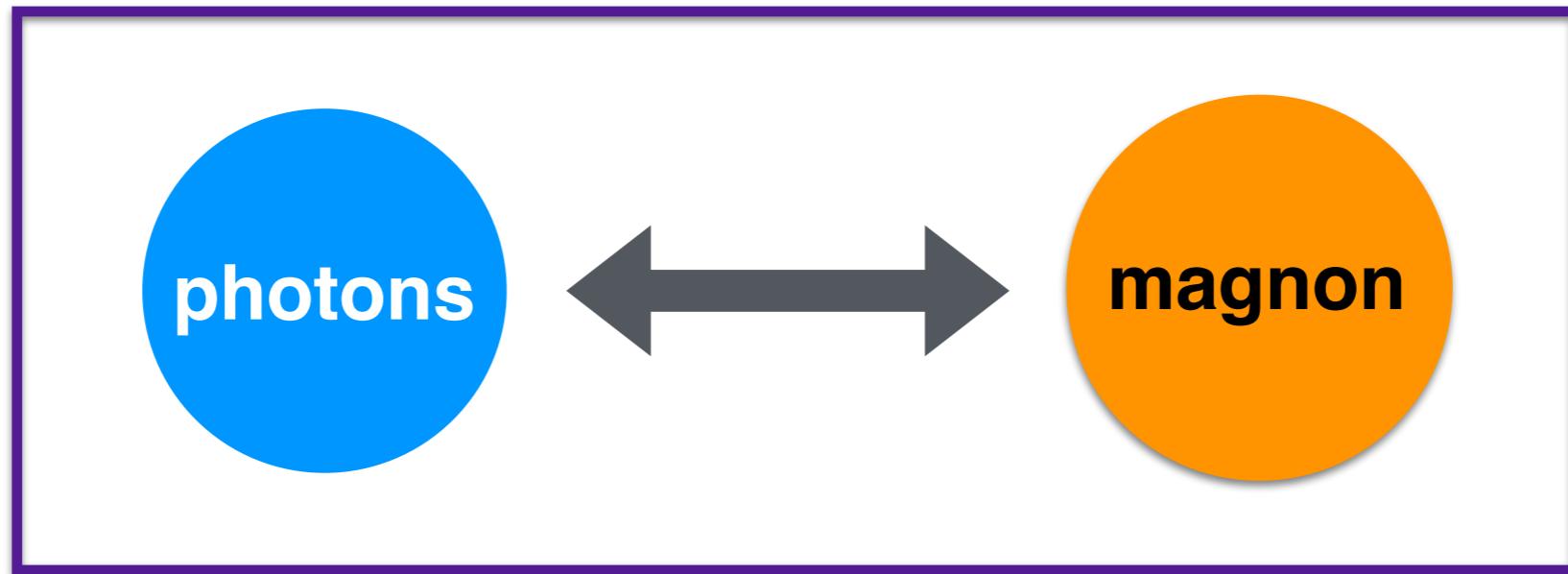
Cavity Optomagnonics

“Box” for the electromagnetic field



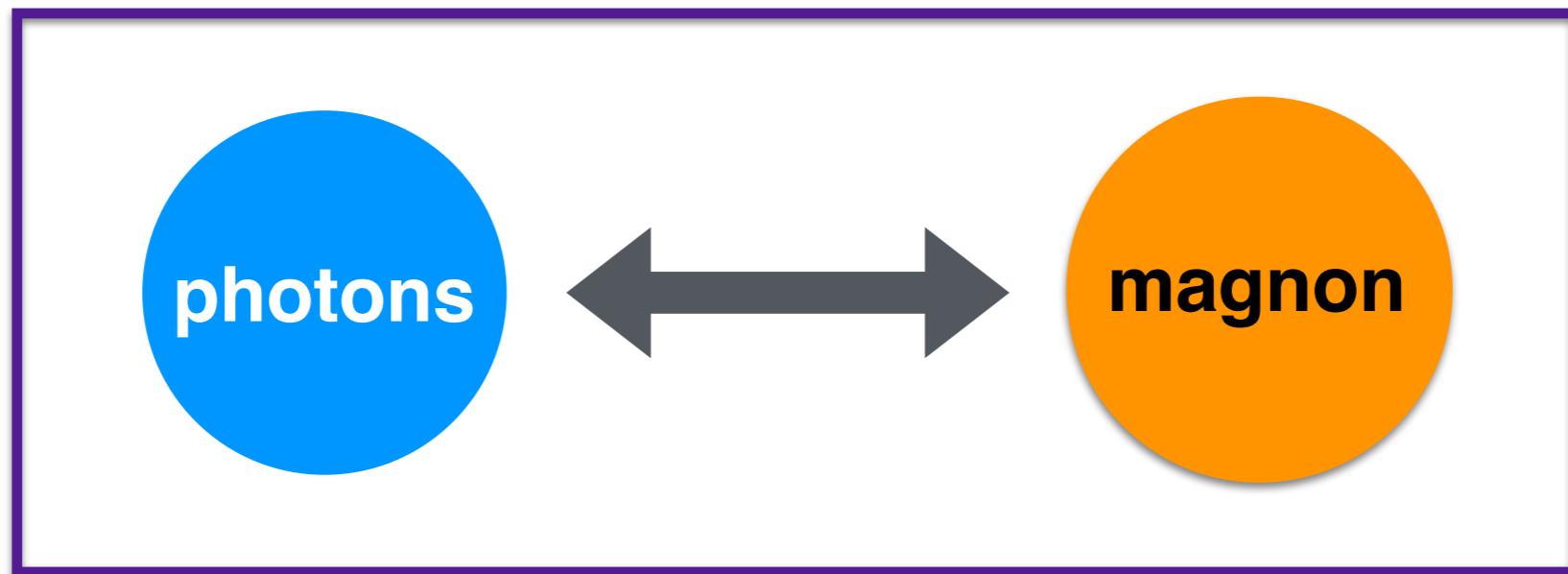
Cavity Optomagnonics

“Box” for the electromagnetic field



cavity-enhanced spin-photon interaction

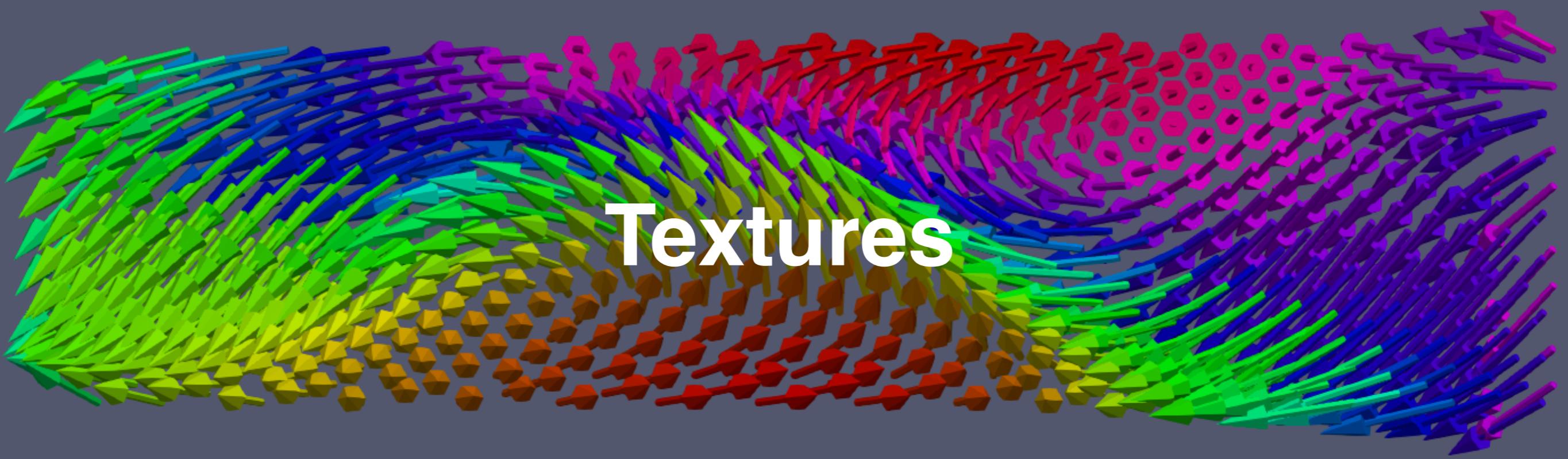
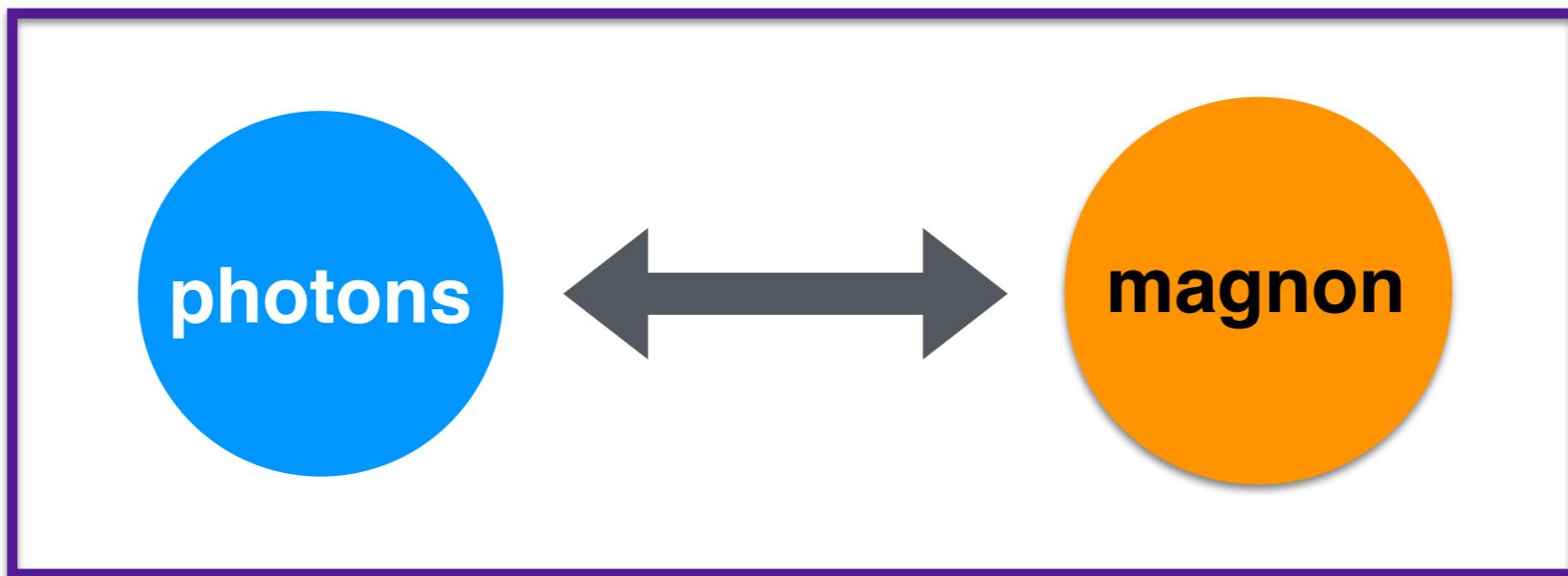
Cavity Optomagnonics



cavity-enhanced spin-photon interaction

cavity QED + magnetism

Cavity Optomagnonics

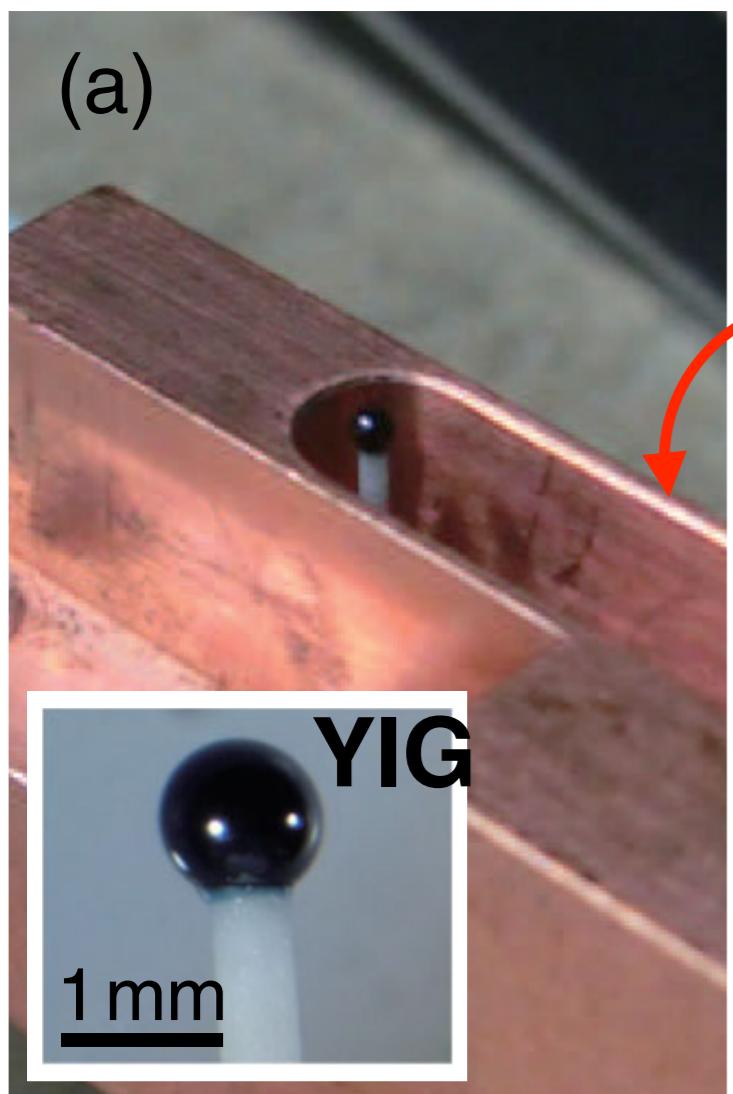


Microwave Regime

Magnons

Microwaves

Strong coupling demonstrated in 2014



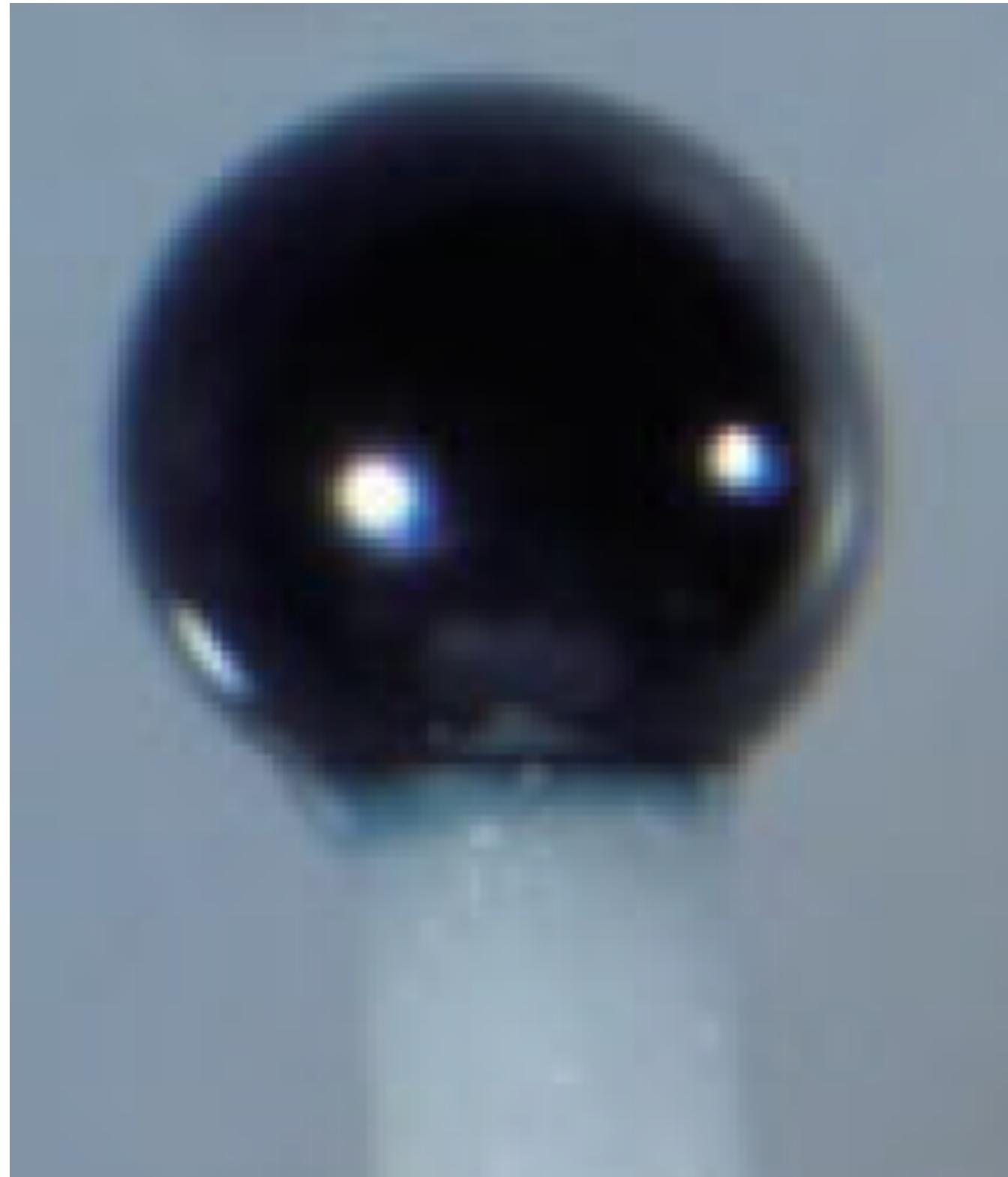
Microwave cavity

Huebl et. al, PRL 111, 127003 (2013)

Zhang et. al PRL 113, 156401 (2014)

Tabuchi et. al PRL 113, 083603 (2014)

YIG



YIG

Yttrium Iron Garnet



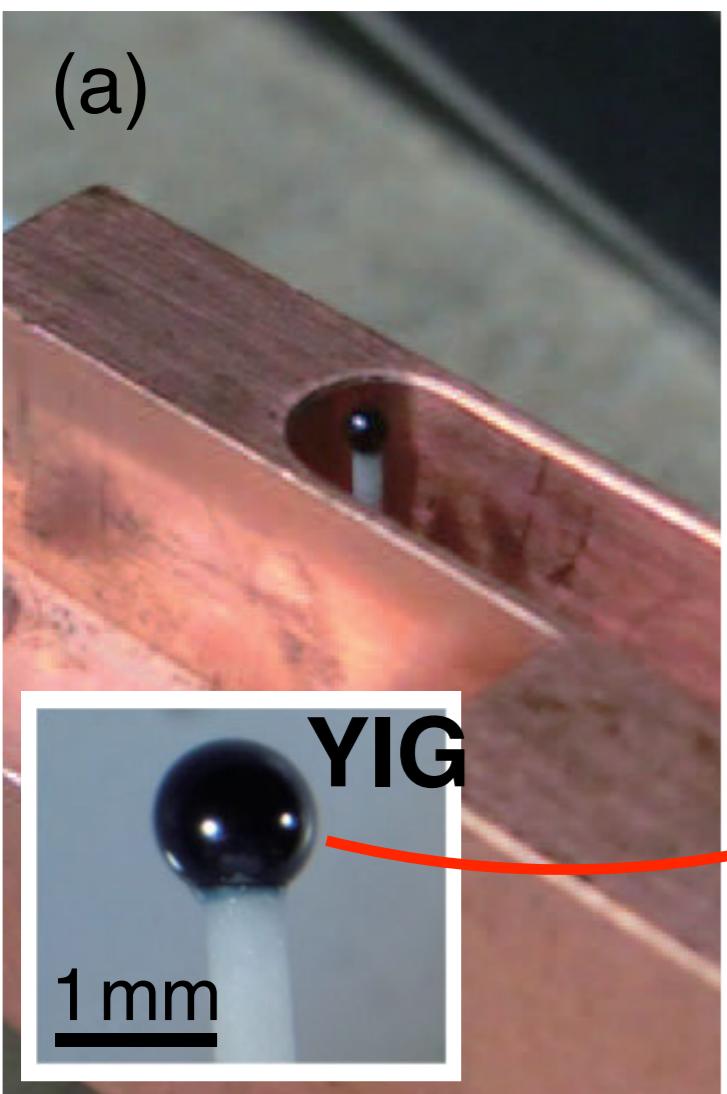
- ferrimagnetic
- insulator
- transparent in the infrared

Microwave Regime

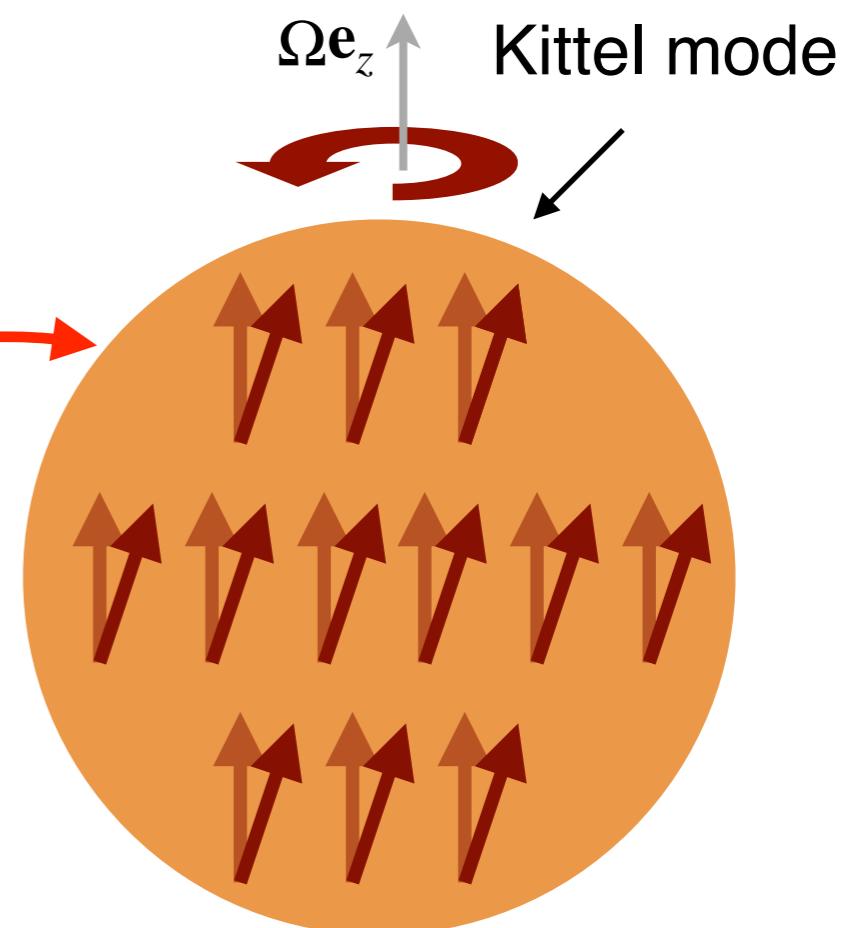
Magnons

Microwaves

Strong coupling demonstrated in 2014



Homogeneous magnon mode



Huebl et. al, PRL 111, 127003 (2013)

Zhang et. al PRL 113, 156401 (2014)

Tabuchi et. al PRL 113, 083603 (2014)

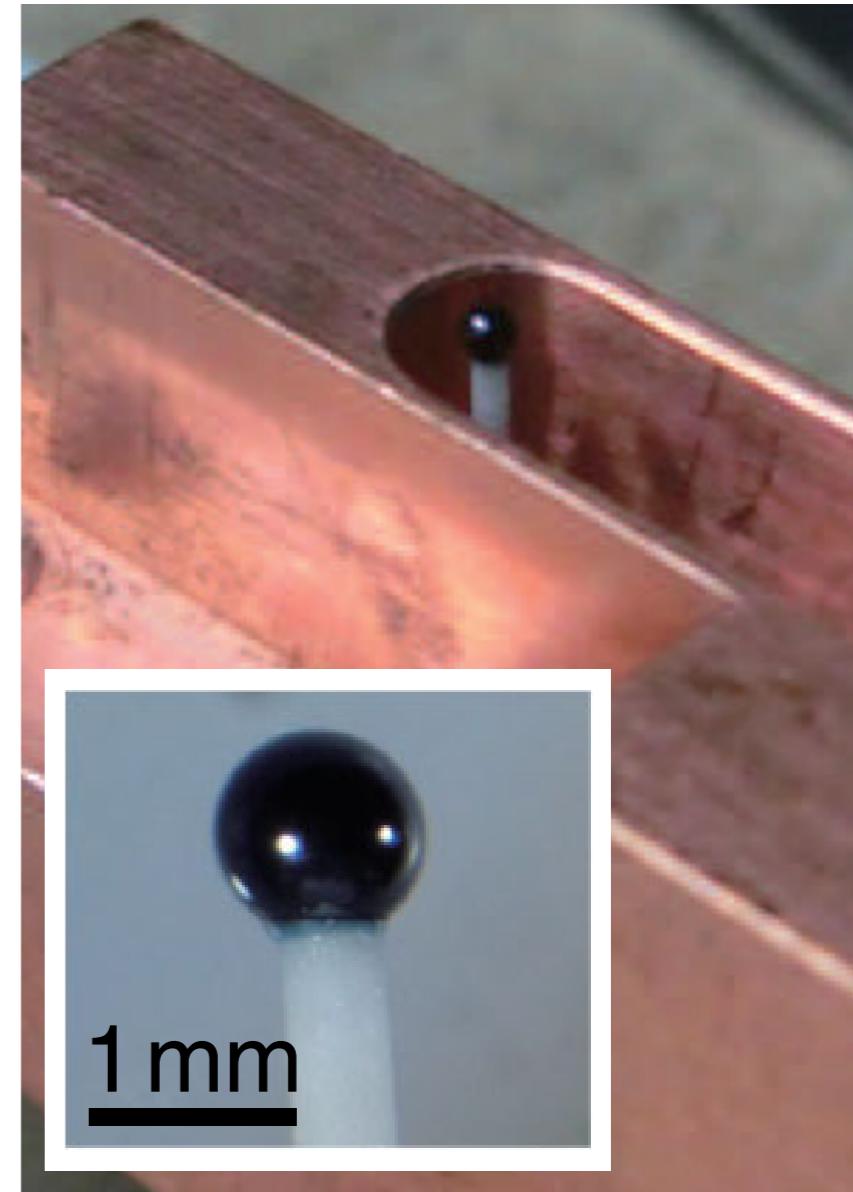
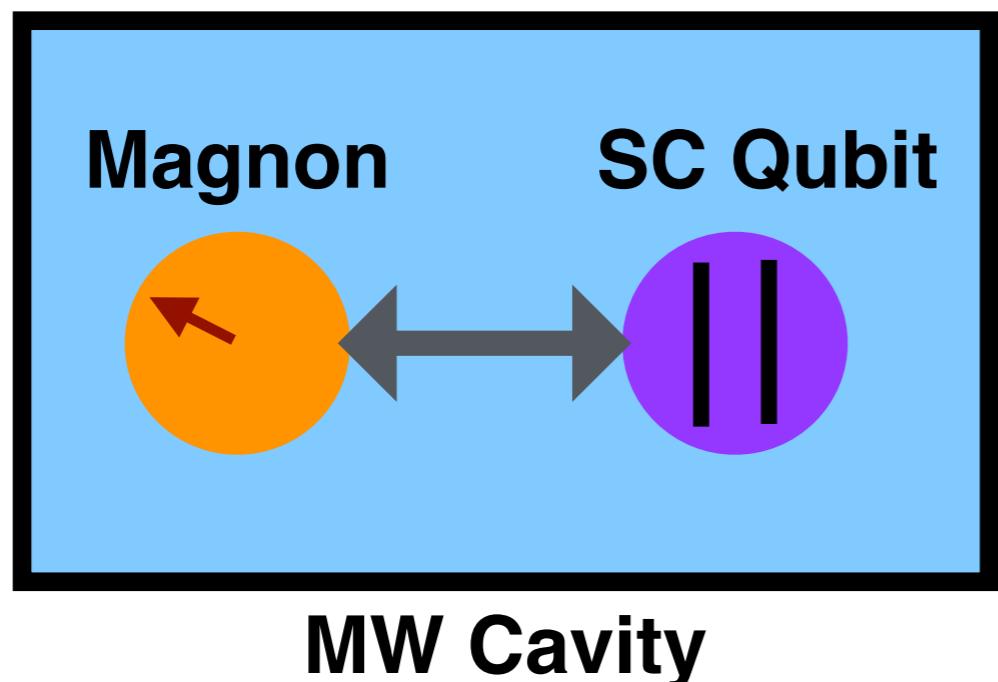
Microwave Regime

QUANTUM INFORMATION

(Science 2015)

Coherent coupling between a ferromagnetic magnon and a superconducting qubit

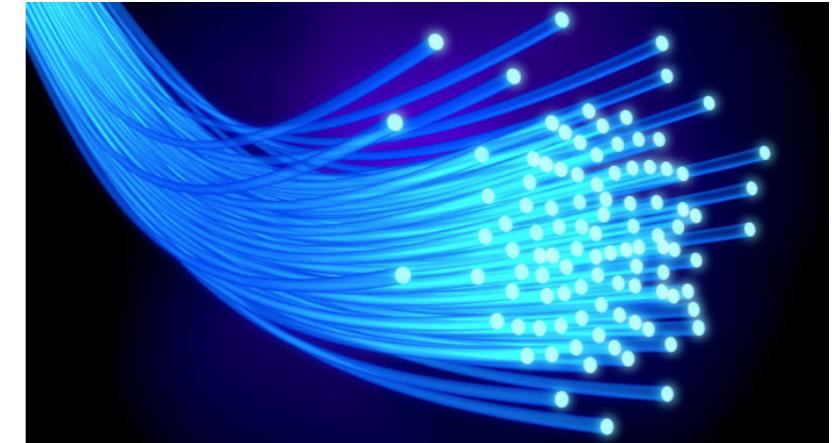
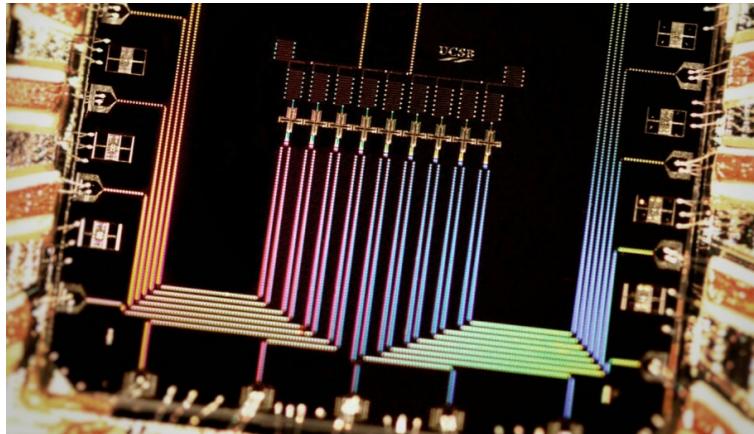
Yutaka Tabuchi,^{1,*} Seiichiro Ishino,¹ Atsushi Noguchi,¹ Toyofumi Ishikawa,¹ Rekishu Yamazaki,¹ Koji Usami,¹ Yasunobu Nakamura^{1,2}



Coupling to Optics?

GHz

THz



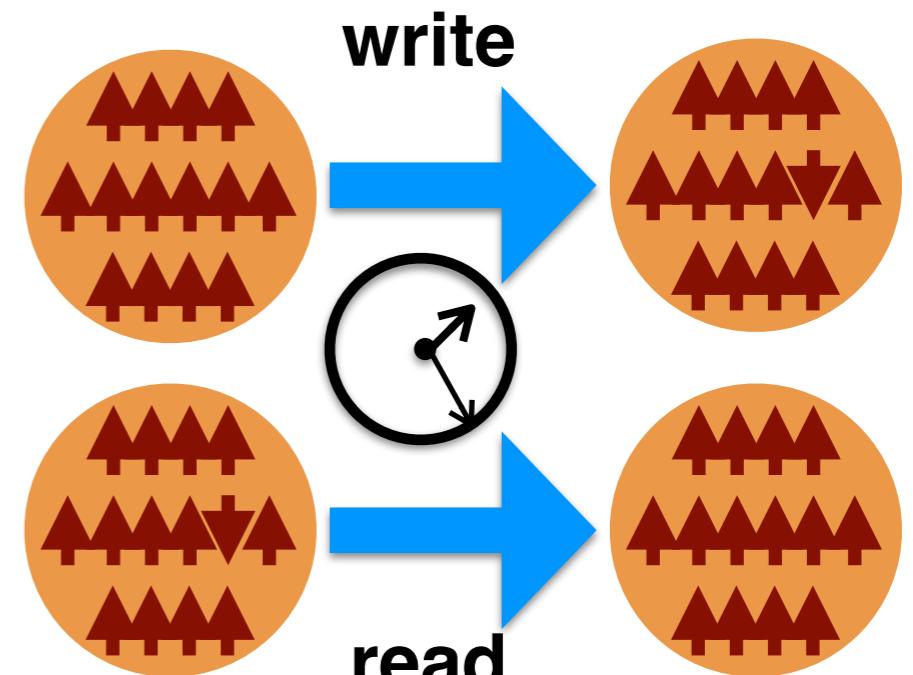
**Motivation:
magnon as a transducer**

Coupling to Optics?



Quantum memory for photons

Afzelius, Gisin, de Riedmatten; Physics Today (2015)

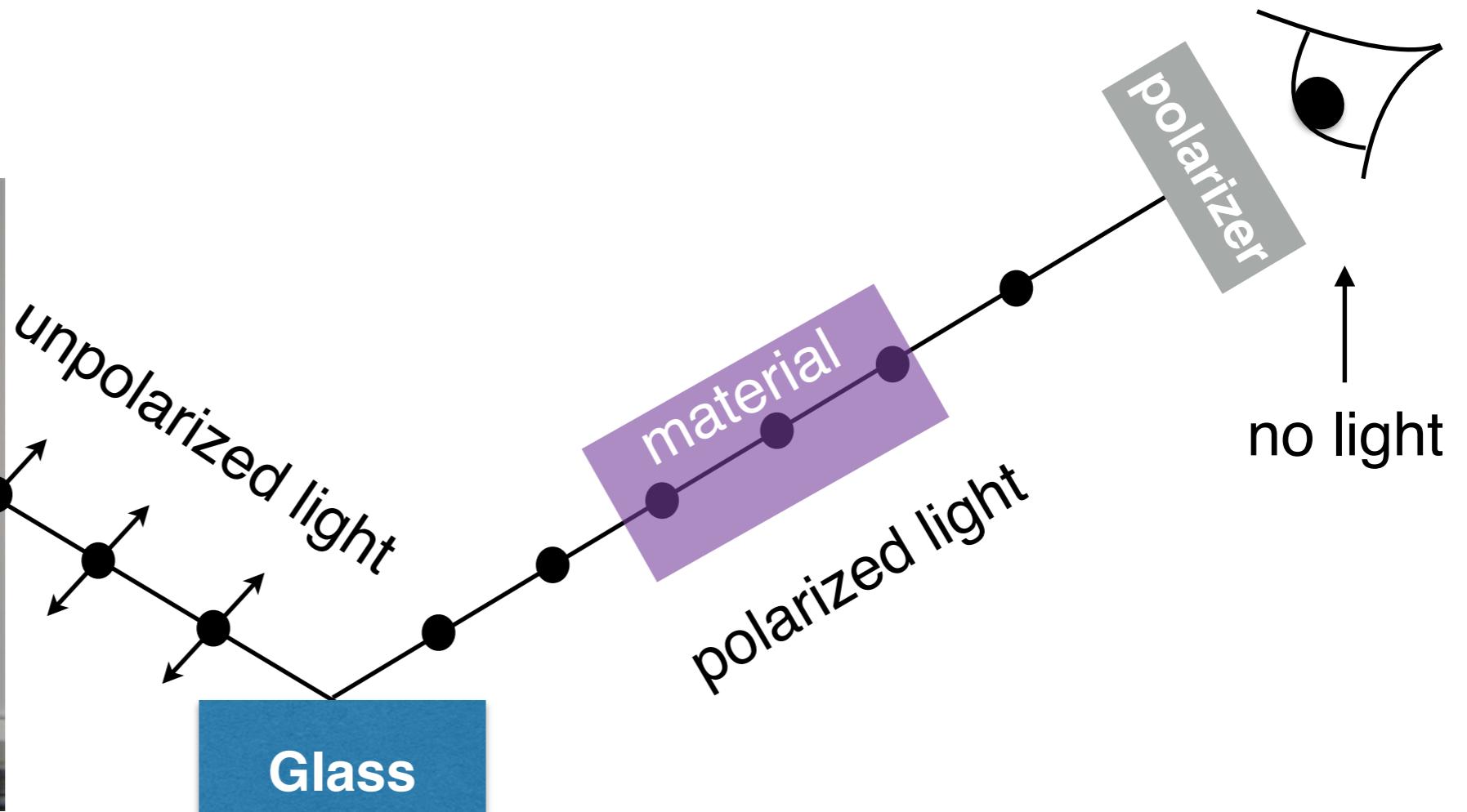


Motivation:
magnon state as a quantum memory

Faraday Effect (1846)



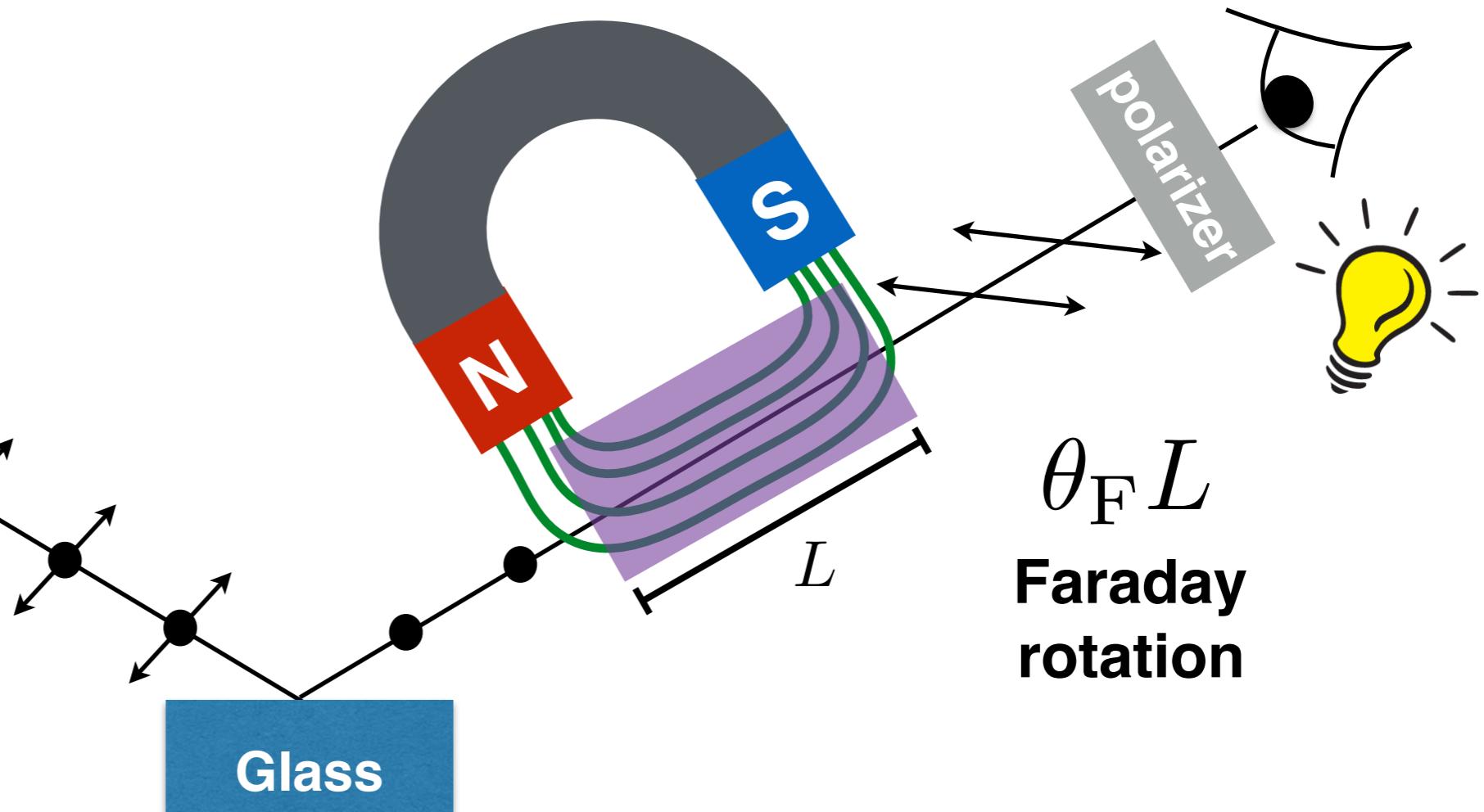
Oil Lamp



Faraday Effect (1846)



Oil Lamp

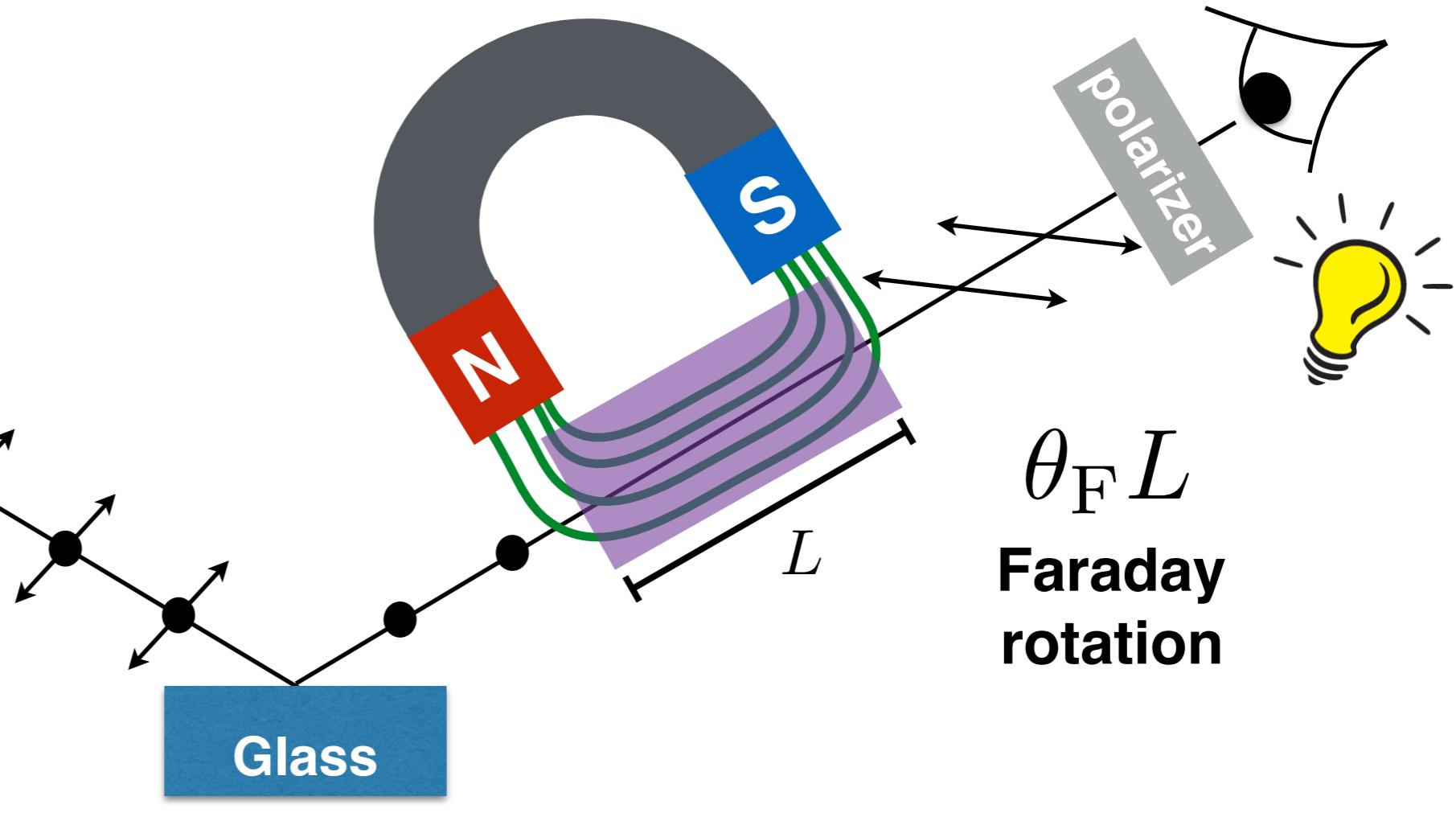


$\theta_F L$
**Faraday
rotation**

Faraday Effect (1846)



Oil Lamp



$\theta_F L$
Faraday
rotation

RELATION OF LIGHT TO THE MAGNETIC FORCE.

15

¶ iii. *General considerations.*

2221. Thus is established, I think for the first time*, a true, direct relation and dependence between light and the magnetic and electric forces; and thus a great

Coupling to Optics?: Faraday Effect

Faraday
rotation

$$\bar{U}_{\text{MO}} = \theta_F \sqrt{\frac{\varepsilon}{\varepsilon_0}} \int d\mathbf{r} \left[\frac{\mathbf{M}(\mathbf{r})}{M_s} \cdot \frac{\varepsilon_0}{2i\omega} [\mathbf{E}^*(\mathbf{r}) \times \mathbf{E}(\mathbf{r})] \right]$$

 magnetization density

optical spin density 

Coupling to Optics?: Faraday Effect

Faraday
rotation

$$\bar{U}_{\text{MO}} = \theta_F \sqrt{\frac{\varepsilon}{\varepsilon_0}} \int d\mathbf{r} \left[\frac{\mathbf{M}(\mathbf{r})}{M_s} \cdot \frac{\varepsilon_0}{2i\omega} [\mathbf{E}^*(\mathbf{r}) \times \mathbf{E}(\mathbf{r})] \right]$$

 magnetization density

optical spin density 

$$\varepsilon_{ij}(\mathbf{M}) = \varepsilon_0 (\varepsilon \delta_{ij} - i f \epsilon_{ijk} M_k)$$

↑
broken time-reversal symmetry

Coupling to Optics?: Faraday Effect

Faraday
rotation

$$\bar{U}_{\text{MO}} = \theta_F \sqrt{\frac{\varepsilon}{\varepsilon_0}} \int d\mathbf{r} \left[\frac{\mathbf{M}(\mathbf{r})}{M_s} \cdot \frac{\varepsilon_0}{2i\omega} [\mathbf{E}^*(\mathbf{r}) \times \mathbf{E}(\mathbf{r})] \right]$$



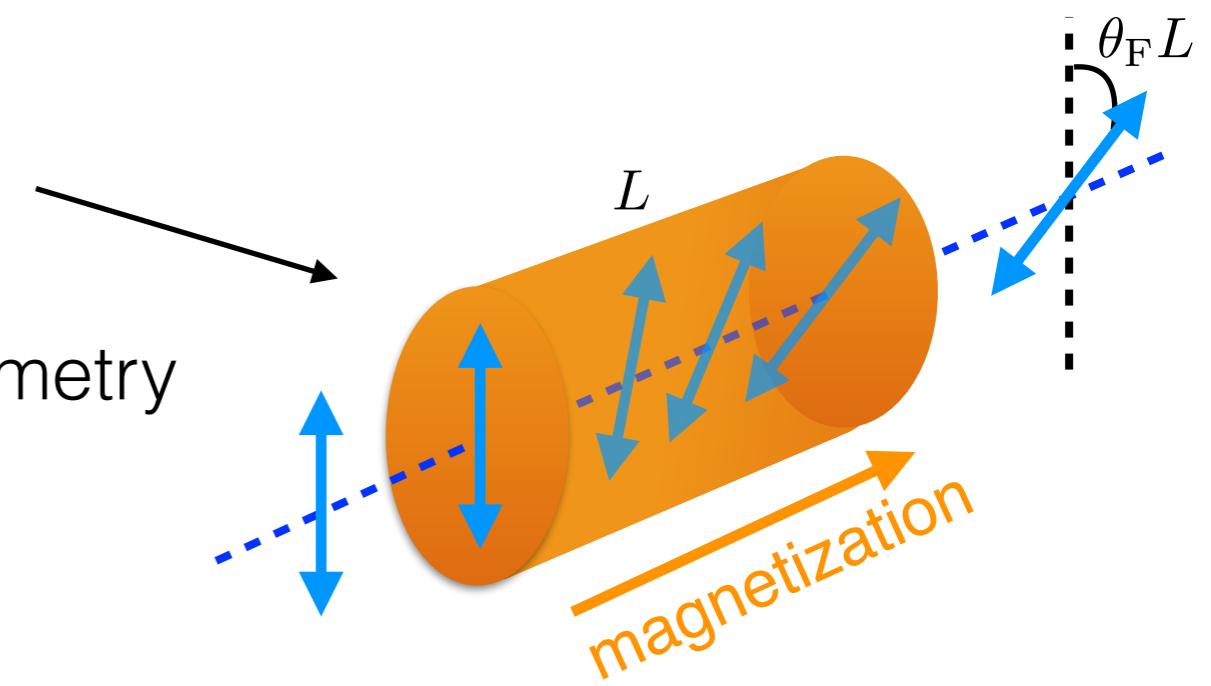
optical
spin density



magnetization
density

$$\varepsilon_{ij}(\mathbf{M}) = \varepsilon_0 (\varepsilon \delta_{ij} - if \epsilon_{ijk} M_k)$$

broken time-reversal symmetry



Optomagnonic Hamiltonian

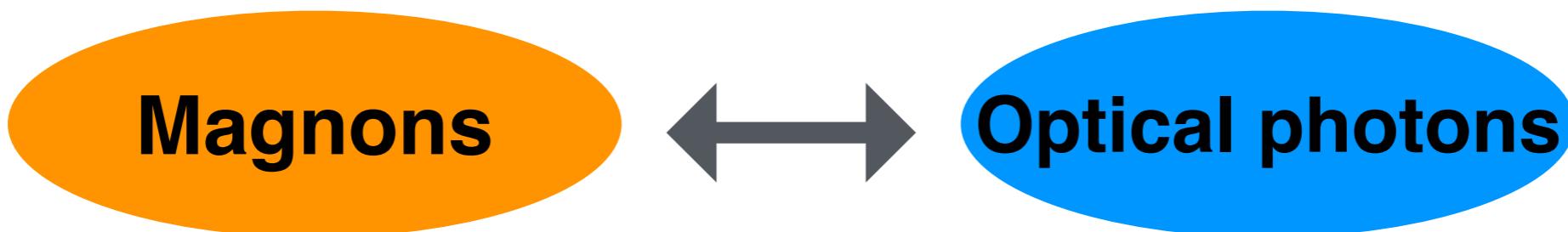
$$\bar{U}_{\text{MO}} = \theta_F \sqrt{\frac{\varepsilon}{\varepsilon_0}} \int d\mathbf{r} \left[\frac{\mathbf{M}(\mathbf{r})}{M_s} \cdot \frac{\varepsilon_0}{2i\omega} [\mathbf{E}^*(\mathbf{r}) \times \mathbf{E}(\mathbf{r})] \right]$$

Quantize:

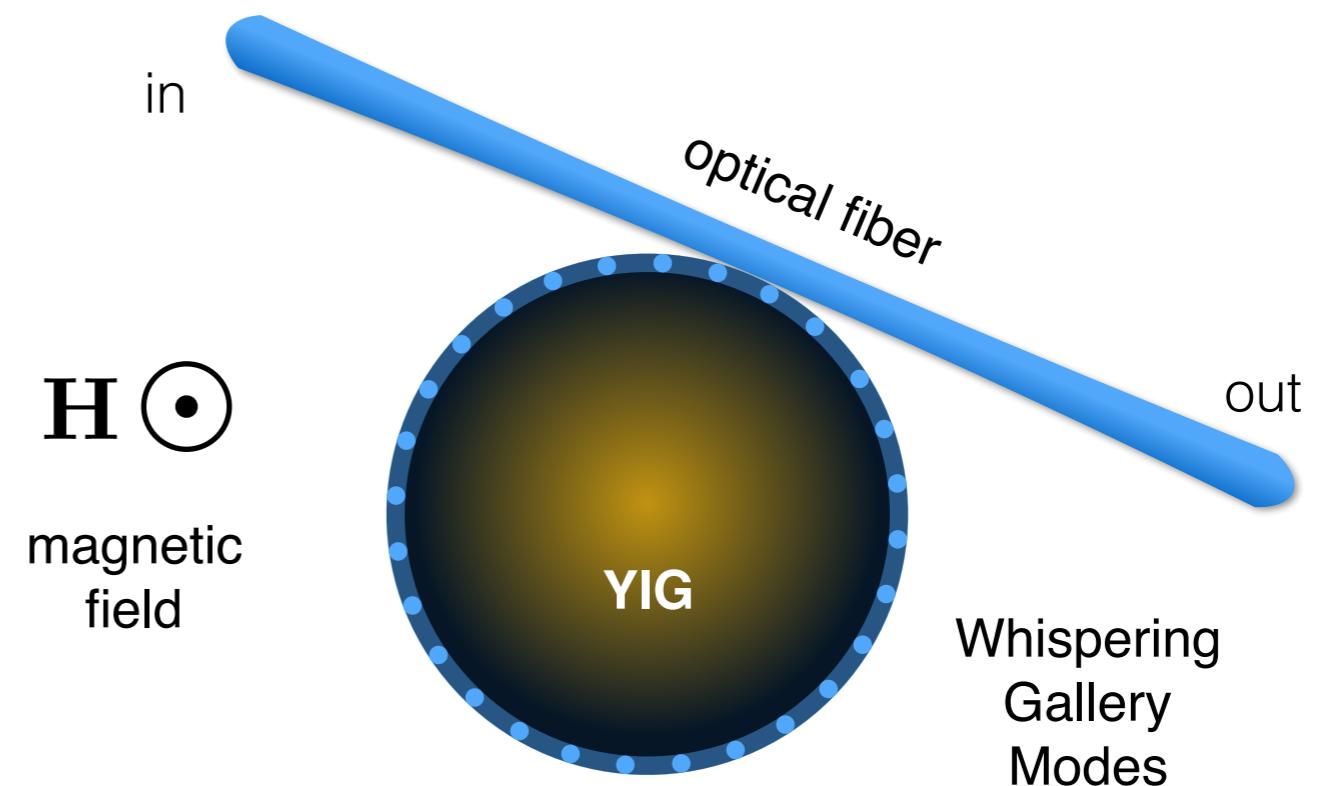
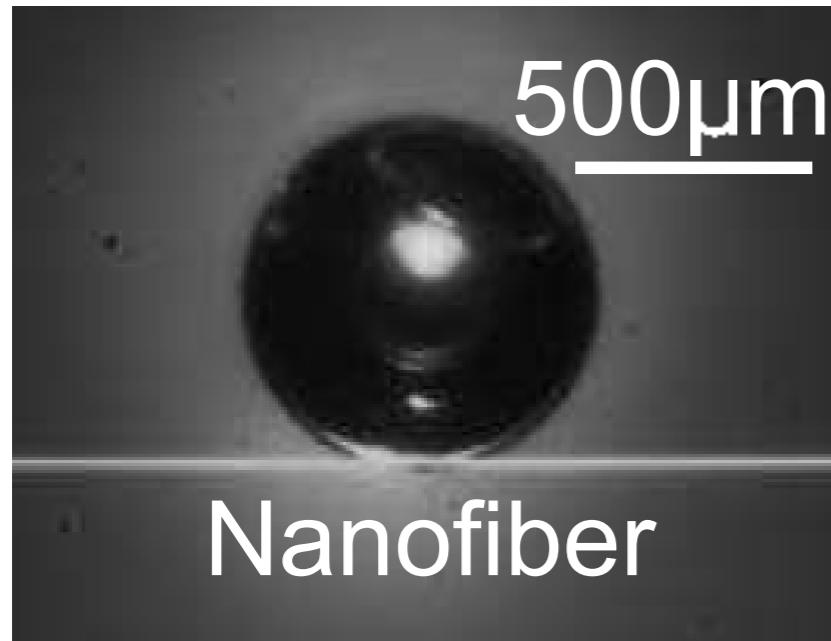


two-photon process

Cavity Optomagnonics



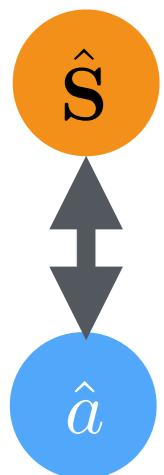
Coherent coupling demonstrated in 2016



- Osada et. al PRL 116, 223601
(Nakamura's group, Tokyo)
- Haigh et. al PRL 117, 133602
(Cambridge Univ / Hitachi)
- Zhang et. al PRL 117, 123605
(Hong Tang's group, Yale)

A cavity enhances the effect

But...

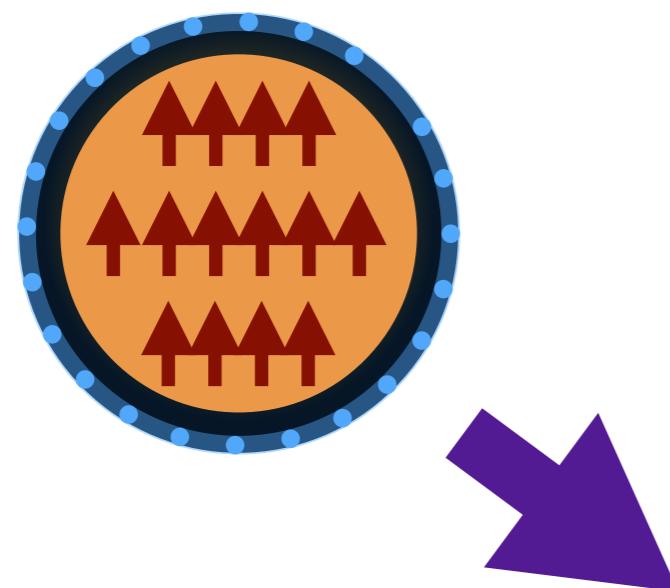


Problem

the state of the art optomagnonic coupling is very small

Some solutions

better overlap of modes



?

Optomagnonics beyond the Kittel mode

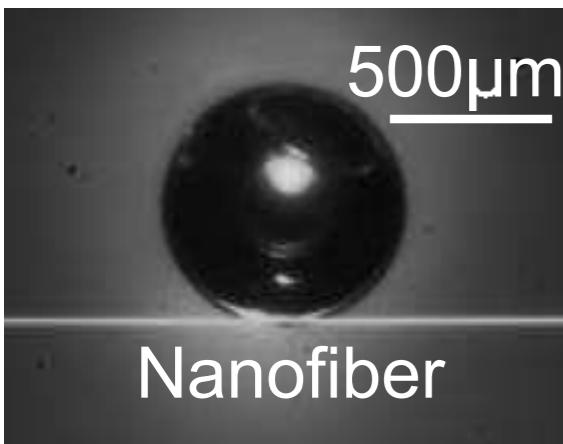
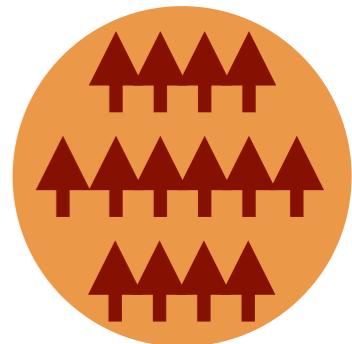


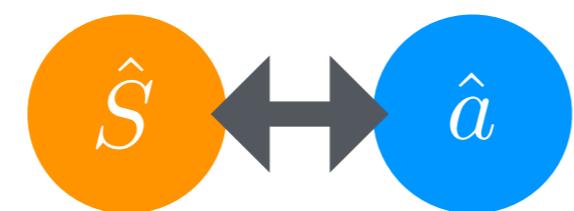
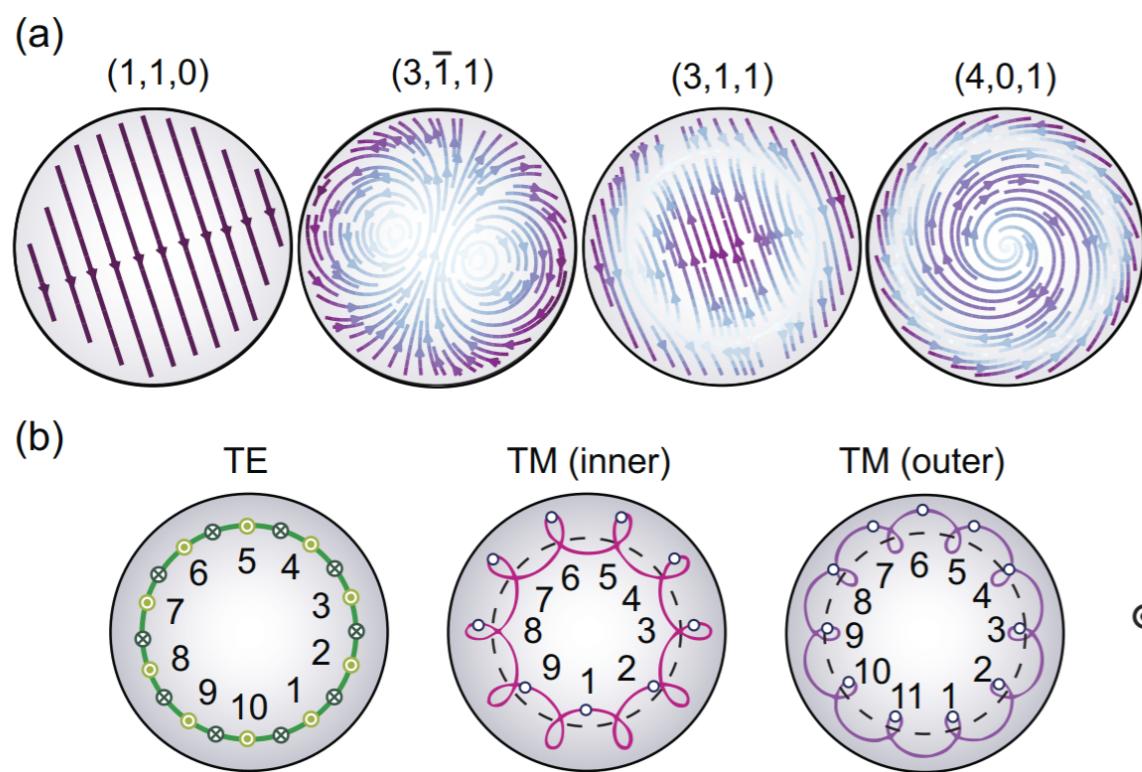
Fig: Osada et. al.
PRL 116, 223601

Optomagnonic coupling demonstrated

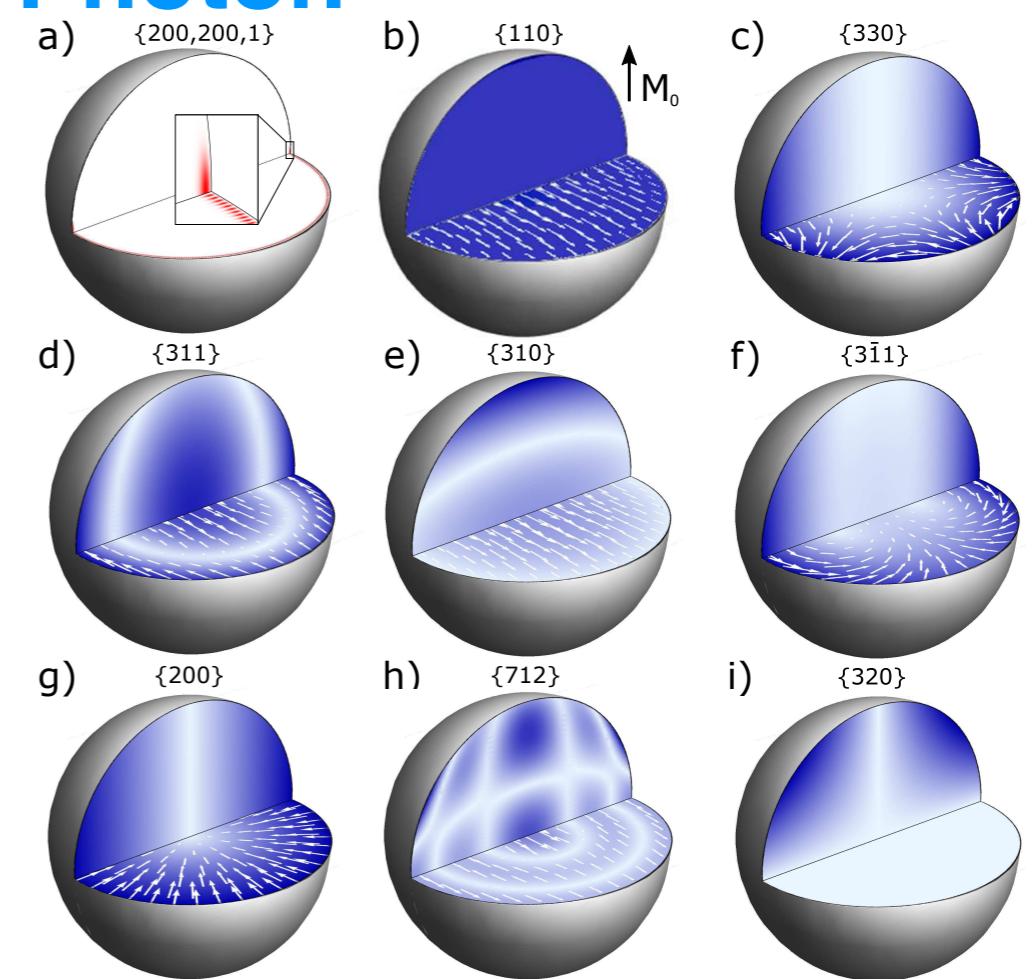
- Non-homogeneous magnon mode
- Homogeneous ground state



Photon Magnon



Photon

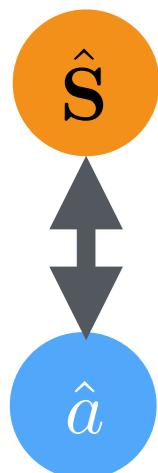


Magnon

But...

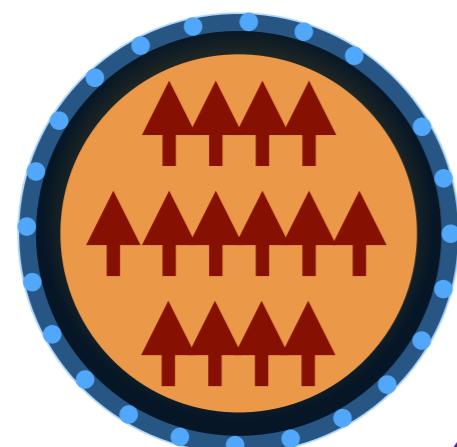
Problem

the state of the art optomagnonic coupling is very small



Some solutions

better overlap of modes



?

smaller systems

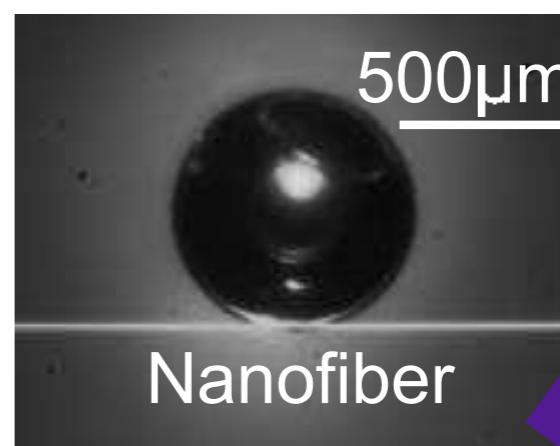
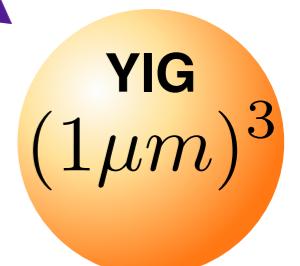
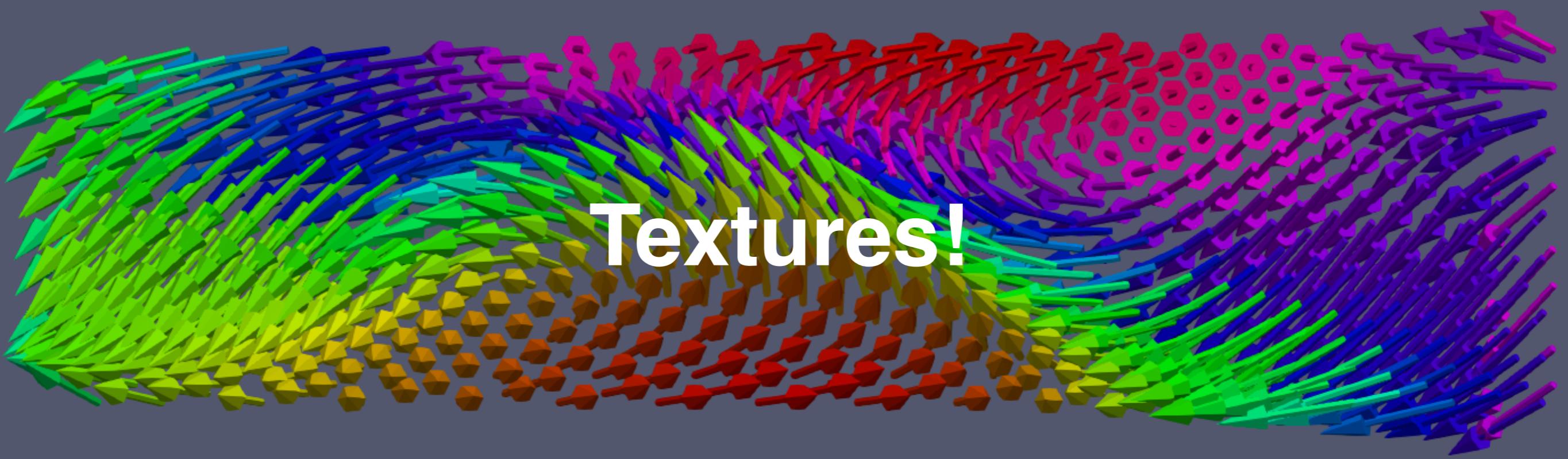
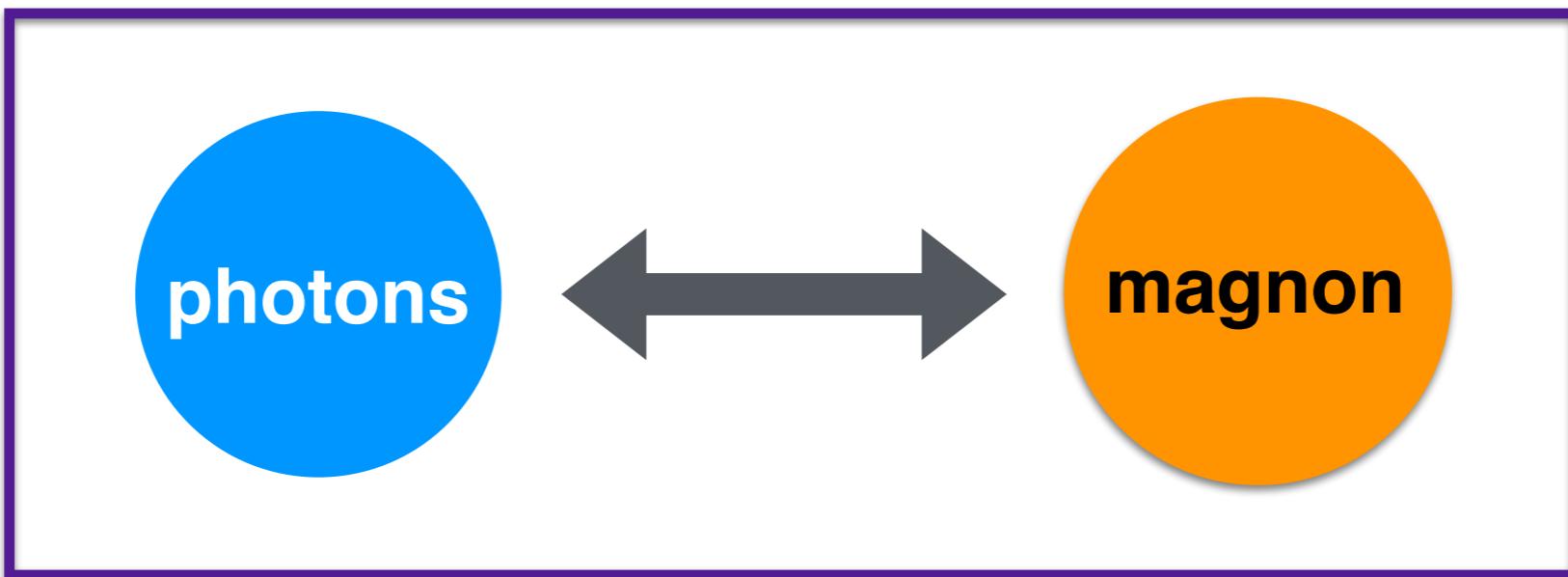
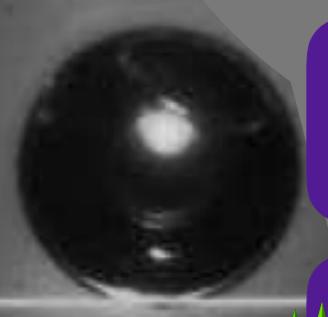


Fig: Osada et. al.
PRL 116, 223601

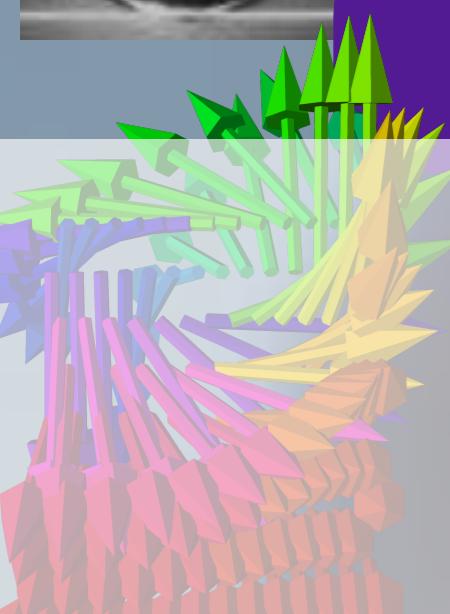


Cavity Optomagnonics





Introduction: cavity optomagnonics

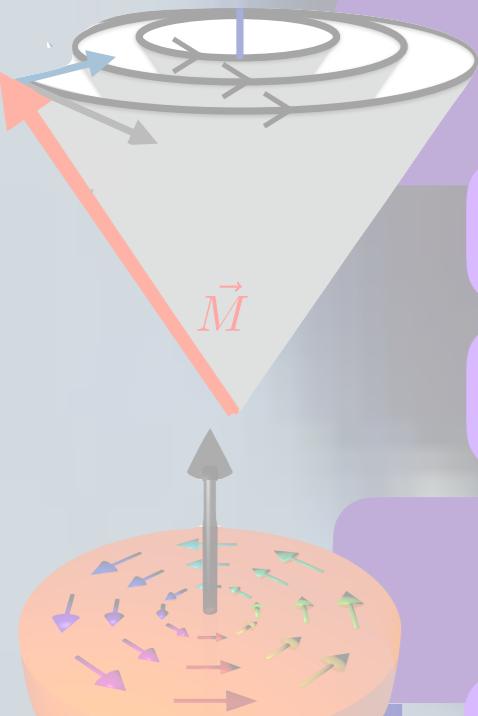


Magnetic textures

Why do they form?

Equilibrium condition

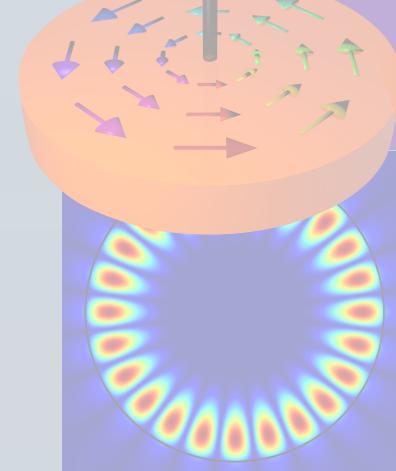
Topology of smooth textures



Dynamics of the magnetization

Landau Lifschitz Gilbert equation

Thiele equation for topological defects



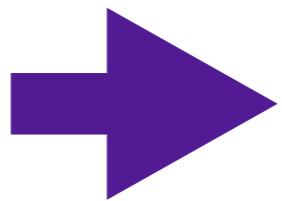
Cavity optomagnonics with magnetic textures

Hamiltonian: Holstein-Primakoff

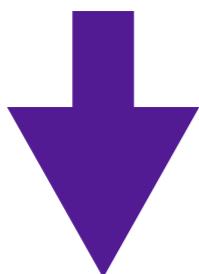
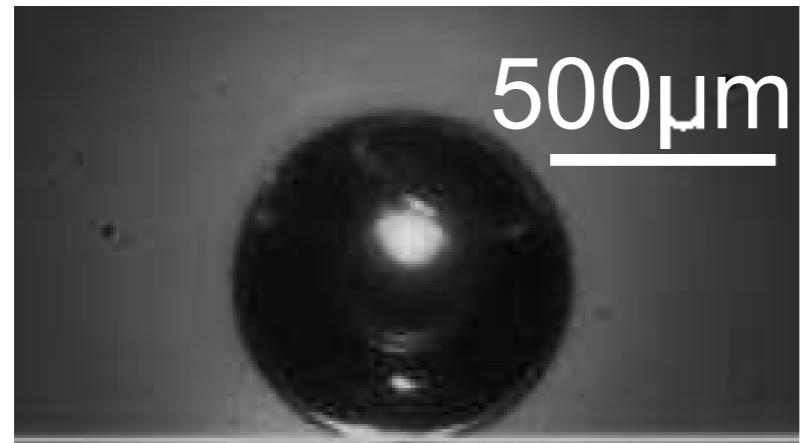
Optomagnetic coupling: gyrotropic mode

Magnetic Textures

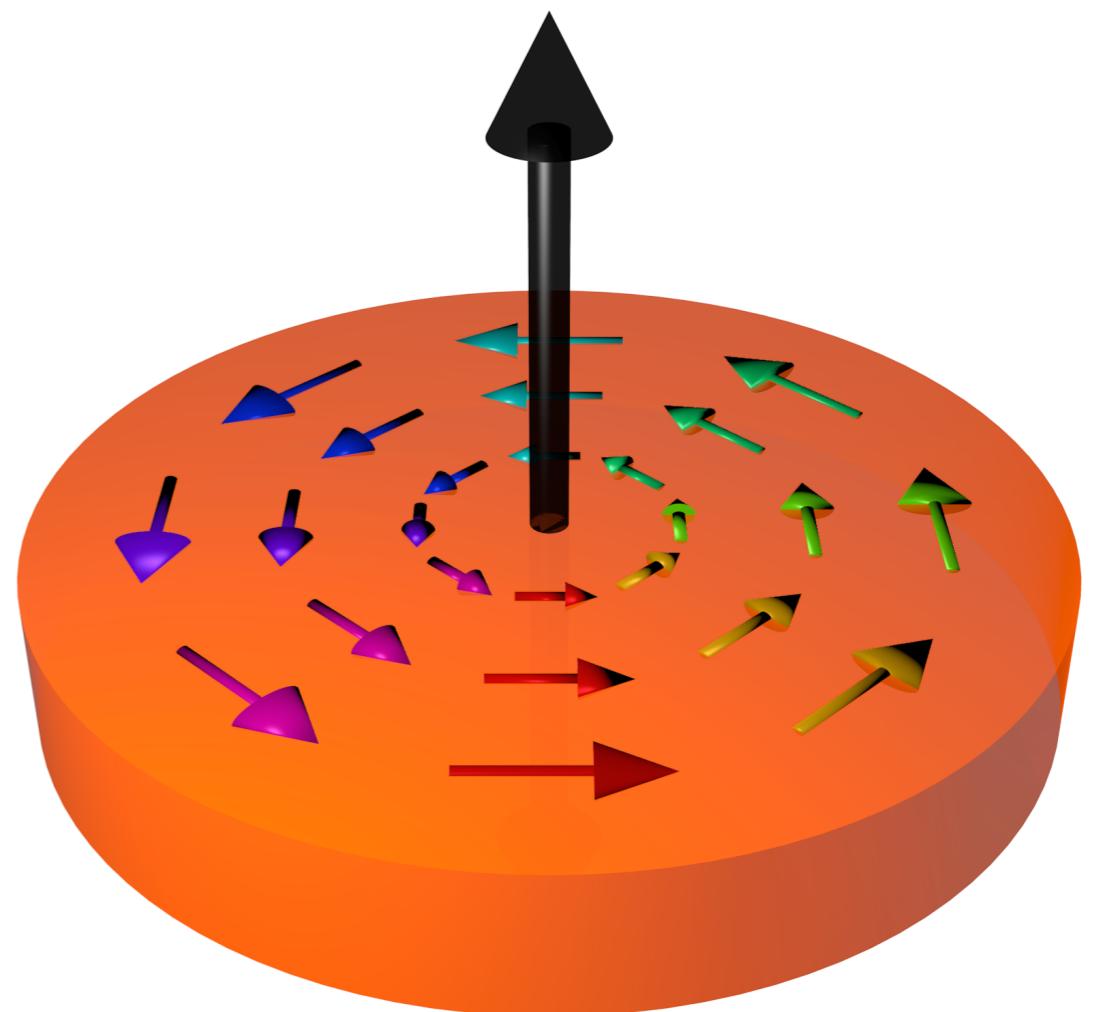
smaller systems



Magnetic textures

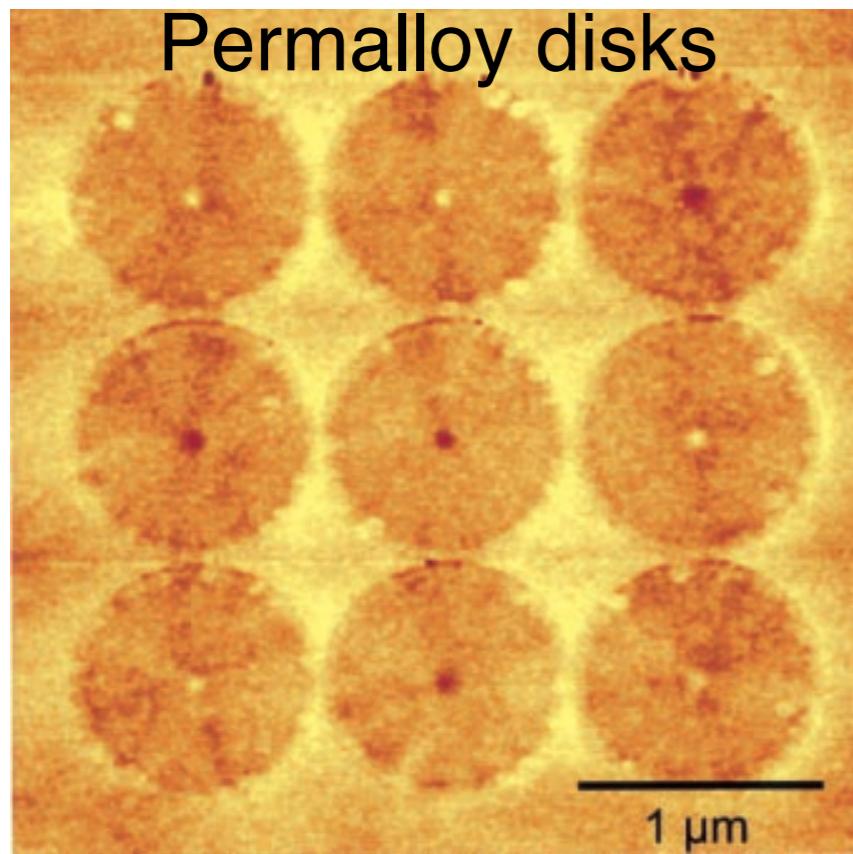


YIG
 $(1\mu m)^3$

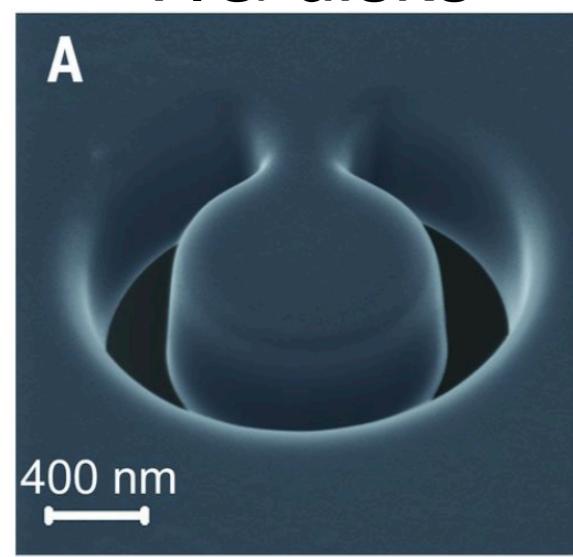


Example: Vortex in a micro disk

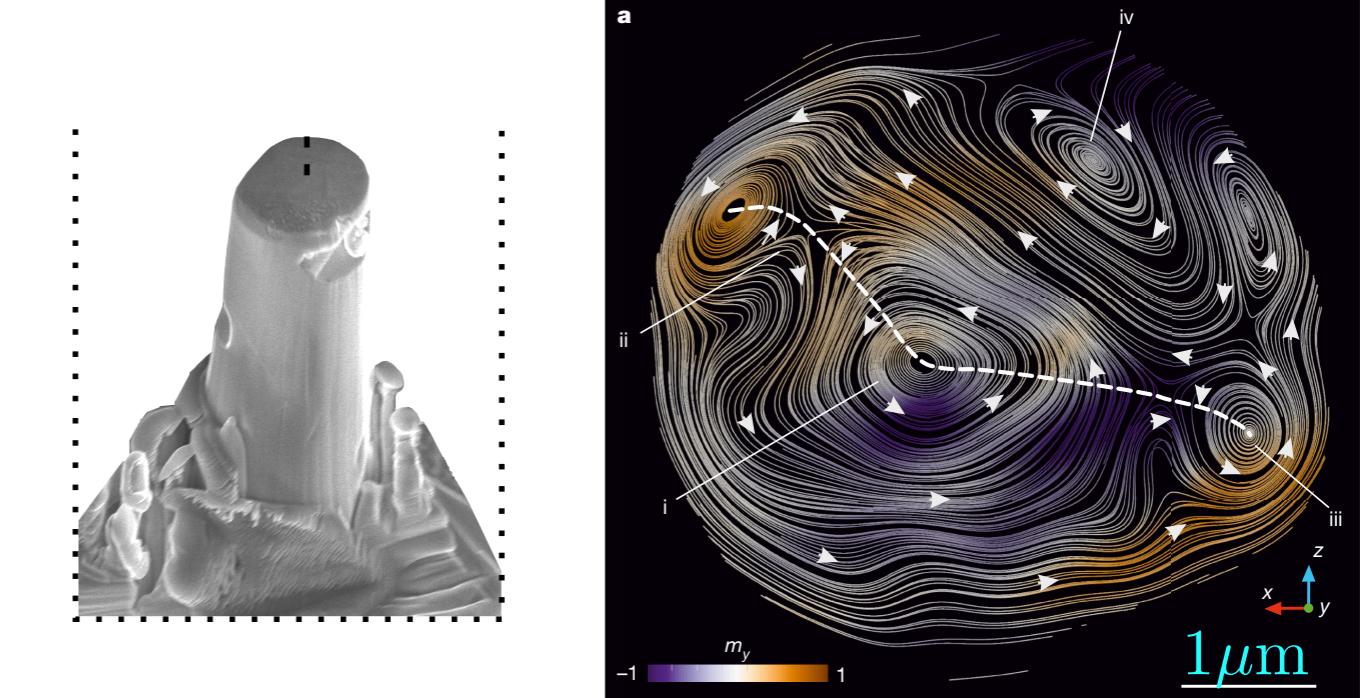
Magnetic Textures: Vortex in Microdisks



T.Shinjo et al, Science 289, 930 (2000)

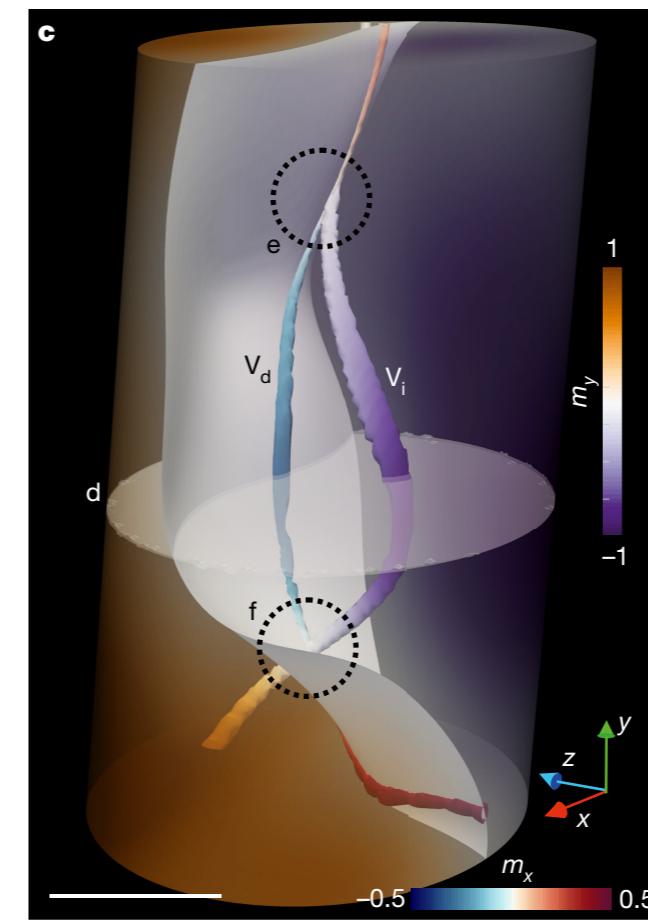


Losby et al, Science 350, 798 (2015)

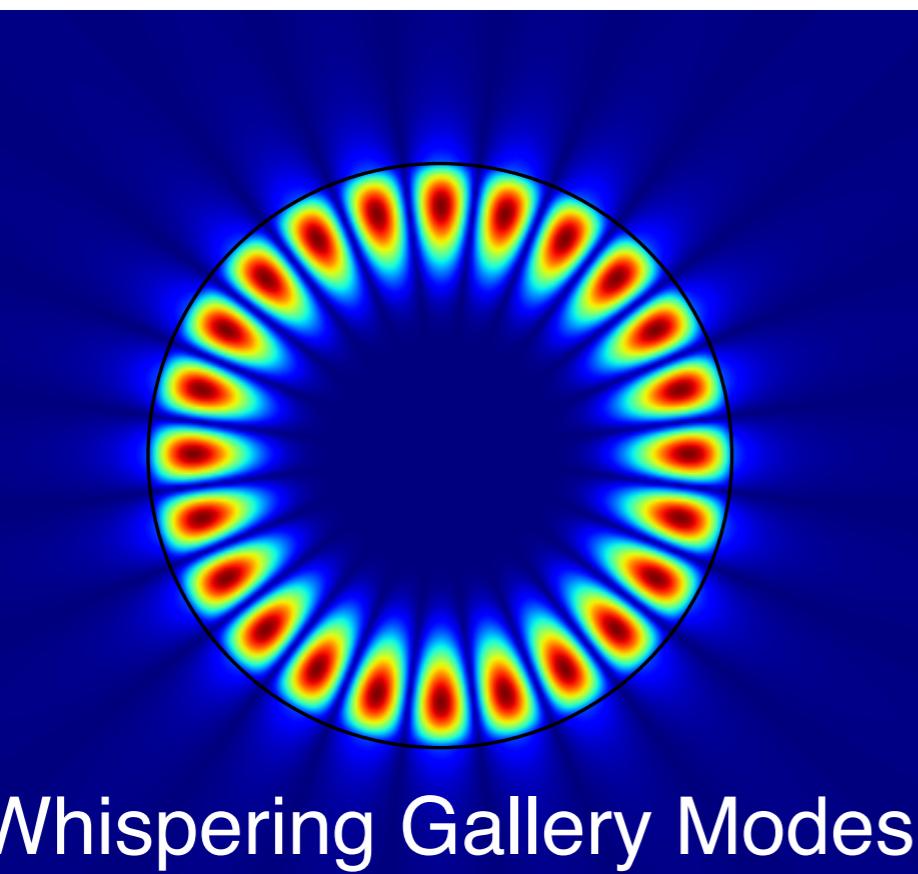
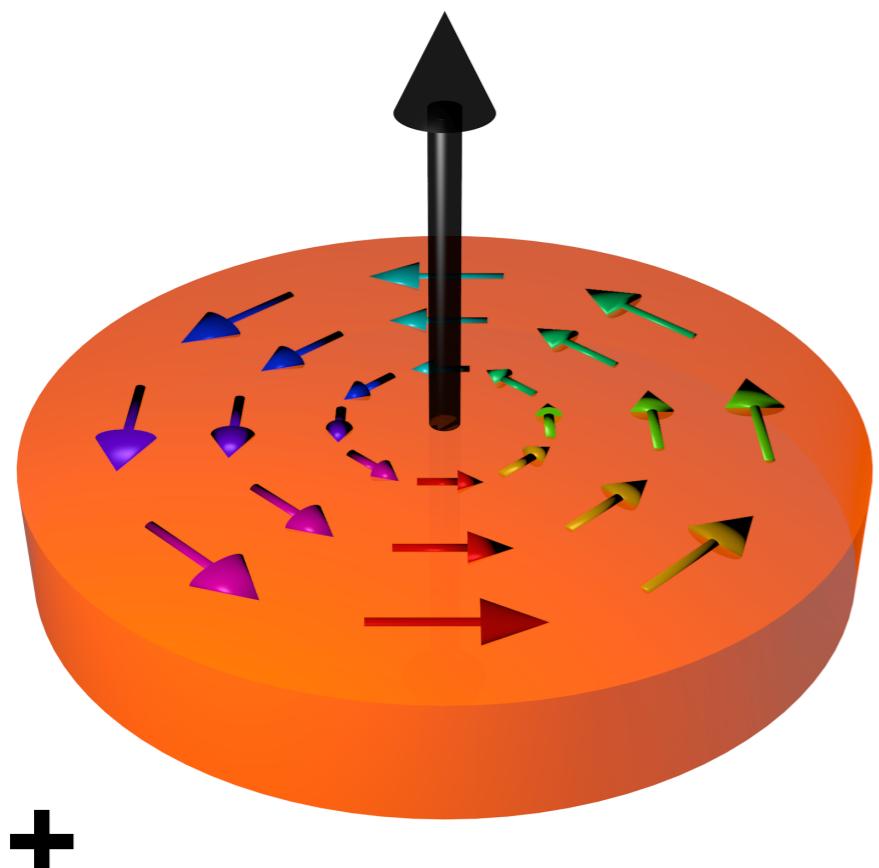


Cobalt Gadolinium
pillars

C. Donally et al,
Nature 547
328 (2017)

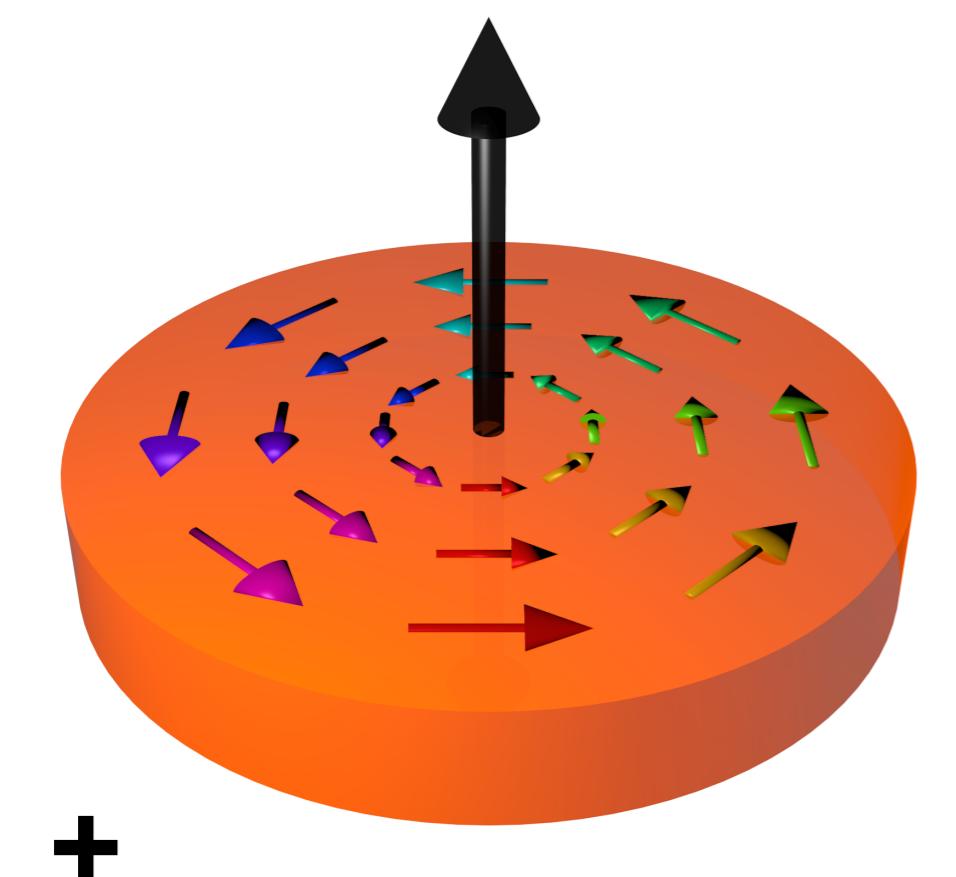


Magnetic Vortex and Optical Cavity

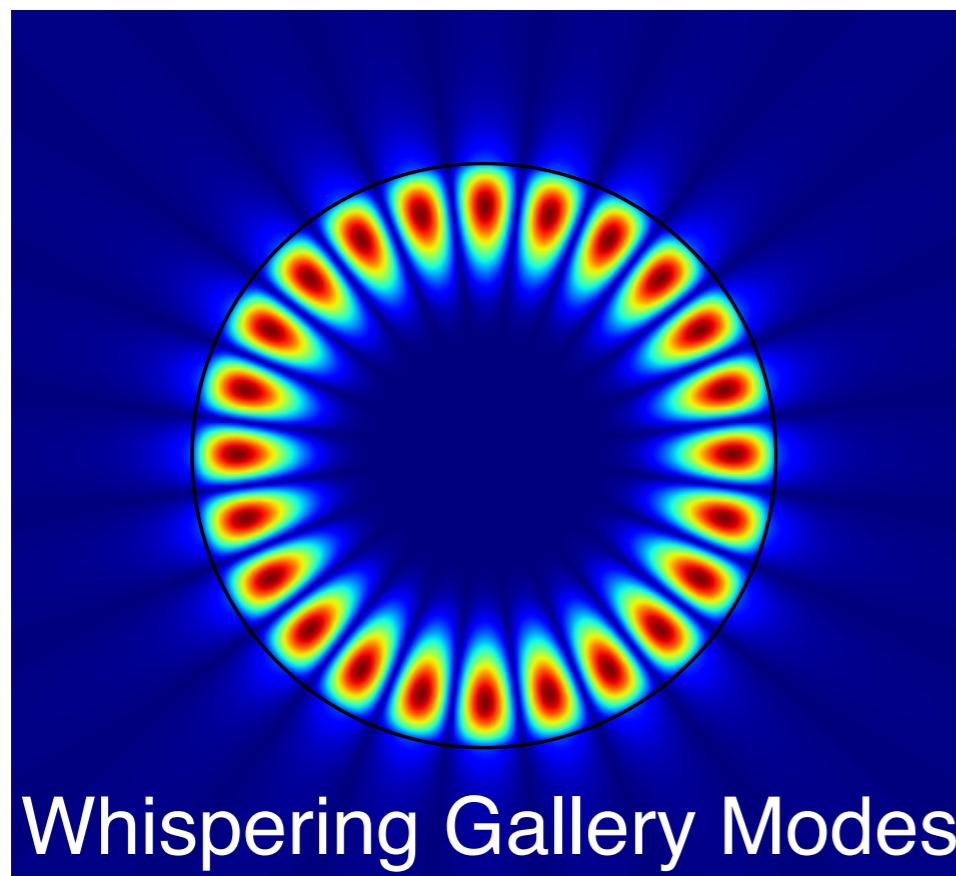


Whispering Gallery Modes

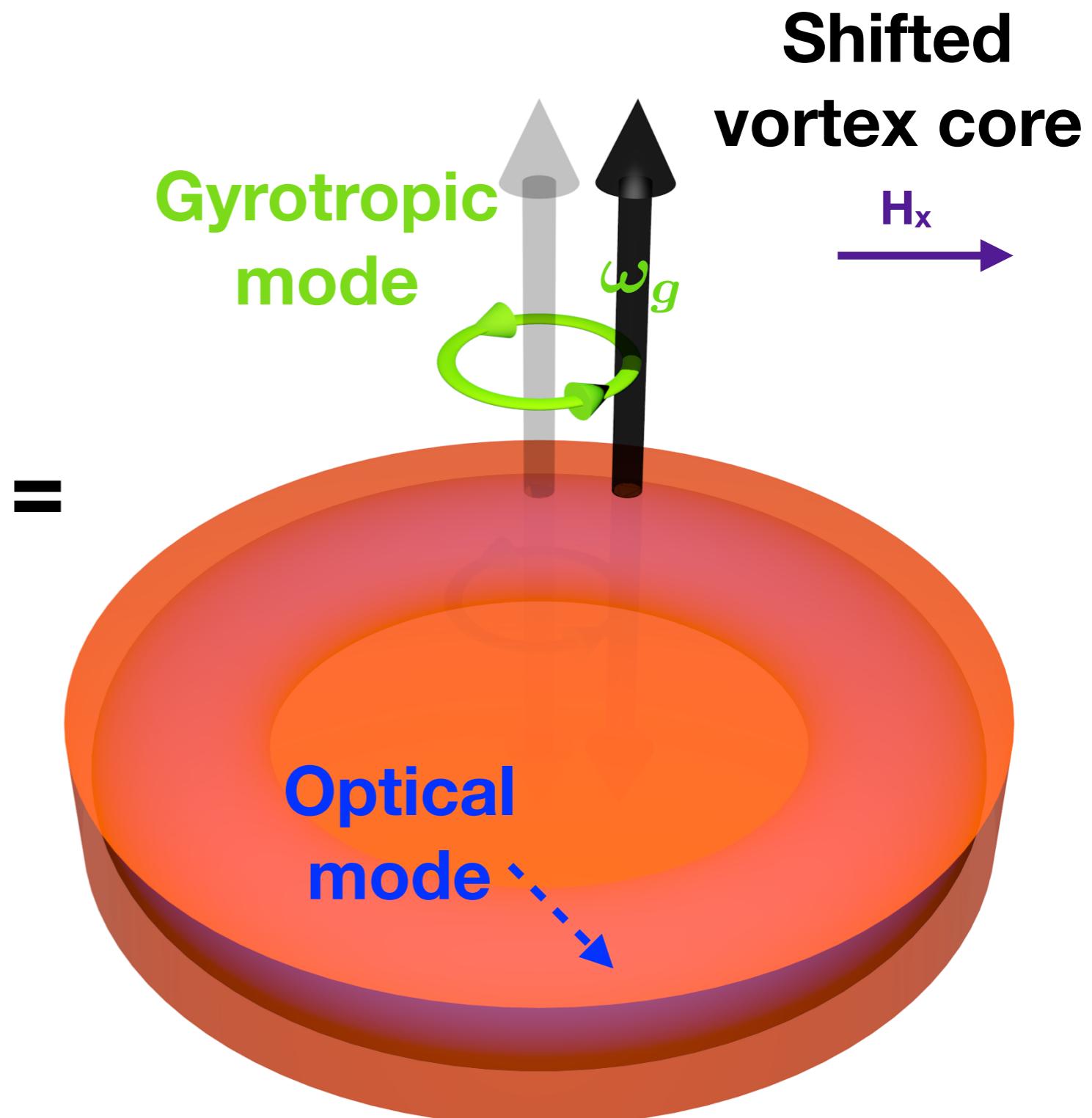
Magnetic Vortex and Optical Cavity



+



Whispering Gallery Modes

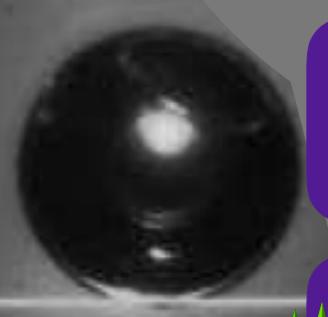


Shifted
vortex core

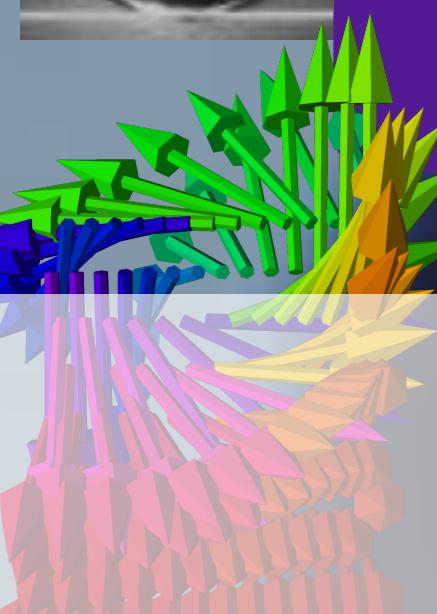
Gyrotropic
mode

Optical
mode

H_x



Introduction: cavity optomagnonics

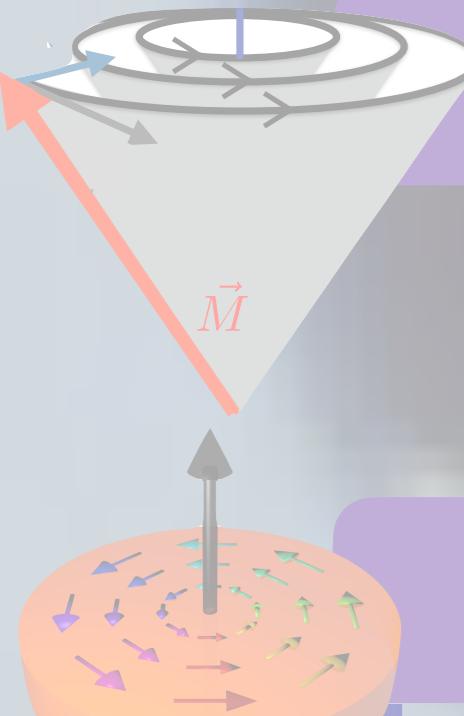


Magnetic textures

Why do they form?

Equilibrium condition

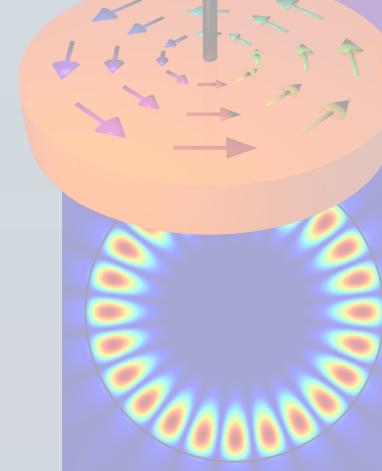
Topology of smooth textures



Dynamics of the magnetization

Landau Lifschitz Gilbert equation

Thiele equation for topological defects



Cavity optomagnonics with magnetic textures

Hamiltonian: Holstein-Primakoff

Optomagnetic coupling: gyrotropic mode

Magnetic Textures: why do they form?

Competition of interactions:

- Exchange energy:
short range, strong
- Dipolar interactions:
long range, weak
- Crystalline anisotropies
short range, defines easy and hard axis
- ...
(e.g. Dzyaloshinskii-Moriya)

Magnetic Textures: why do they form?

Magnetic Textures: why do they form?

$$E = \int_V d^3r \left[\frac{A}{M_s} \sum_{i=x,y,z} |\nabla M_i|^2 + U_{\text{an}}(\mathbf{M}) - \mu_0 \mathbf{M} \cdot \mathbf{H}_a - \frac{\mu_0}{2} \mathbf{M} \cdot \mathbf{H}_d \right]$$

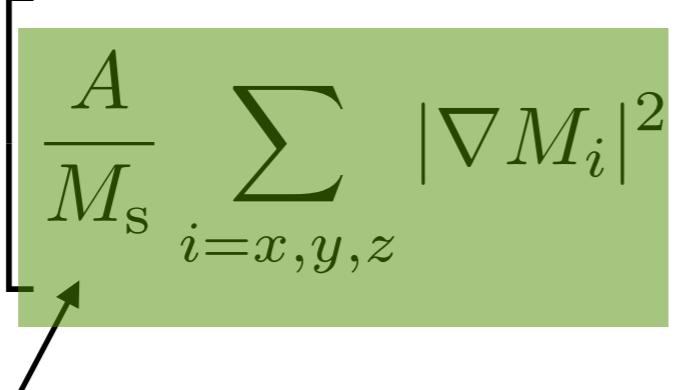
exchange

external field

anisotropy

dipolar

saturation magnetization

$$|\mathbf{M}(\mathbf{r})| = M_s$$


An arrow points from the text "saturation magnetization" to the term $\frac{A}{M_s} \sum_{i=x,y,z} |\nabla M_i|^2$ inside the green box.

Magnetic Textures: why do they form?

obtained from
Heisenberg

$$-\frac{J}{2} \sum_{} \hat{\mathbf{S}}_i \cdot \hat{\mathbf{S}}_j$$

Magnetic Textures: why do they form?

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↑

anisotropy

external field

exchange

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Magnetic Textures: why do they form?

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H_d magnetostatic: demagnetizing (stray) fields

$$\nabla \times \mathbf{H}_d = 0 \quad \rightarrow \quad \mathbf{H}_d = -\nabla \phi \quad \nabla^2 \phi = \nabla \cdot \mathbf{M}$$

$$-\nabla \cdot \mathbf{H}_d = \nabla \cdot \mathbf{M}$$

Magnetic Textures: why do they form?

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Poisson equation, nonlocal

$$\phi = -\frac{1}{4\pi} \int_V d^3r' \frac{\nabla \cdot \mathbf{M}}{|\mathbf{r} - \mathbf{r}'|} + \frac{1}{4\pi} \int_{\partial V} d^2r' \frac{\hat{\mathbf{n}}' \cdot \mathbf{M}}{|\mathbf{r} - \mathbf{r}'|}$$

Magnetic Textures: why do they form?

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► The magnetic energy is a non trivial functional of the magnetization

Magnetic Textures: why do they form?

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exchange

dipolar

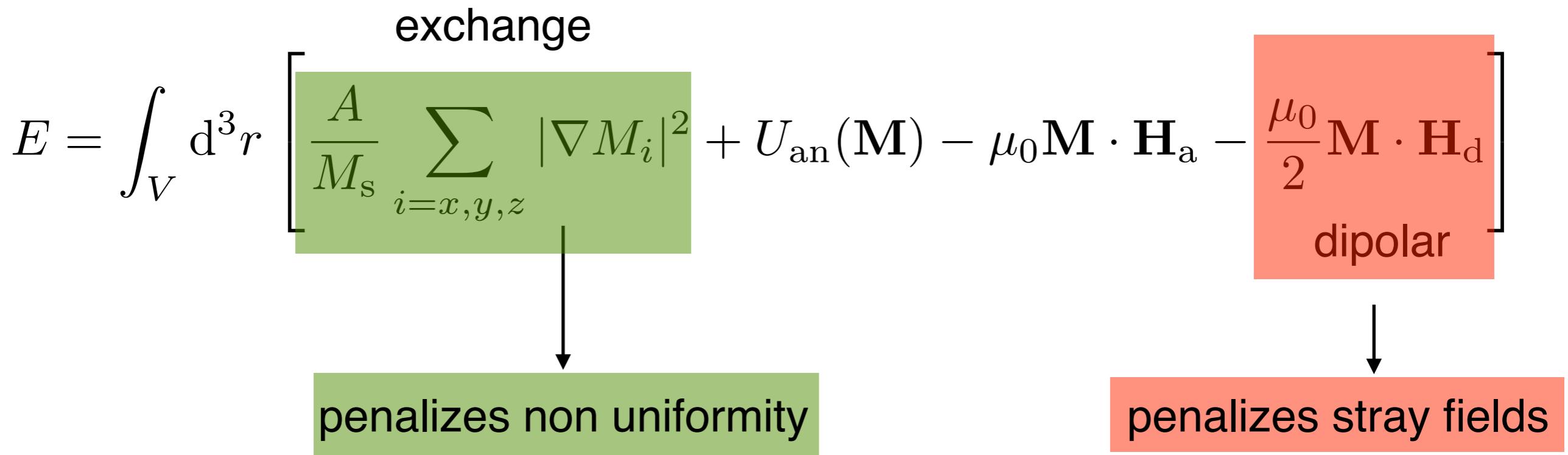
↓

penalizes non uniformity

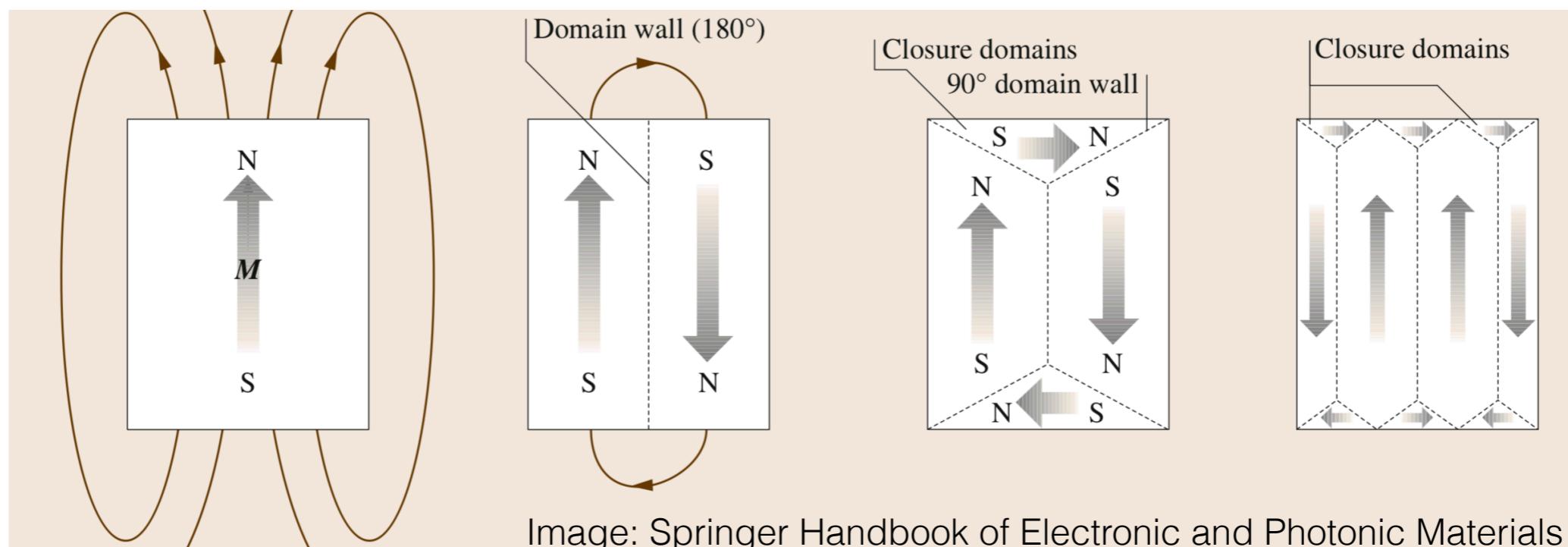
↓

penalizes stray fields

Magnetic Textures: why do they form?

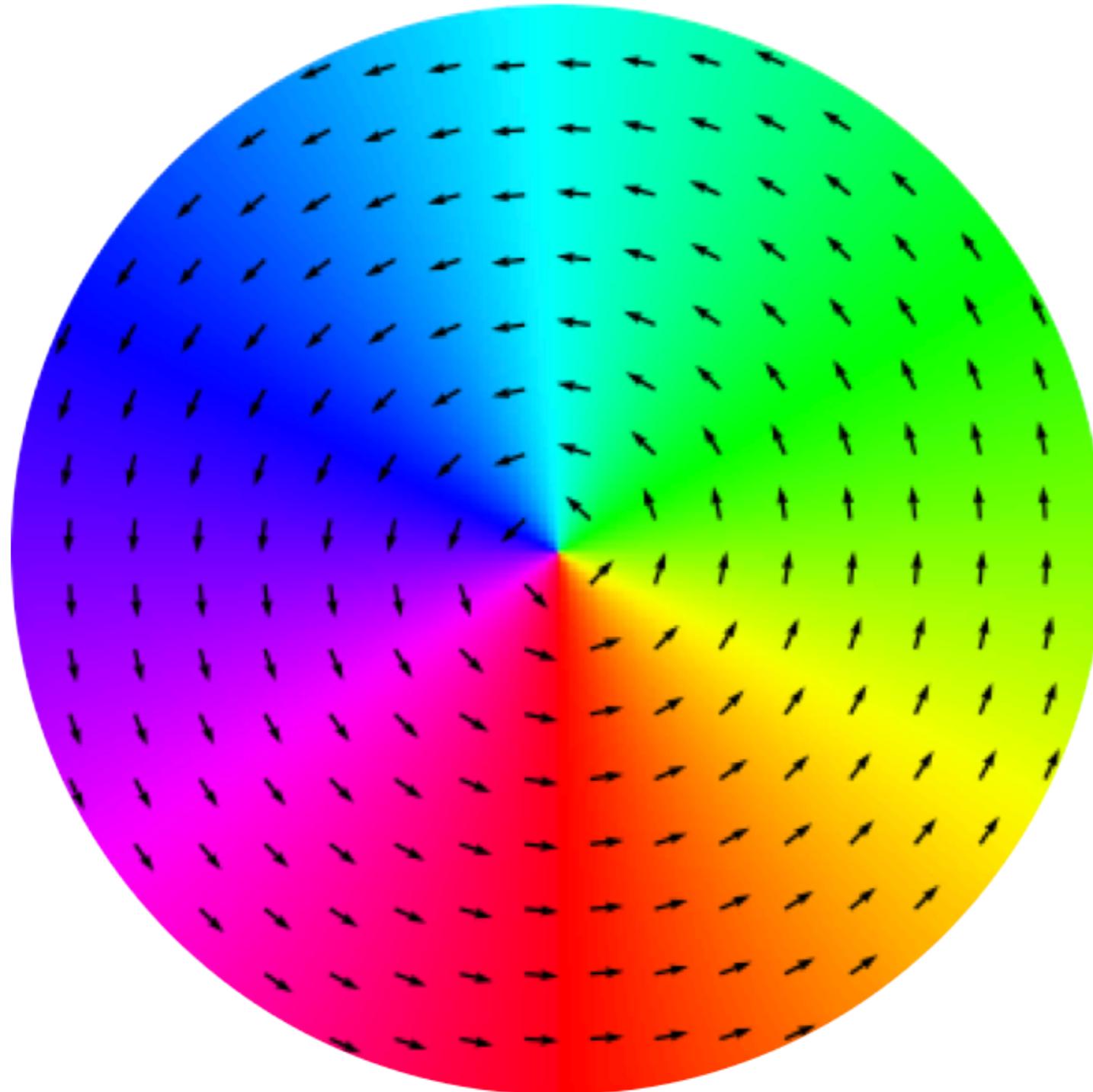


Equilibrium: Flux-closure configurations

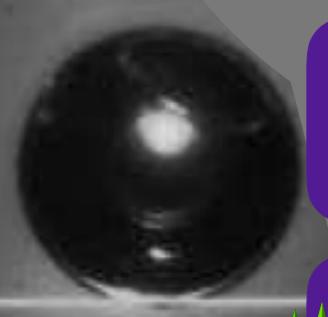


Magnetic Textures: why do they form?

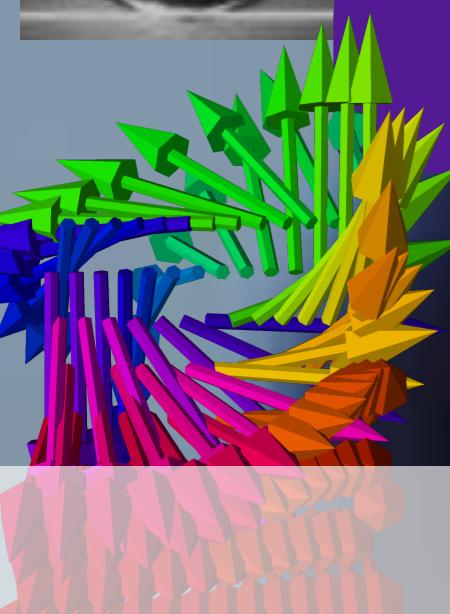
Vortex!



Microscale: Exchange energy \sim dipolar energy



Introduction: cavity optomagnonics



Magnetic textures

Why do they form?

Equilibrium condition

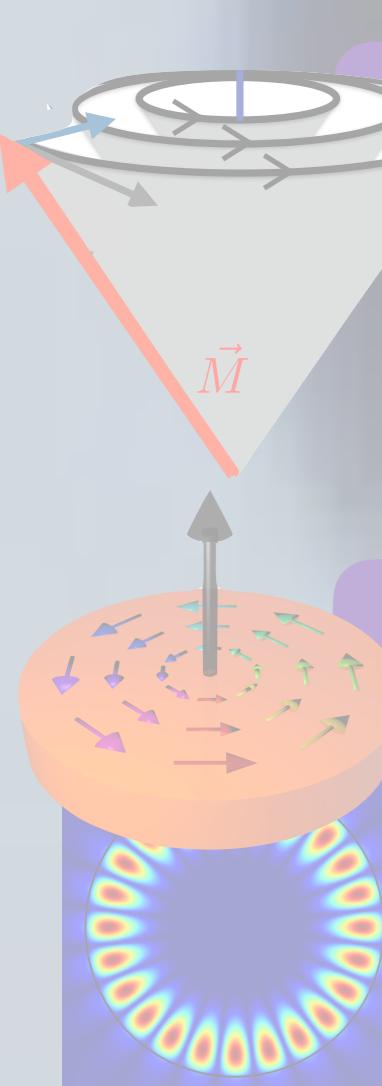
Topology of smooth textures



Dynamics of the magnetization

Landau Lifschitz Gilbert equation

Thiele equation for topological defects



Cavity optomagnonics with magnetic textures

Hamiltonian: Holstein-Primakoff

Optomagnetic coupling: gyrotropic mode

Magnetic Textures: Equilibrium

The magnetic energy is a non trivial functional of the magnetization

$$E = \int_V d^3r \left[\frac{A}{M_s} \sum_{i=x,y,z} |\nabla M_i|^2 + U_{\text{an}}(\mathbf{M}) - \mu_0 \mathbf{M} \cdot \mathbf{H}_a - \frac{\mu_0}{2} \mathbf{M} \cdot \mathbf{H}_d \right]$$

Equilibrium condition: minimize the energy functional

$$\delta E = -\mu_0 \int_V d^3r \mathbf{H}_{\text{eff}} \cdot \delta \mathbf{M} - \frac{2A}{\mu_0 M_s^2} \oint_{\partial V} d^2r \frac{\partial \mathbf{M}}{\partial \mathbf{n}} \cdot \delta \mathbf{M} = 0$$

Magnetic Textures: Equilibrium

The magnetic energy is a non trivial functional of the magnetization

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Effective magnetic field

$$\mathbf{H}_{\text{eff}} = \mathbf{H}_a + \mathbf{H}_d + \mathbf{H}_{\text{an}} + \mathbf{H}_{\text{ex}}$$

$$\mathbf{H}_{\text{an}} = -\frac{1}{\mu_0} \frac{\partial U_{\text{an}}}{\partial \mathbf{M}}$$

$$\mathbf{H}_{\text{ex}} = \frac{2A}{\mu_0 M_s^2} \nabla^2 \mathbf{M}$$

Magnetic Textures: Equilibrium

The magnetic energy is a non trivial functional of the magnetization

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Effective magnetic field

$$\mathbf{H}_{\text{eff}} = \mathbf{H}_a + \mathbf{H}_d + \mathbf{H}_{\text{an}} + \mathbf{H}_{\text{ex}}$$

imposing the constraint:

$$|\mathbf{M}(\mathbf{r})| = M_s$$



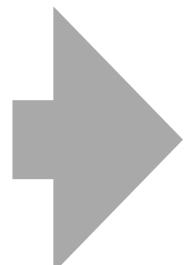
$$\delta \mathbf{M} = \mathbf{M} \times \delta \mathbf{v}$$

arbitrary vector

Magnetic Textures: Equilibrium

$$\delta E = -\mu_0 \int_V d^3r \mathbf{H}_{\text{eff}} \cdot \delta \mathbf{M} - \frac{2A}{\mu_0 M_s^2} \oint_{\partial V} d^2r \frac{\partial \mathbf{M}}{\partial \mathbf{n}} \cdot \delta \mathbf{M} = 0$$

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Magnetic Textures: Equilibrium

$$\delta E = -\mu_0 \int_V d^3r \mathbf{H}_{\text{eff}} \cdot \delta \mathbf{M} - \frac{2A}{\mu_0 M_s^2} \oint_{\partial V} d^2r \frac{\partial \mathbf{M}}{\partial \mathbf{n}} \cdot \delta \mathbf{M} = 0$$

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equilibrium condition


$$\mathbf{M} \times \mathbf{H}_{\text{eff}}(\mathbf{M}) = 0$$

$$\left. \frac{\partial \mathbf{M}}{\partial \mathbf{n}} \right|_{\delta V} = 0$$

$$\mathbf{H}_{\text{eff}} = \mathbf{H}_a + \mathbf{H}_d + \mathbf{H}_{\text{an}} + \mathbf{H}_{\text{ex}}$$

Very difficult to solve in general

→ micromagnetic simulations

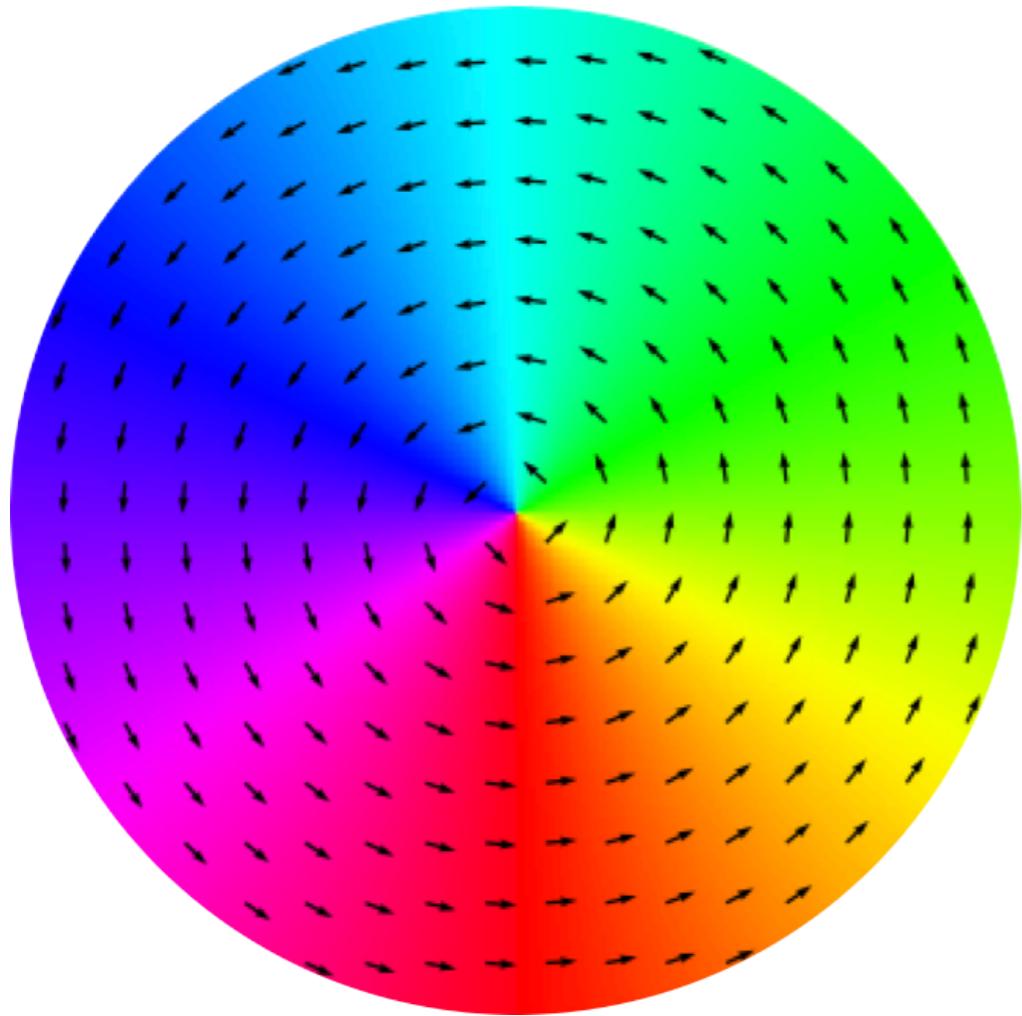
computationally costly

Example: Vortex in thin microdisk

Height of disk

$$h < L_{\text{ex}} = \frac{1}{M_s} \sqrt{\frac{2A}{\mu_0}}$$

2D description



$$\mathbf{H}_a = 0$$

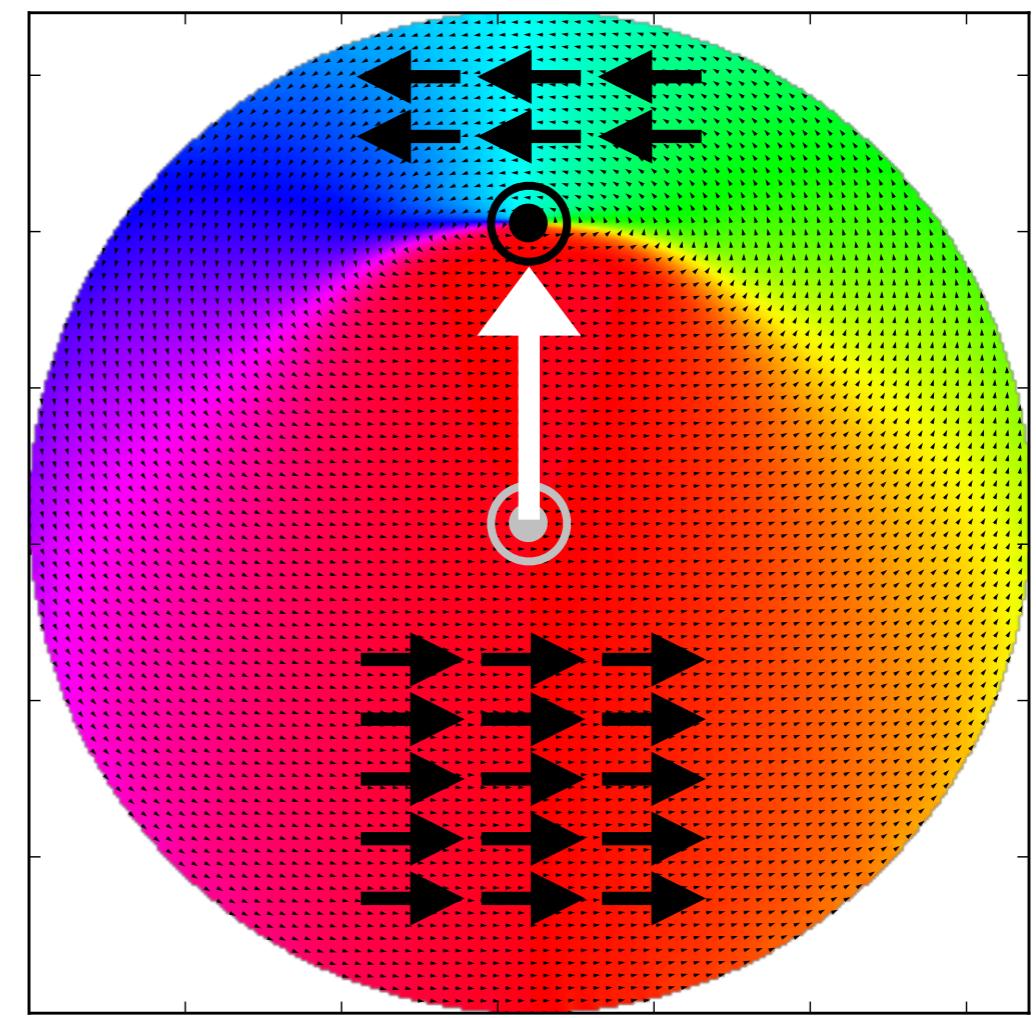
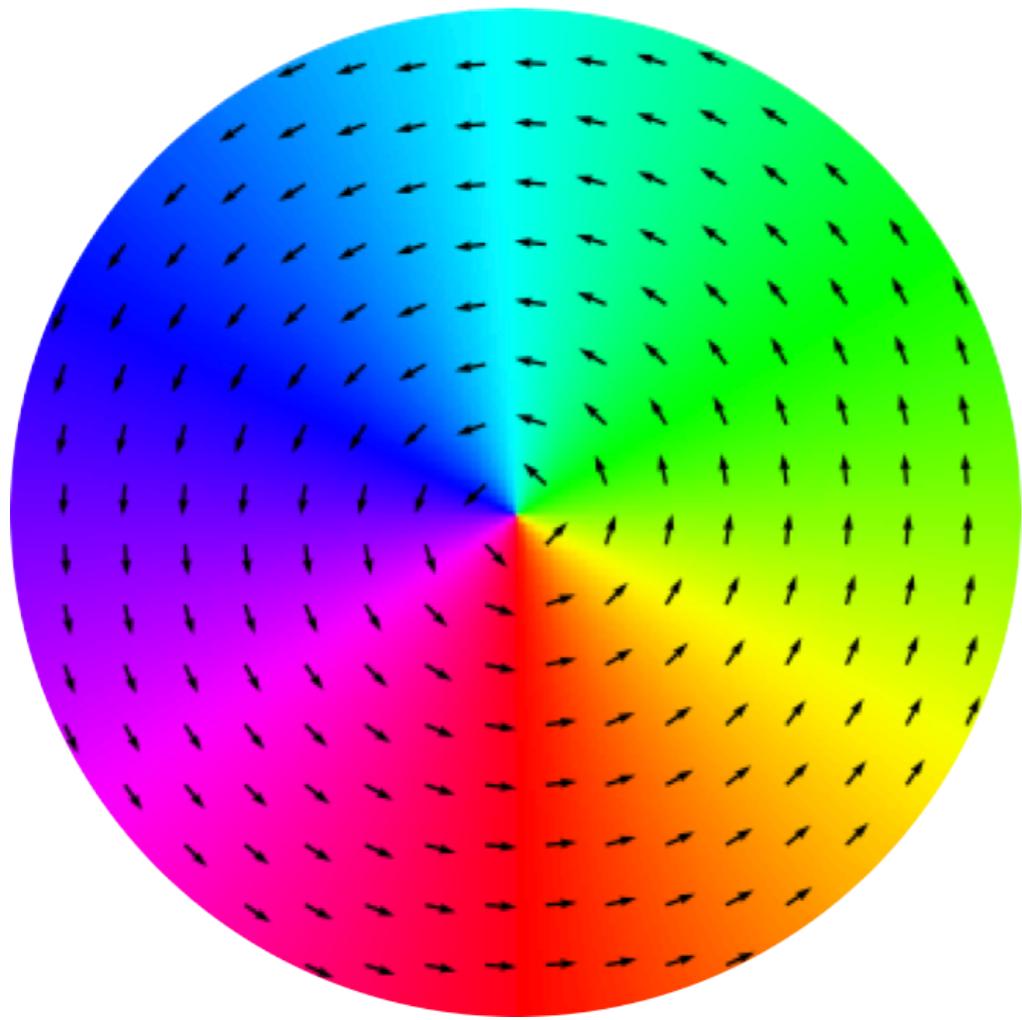
Example: Vortex in thin microdisk

Height of disk

$$h < L_{\text{ex}} = \frac{1}{M_s} \sqrt{\frac{2A}{\mu_0}}$$

2D description

Position tunable by a magnetic field

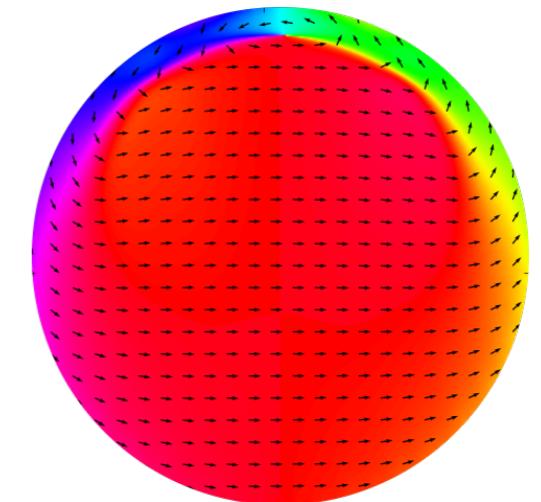
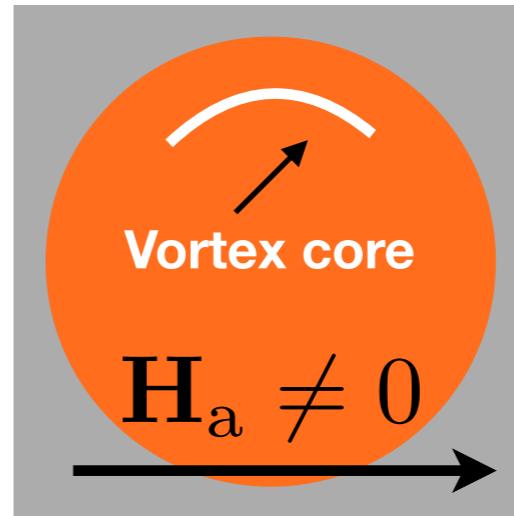
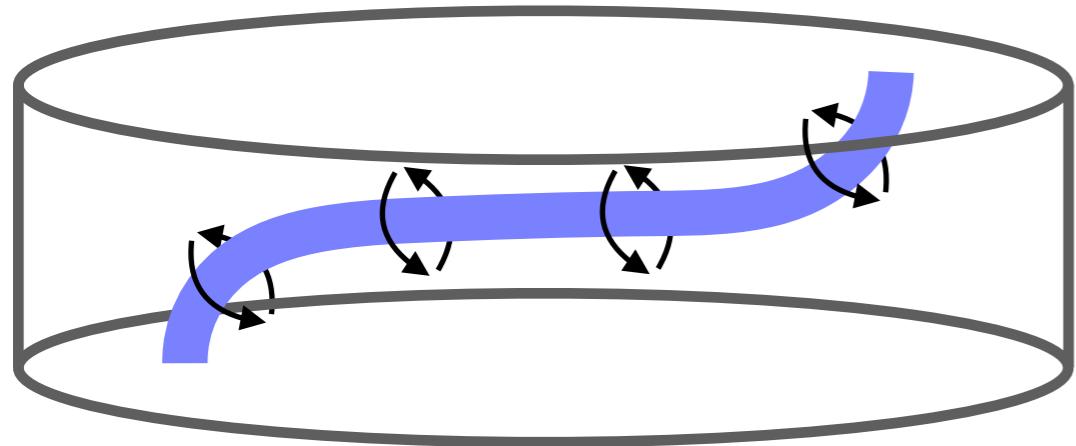


$H_a = 0$

$H_a \neq 0$

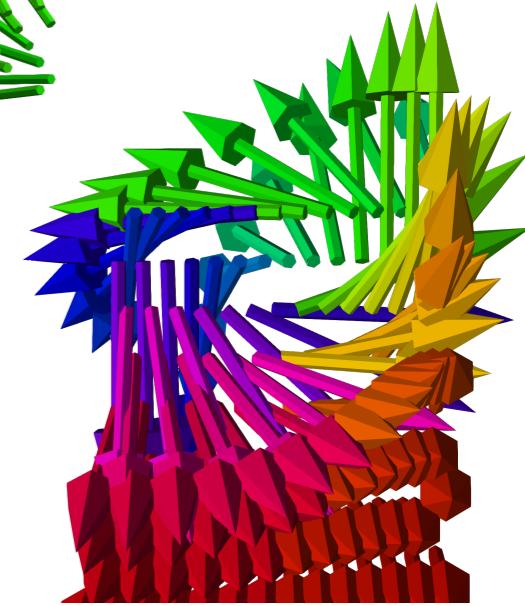
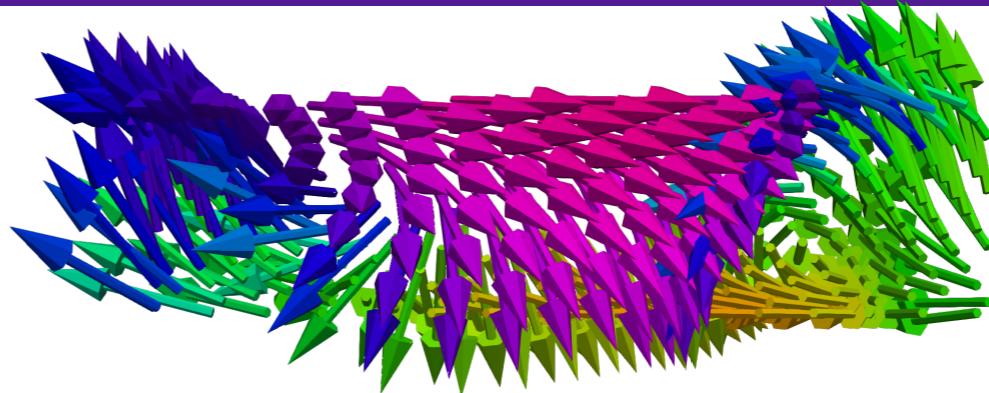
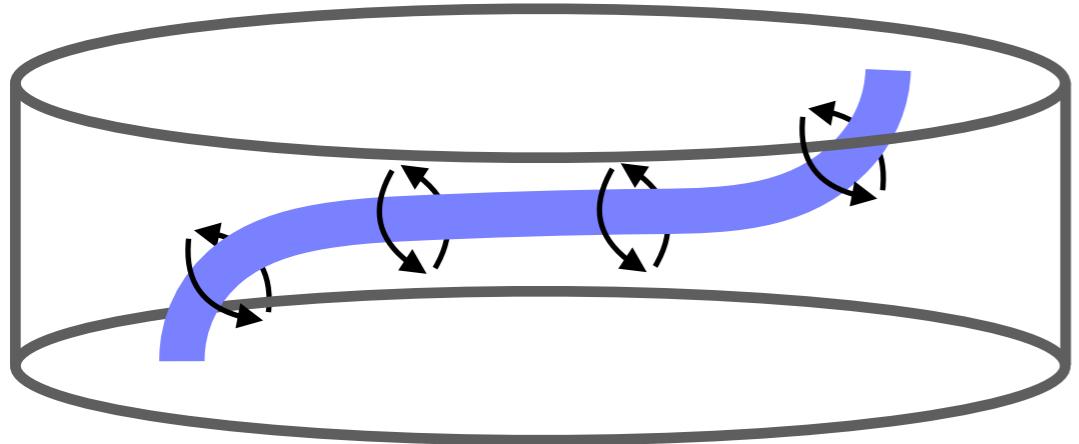
Vortex in a thick microdisk

Vortex

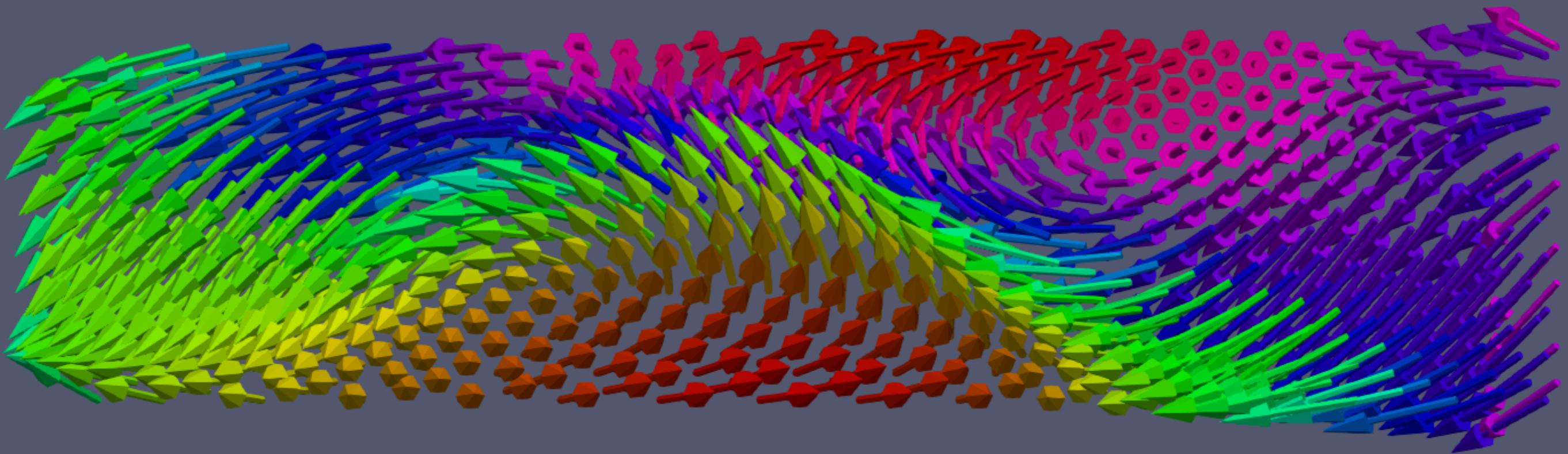


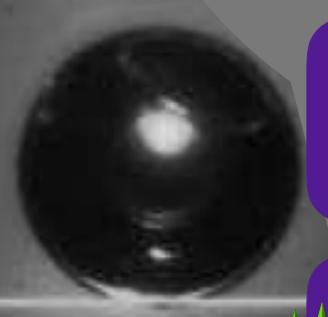
Vortex in a thick microdisk

Vortex

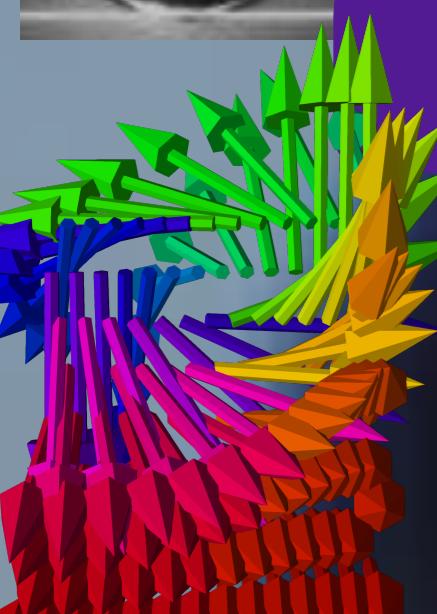


Definitely not 2D!!





Introduction: cavity optomagnonics

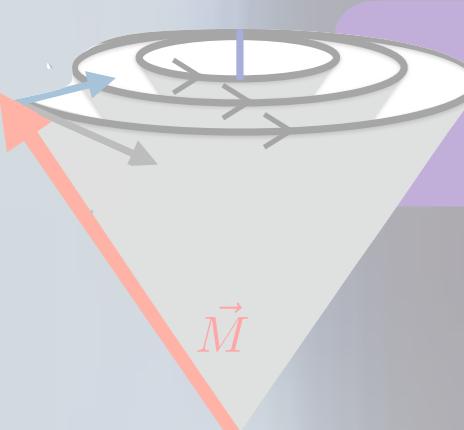


Magnetic textures

Why do they form?

Equilibrium condition

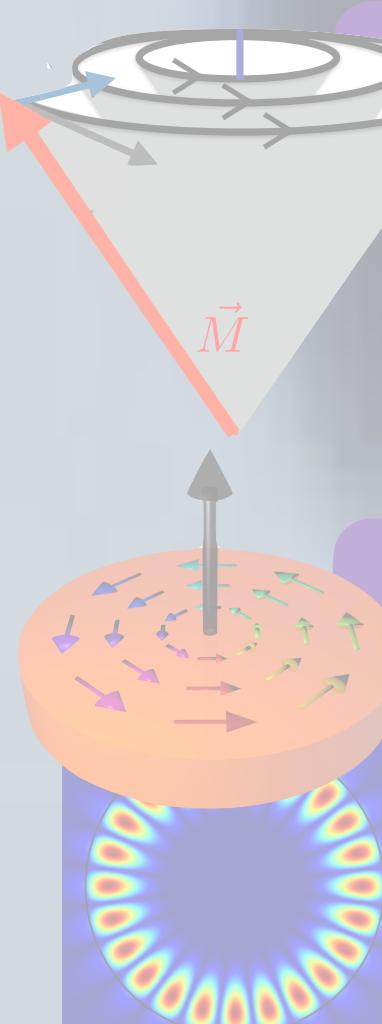
Topology of smooth textures



Dynamics of the magnetization

Landau Lifschitz Gilbert equation

Thiele equation for topological defects



Cavity optomagnonics with magnetic textures

Hamiltonian: Holstein-Primakoff

Optomagnetic coupling: gyrotropic mode

Topology of smooth magnetic textures

Two relevant topological numbers

Easy-plane spins:

$$f : S^1 \rightarrow S^1 \quad \text{e.g.:} \quad f \equiv \mathbf{m} = (\cos \phi(x), \sin \phi(x), 0)$$



$$w_{S^1} = \frac{1}{2\pi} \oint dx \partial_x \phi \quad \text{“winding number” } q$$

Topology of smooth magnetic textures

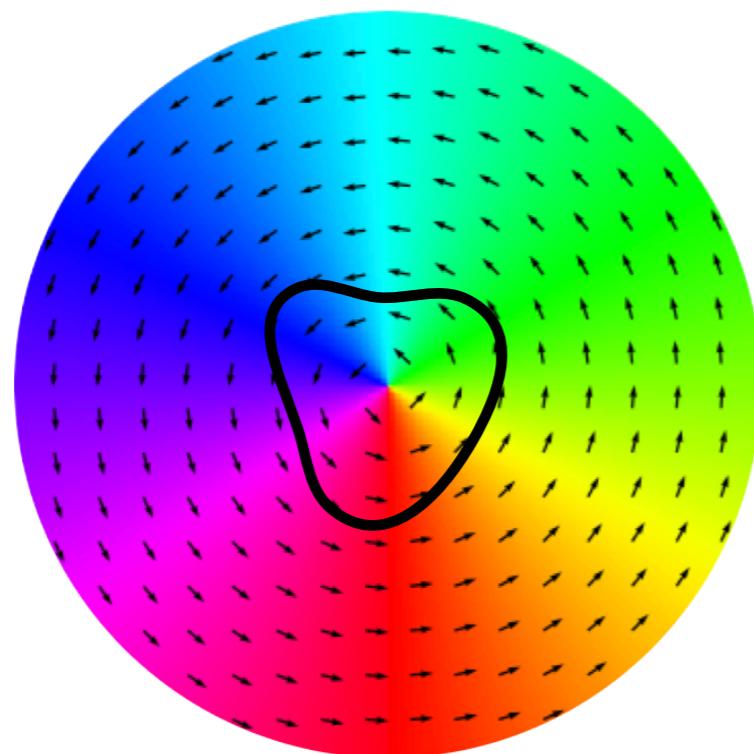
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E.g. 2D vortex

$$q = 1$$

Topology of smooth magnetic textures

Two relevant topological numbers

Easy-plane spins:

see e.g. H.B Braun, Adv. in Phys. 61, 1 (2011)

$$f : S^1 \rightarrow S^1 \quad \text{e.g.:} \quad f \equiv \mathbf{m} = (\cos \phi(x), \sin \phi(x), 0)$$



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Bloch-sphere spins:

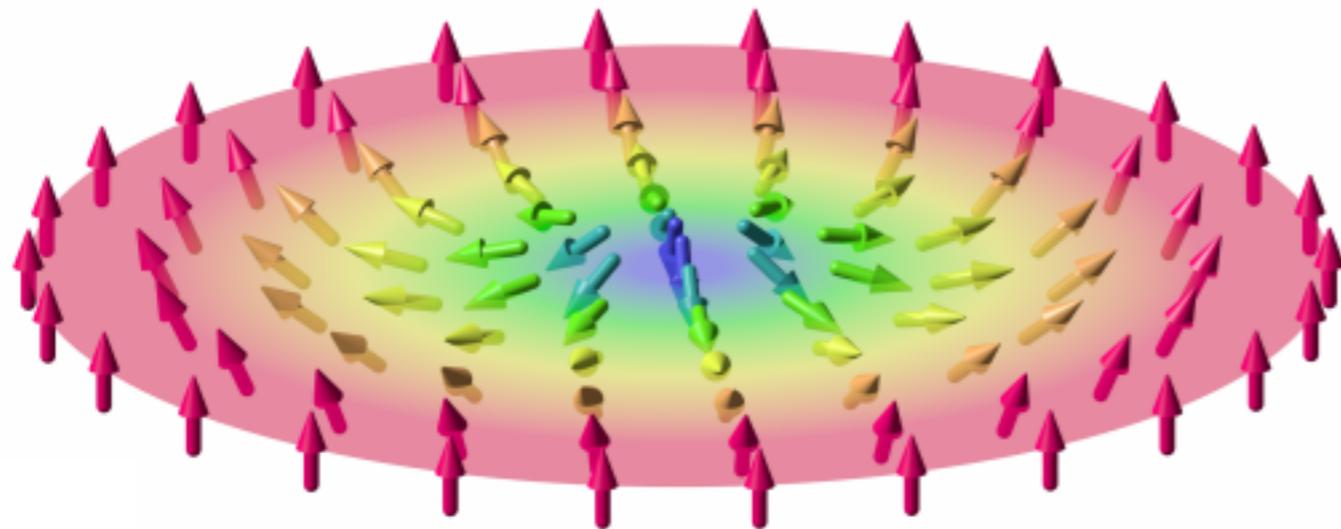
$$f : S^2 \rightarrow S^2 \quad \text{e.g.:} \quad f \equiv \mathbf{m} = (m_1(x_1, x_2), m_2(x_1, x_2), m_3(x_1, x_2))$$



$$w_{S^2} = \frac{1}{4\pi} \int_M \mathbf{m} \cdot (\partial_1 \mathbf{m} \times \partial_2 \mathbf{m}) dx_1 dx_2 \quad \text{"skyrmion number"}$$

Topology of smooth magnetic textures

Skyrmion in a magnetic thin film



$$q = 1$$

$$p = -1$$

Image: Karin Everschor-Sitte (Wikipedia)

$$w_{S^2} = \frac{1}{4\pi} \int_M \mathbf{m} \cdot (\partial_1 \mathbf{m} \times \partial_2 \mathbf{m}) dx_1 dx_2$$

Skyrmion number

$$w_{S^2} = p \quad w_{S^1} = p q$$

polarity:  or 
+1 -1

Topology of smooth magnetic textures

Skyrmion in a magnetic thin film

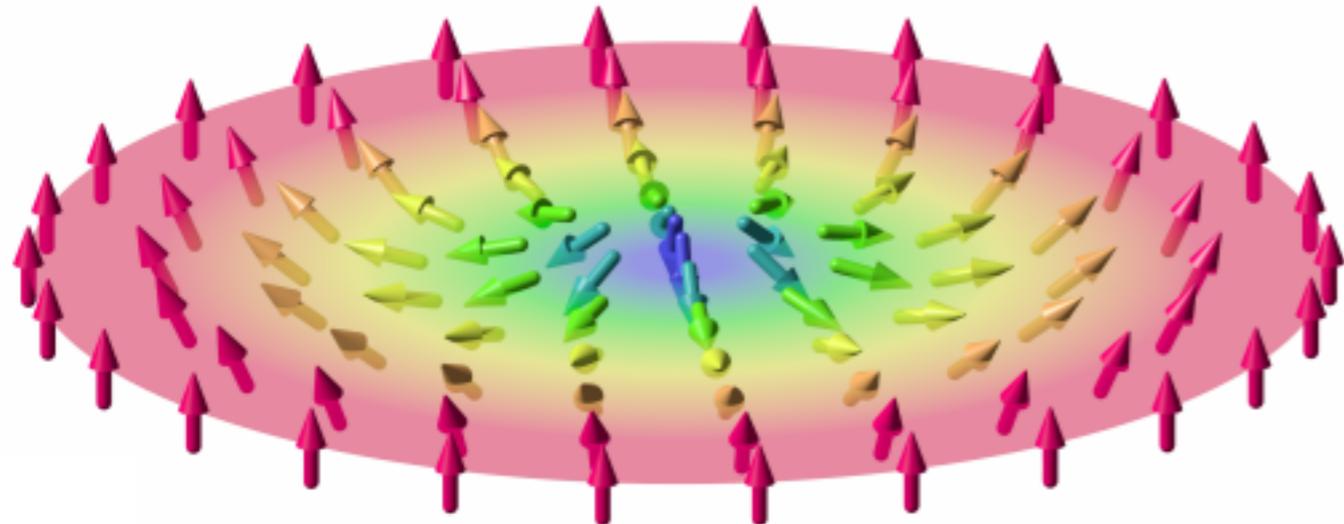


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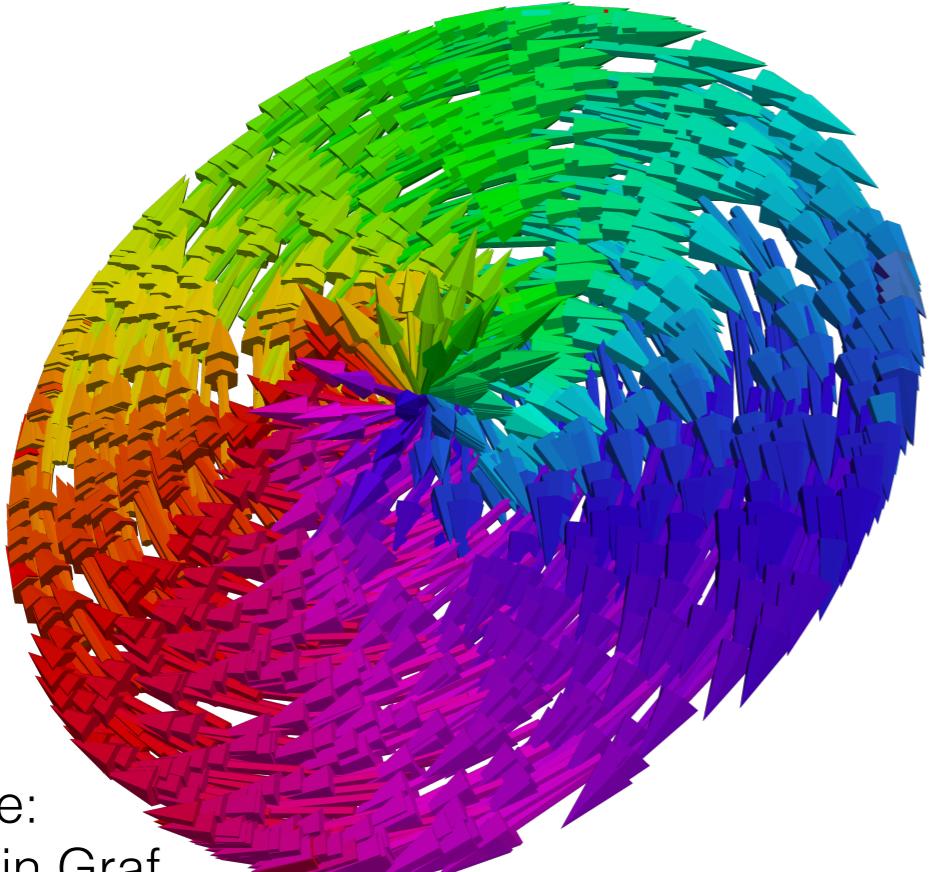
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Vortex in a magnetic thin microdisk



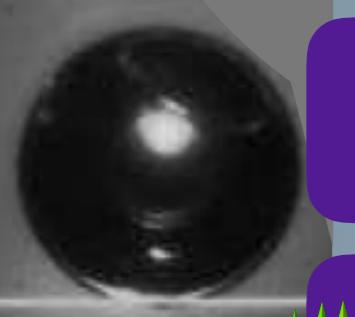
(Meron)

Image:
Jasmin Graf

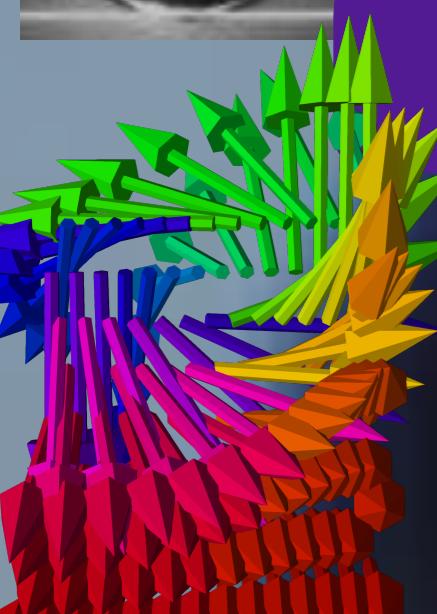
Skyrmion number

$$w_{S^2} = \frac{p q}{2}$$

“half a skyrmion”



Introduction: cavity optomagnonics

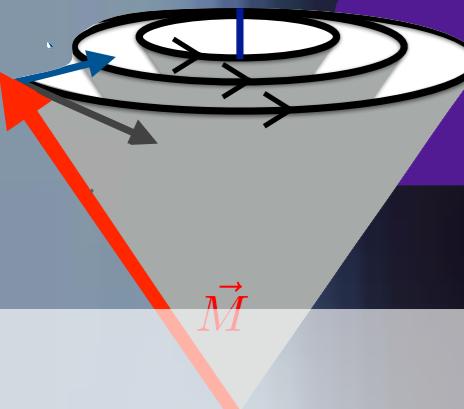


Magnetic textures

Why do they form?

Equilibrium condition

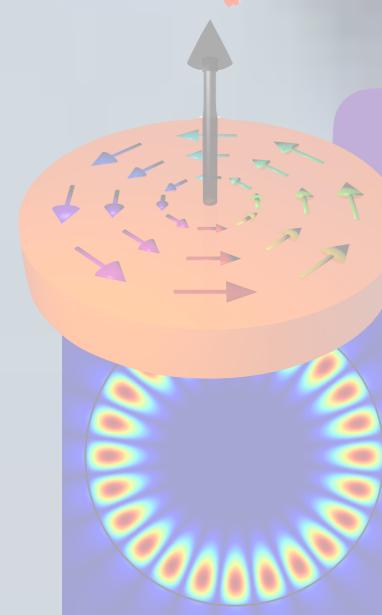
Topology of smooth textures



Dynamics of the magnetization

Landau Lifschitz Gilbert equation

Thiele equation for topological defects



Cavity optomagnonics with magnetic textures

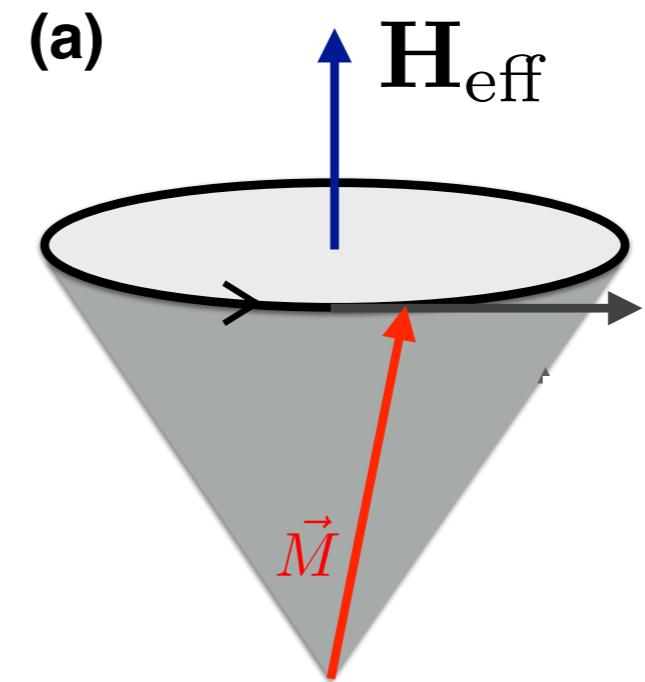
Hamiltonian: Holstein-Primakoff

Optomagnetic coupling: gyrotropic mode

Dynamics of \mathbf{M} : Landau Lifschitz equation

Precession of the magnetization

$$\frac{\partial \mathbf{M}}{\partial t} = -\gamma \mathbf{M} \times \mathbf{H}_{\text{eff}}$$



Dynamics of \mathbf{M} : Landau Lifschitz equation

Precession of the magnetization

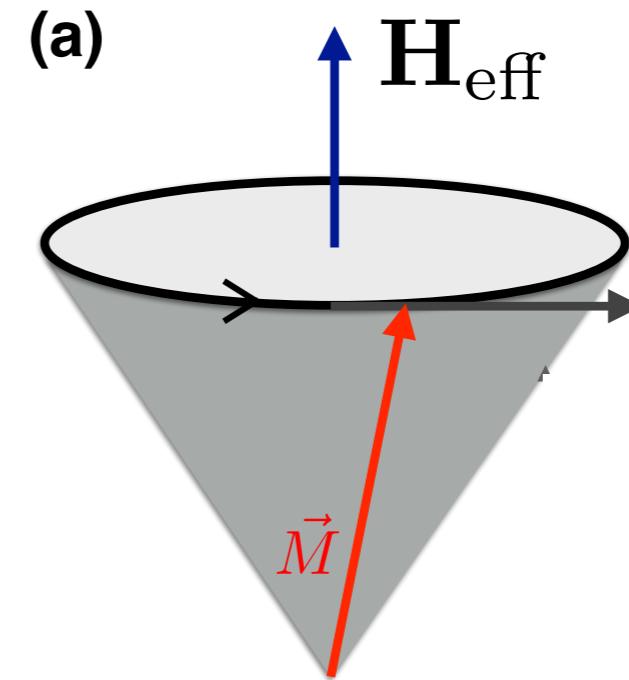
$$\frac{\partial \mathbf{M}}{\partial t} = -\gamma \mathbf{M} \times \mathbf{H}_{\text{eff}}$$

Can be derived from
the Lagrangian

$$L = \frac{M_s}{\gamma} \int_V d^3r (1 - \cos \theta) \dot{\phi} - E$$

geometrical phase

$$\frac{\mathbf{M}}{M_s} = \mathbf{m} = (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta)$$



Dynamics of \mathbf{M} : Landau Lifschitz equation

Precession of the magnetization

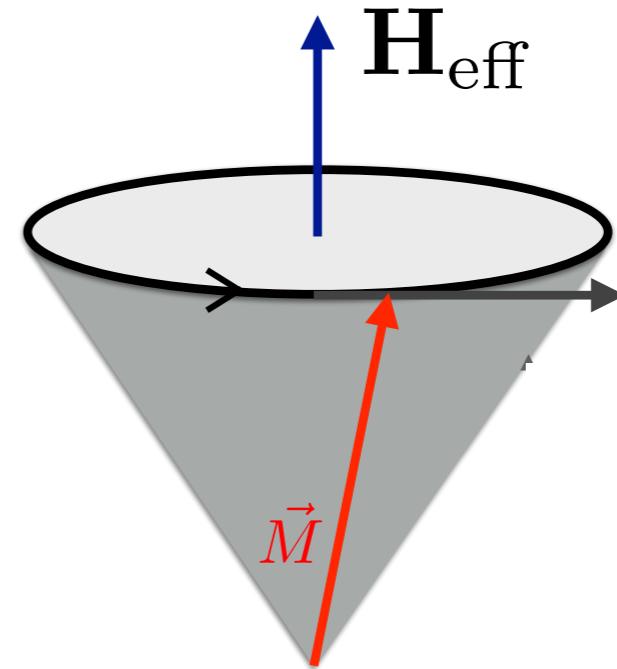
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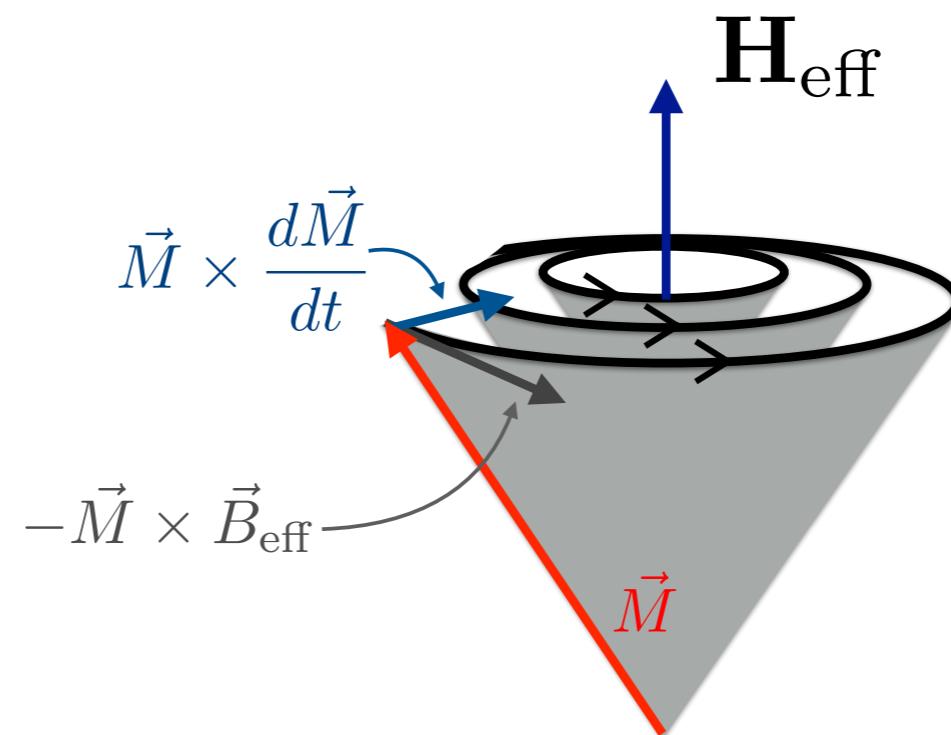
or from the quantum Hamiltonian via Heisenberg EOM

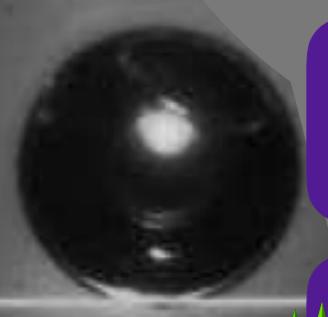
Dynamics of \mathbf{M} : Landau Lifschitz Gilbert equation

Damped precession

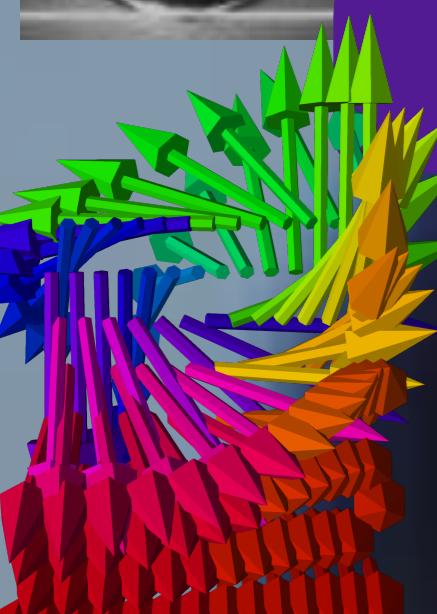
$$\frac{\partial \mathbf{M}}{\partial t} = -\gamma \mathbf{M} \times \mathbf{H}_{\text{eff}} + \frac{\alpha}{M_s} \left(\mathbf{M} \times \frac{\partial \mathbf{M}}{\partial t} \right)$$

Gilbert damping





Introduction: cavity optomagnonics

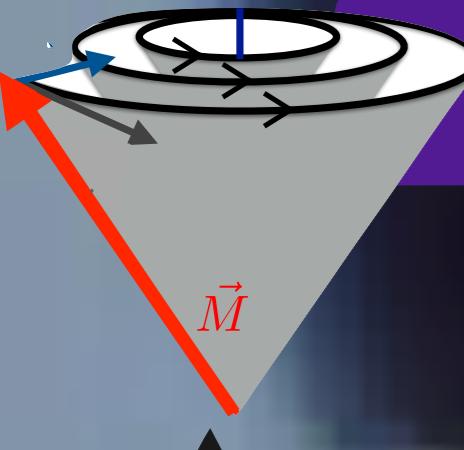


Magnetic textures

Why do they form?

Equilibrium condition

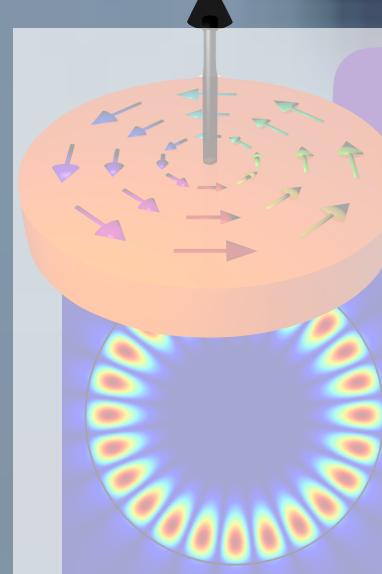
Topology of smooth textures



Dynamics of the magnetization

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Cavity optomagnonics with magnetic textures

Hamiltonian: Holstein-Primakoff

Optomagnetic coupling: gyrotropic mode

Dynamics of the vortex: Thiele equation

Rigid translation: vortex parametrized just by the core's position

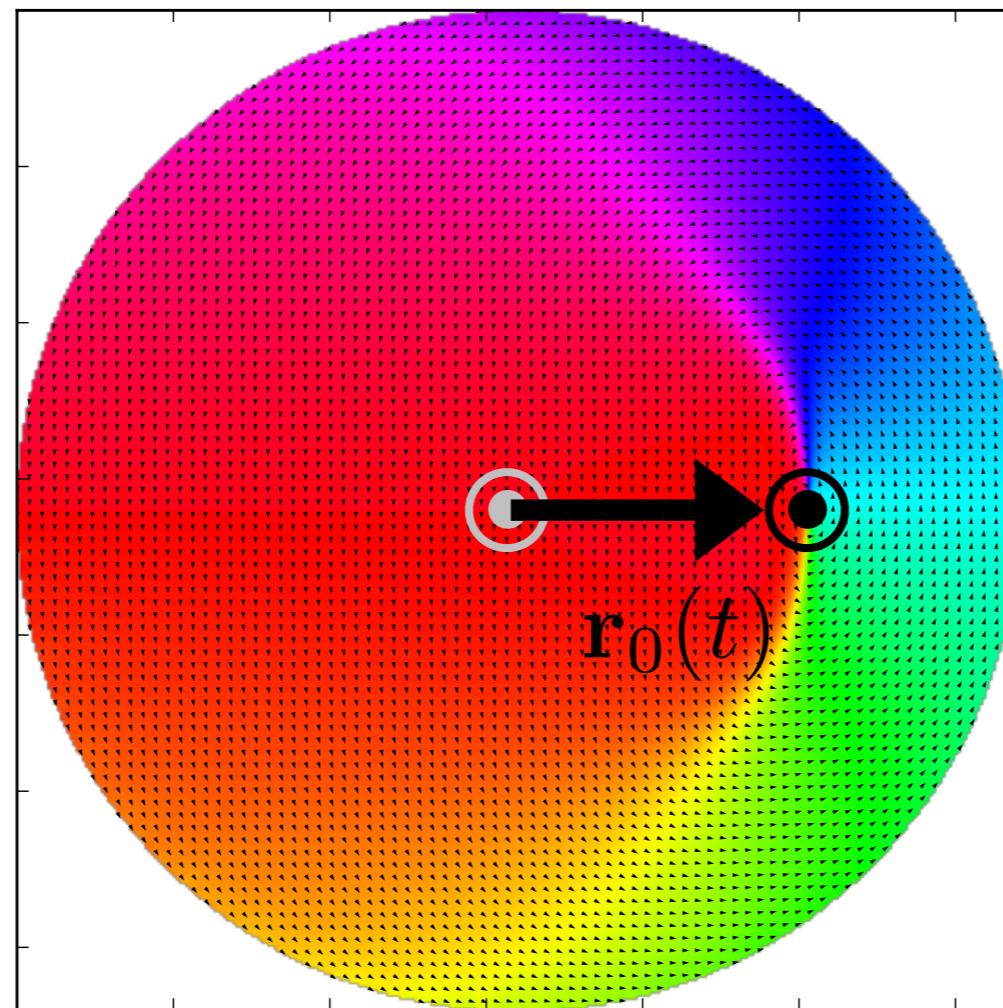
$$\mathbf{m}(\mathbf{r}, t) = \mathbf{m}(\mathbf{r} - \mathbf{r}_0(t))$$

$$\mathbf{m} = (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta)$$



$$\theta(\mathbf{r}, t) = \theta(\mathbf{r} - \mathbf{r}_0(t))$$

$$\phi(\mathbf{r}, t) = \phi(\mathbf{r} - \mathbf{r}_0(t))$$



Dynamics of the vortex: Thiele equation

Rigid translation: vortex parametrized just by the core's position

$$\mathbf{m}(\mathbf{r}, t) = \mathbf{m}(\mathbf{r} - \mathbf{r}_0(t))$$

$$\mathbf{m} = (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta)$$



$$\theta(\mathbf{r}, t) = \theta(\mathbf{r} - \mathbf{r}_0(t))$$

$$\phi(\mathbf{r}, t) = \phi(\mathbf{r} - \mathbf{r}_0(t))$$



need only the EOM for $\mathbf{r}_0(t)$

Thiele derived it from the LLG equation:

$$\mathbf{H}_{\text{eff}} = \frac{1}{\gamma} (\mathbf{m} \times \dot{\mathbf{m}} + \alpha \dot{\mathbf{m}}) \cdot$$

$$\dot{\mathbf{m}} = - \sum_i v_i \frac{\partial \mathbf{m}}{\partial r_i}$$

$$\text{with } \mathbf{v} = \dot{\mathbf{r}}_0$$

A.A. Thiele, PRL 230 (1972)

Dynamics of the vortex: Thiele equation

$$\mathbf{m}(\mathbf{r}, t) = \mathbf{m}(\mathbf{r} - \mathbf{r}_0(t))$$

$$\mathbf{H}_{\text{eff}} = \frac{1}{\gamma} (\mathbf{m} \times \dot{\mathbf{m}} + \alpha \ddot{\mathbf{m}})$$

$$\dot{\mathbf{m}} = - \sum_i v_i \frac{\partial \mathbf{m}}{\partial r_i} \quad \mathbf{v} = \dot{\mathbf{r}}_0$$

The force on the “particle” is

$$F_i = - \frac{dE}{dr_{0i}} = \mu_0 M_s \int_V d^3r \mathbf{H}_{\text{eff}} \cdot \frac{\partial \mathbf{m}}{\partial r_{0i}} = - \mu_0 M_s \int_V d^3r \mathbf{H}_{\text{eff}} \cdot \frac{\partial \mathbf{m}}{\partial r_i}$$

Dynamics of the vortex: Thiele equation

$$\mathbf{m}(\mathbf{r}, t) = \mathbf{m}(\mathbf{r} - \mathbf{r}_0(t))$$

$$\mathbf{H}_{\text{eff}} = \frac{1}{\gamma} (\mathbf{m} \times \dot{\mathbf{m}} + \alpha \ddot{\mathbf{m}})$$

$$\dot{\mathbf{m}} = - \sum_i v_i \frac{\partial \mathbf{m}}{\partial r_i}$$

$$\mathbf{v} = \dot{\mathbf{r}}_0$$

The force on the “particle” is

$$F_i = -\frac{dE}{dr_{0i}} = \mu_0 M_s \int_V d^3r \mathbf{H}_{\text{eff}} \cdot \frac{\partial \mathbf{m}}{\partial r_{0i}} = -\mu_0 M_s \int_V d^3r \mathbf{H}_{\text{eff}} \cdot \frac{\partial \mathbf{m}}{\partial r_i}$$



$$F_i = -\frac{\mu_0 M_s}{\gamma} \int_V d^3r \sum_j v_j \mathbf{m} \cdot \left(\frac{\partial \mathbf{m}}{\partial r_i} \times \frac{\partial \mathbf{m}}{\partial r_j} \right) + \alpha \frac{\mu_0 M_s}{\gamma} \int_V d^3r \sum_j v_j \left(\frac{\partial \mathbf{m}}{\partial r_i} \cdot \frac{\partial \mathbf{m}}{\partial r_j} \right)$$

Gyrotropic force

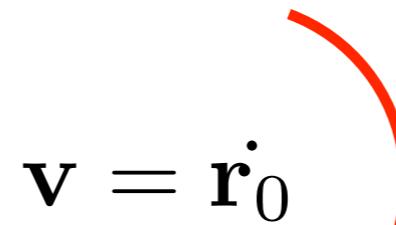
Dissipative force

Dynamics of the vortex: Thiele equation

$$\mathbf{m}(\mathbf{r}, t) = \mathbf{m}(\mathbf{r} - \mathbf{r}_0(t))$$

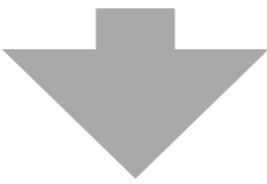
$$\mathbf{H}_{\text{eff}} = \frac{1}{\gamma} (\mathbf{m} \times \dot{\mathbf{m}} + \alpha \ddot{\mathbf{m}})$$

$$\dot{\mathbf{m}} = - \sum_i v_i \frac{\partial \mathbf{m}}{\partial r_i}$$

$$\mathbf{v} = \dot{\mathbf{r}}_0$$


The force on the “particle” is

$$F_i = -\frac{dE}{dr_{0i}} = \mu_0 M_s \int_V d^3r \mathbf{H}_{\text{eff}} \cdot \frac{\partial \mathbf{m}}{\partial r_{0i}} = -\mu_0 M_s \int_V d^3r \mathbf{H}_{\text{eff}} \cdot \frac{\partial \mathbf{m}}{\partial r_i}$$



$$F_i = -\frac{\mu_0 M_s}{\gamma} \int_V d^3r \sum_j v_j \mathbf{m} \cdot \left(\frac{\partial \mathbf{m}}{\partial r_i} \times \frac{\partial \mathbf{m}}{\partial r_j} \right) + \alpha \frac{\mu_0 M_s}{\gamma} \int_V d^3r \sum_j v_j \left(\frac{\partial \mathbf{m}}{\partial r_i} \cdot \frac{\partial \mathbf{m}}{\partial r_j} \right)$$

Gyrotropic force \mathbf{F}_g

A.A. Thiele, PRL 230 (1972)

Dynamics of the vortex: Thiele equation

$$\mathbf{m}(\mathbf{r}, t) = \mathbf{m}(\mathbf{r} - \mathbf{r}_0(t)) \quad \mathbf{v} = \dot{\mathbf{r}}_0$$

Gyrotropic force on the topological defect

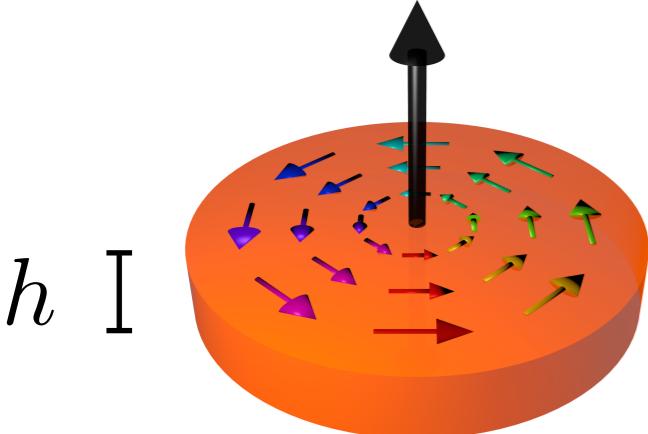
$$\mathbf{F}_g = \mathbf{G} \times \mathbf{v}$$

with

$$G_i = \frac{\mu_0 M_s}{|\gamma|} \frac{\epsilon_{ijk}}{2} \int_V d^3 r \mathbf{m} \cdot \left(\frac{\partial \mathbf{m}}{\partial r_j} \times \frac{\partial \mathbf{m}}{\partial r_k} \right)$$

skyrmion number!

for the vortex:



$$G_z = h \frac{\mu_0 M_s}{2|\gamma|} pq$$

like a Lorentz force!

Dynamics of the vortex: Gyrotropic mode

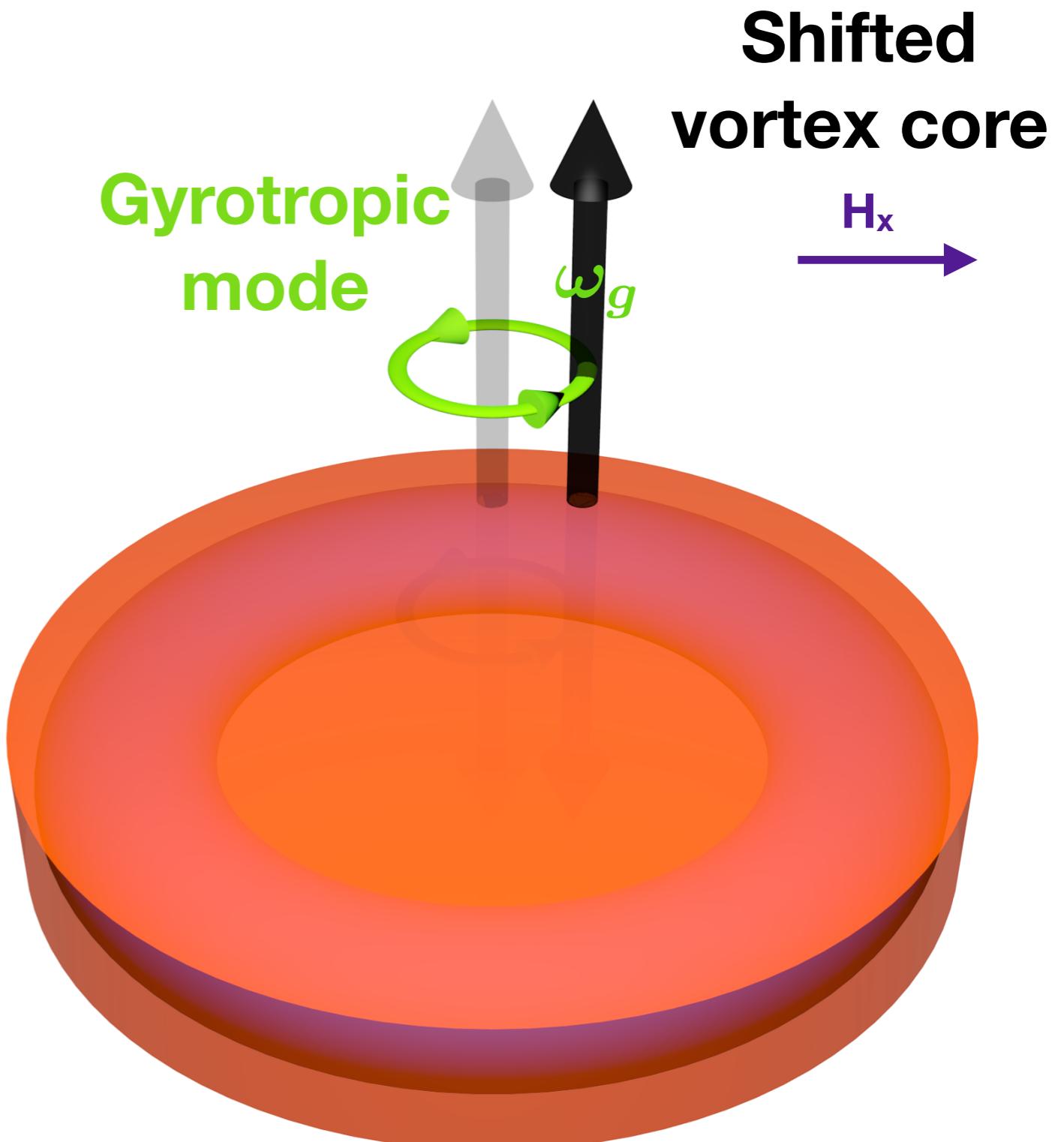
$$\mathbf{F}_g = \mathbf{G} \times \mathbf{v}$$

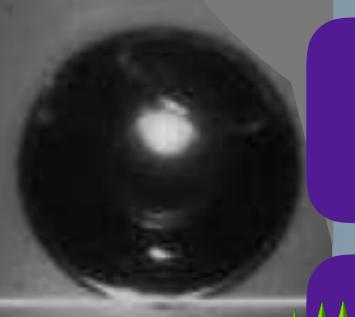
$$G_z = h \frac{\mu_0 M_s}{2|\gamma|} pq$$

like a Lorentz force!

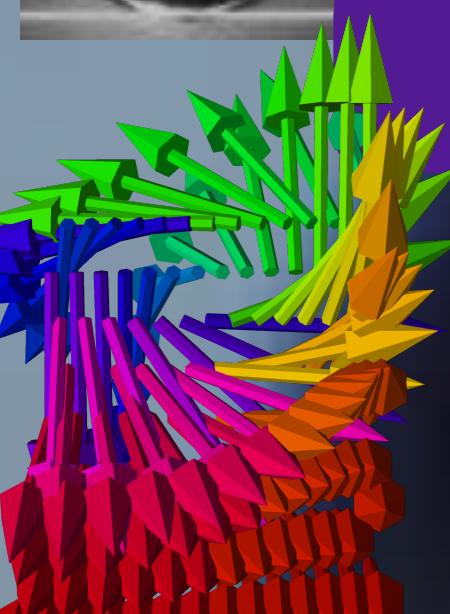
The vortex acts like
a charge in a magnetic
field:

circular motion of the core
around its equilibrium position





Introduction: cavity optomagnonics

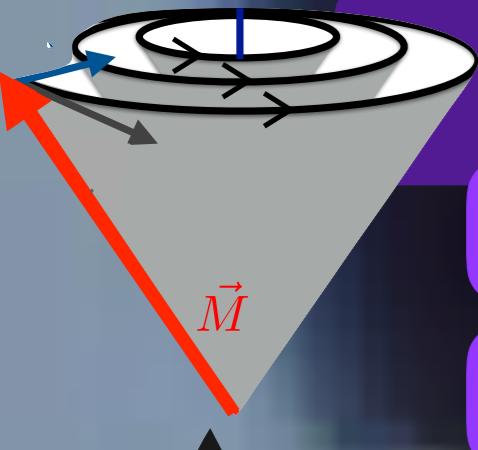


Magnetic textures

Why do they form?

Equilibrium condition

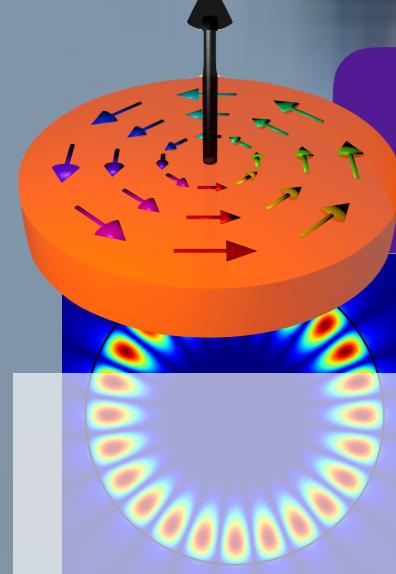
Topology of smooth textures



Dynamics of the magnetization

Landau Lifschitz Gilbert equation

Thiele equation for topological defects



Cavity optomagnonics with magnetic textures

Hamiltonian: Holstein-Primakoff

Optomagnetic coupling: gyrotropic mode

Optomagnonic Hamiltonian

$$\bar{U}_{\text{MO}} = \theta_F \sqrt{\frac{\varepsilon}{\varepsilon_0}} \int d\mathbf{r} \left[\frac{\mathbf{M}(\mathbf{r})}{M_s} \cdot \frac{\varepsilon_0}{2i\omega} [\mathbf{E}^*(\mathbf{r}) \times \mathbf{E}(\mathbf{r})] \right]$$

Quantize:



two-photon process

Optomagnonics beyond the Kittel mode

$$H_{\text{MO}} = -i \frac{\theta_F \lambda_n}{2\pi} \frac{\varepsilon_0 \varepsilon}{2} \int d\mathbf{r} \ \mathbf{m}(\mathbf{r}, t) \cdot [\mathbf{E}^*(\mathbf{r}, t) \times \mathbf{E}(\mathbf{r}, t)]$$

Optomagnonics beyond the Kittel mode

$$H_{\text{MO}} = -i \frac{\theta_F \lambda_n}{2\pi} \frac{\varepsilon_0 \varepsilon}{2} \int d\mathbf{r} \ \mathbf{m}(\mathbf{r}, t) \cdot [\mathbf{E}^*(\mathbf{r}, t) \times \mathbf{E}(\mathbf{r}, t)]$$

$$\mathbf{m}(\mathbf{r}, t) = \mathbf{m}_0(\mathbf{r}) + \delta\mathbf{m}(\mathbf{r}, t)$$

small

Optomagnonics beyond the Kittel mode

$$H_{\text{MO}} = -i \frac{\theta_F \lambda_n}{2\pi} \frac{\varepsilon_0 \varepsilon}{2} \int d\mathbf{r} \ \mathbf{m}(\mathbf{r}, t) \cdot [\mathbf{E}^*(\mathbf{r}, t) \times \mathbf{E}(\mathbf{r}, t)]$$

$$\mathbf{m}(\mathbf{r}, t) = \mathbf{m}_0(\mathbf{r}) + \delta\mathbf{m}(\mathbf{r}, t)$$

Quantize: Holstein Primakoff to first order

$$\delta\mathbf{m}(\mathbf{r}, t) \rightarrow \frac{1}{2} \sum_{\gamma} \left(\delta\mathbf{m}_{\gamma}(\mathbf{r}) \hat{b}_{\gamma} e^{-i\omega_{\gamma}t} + \delta\mathbf{m}_{\gamma}^*(\mathbf{r}) \hat{b}_{\gamma}^\dagger e^{i\omega_{\gamma}t} \right)$$

↓

mode functions

↑ magnon mode index ↘ bosonic operator

Optomagnonics beyond the Kittel mode

$$H_{\text{MO}} = -i \frac{\theta_F \lambda_n}{2\pi} \frac{\varepsilon_0 \varepsilon}{2} \int d\mathbf{r} \ \mathbf{m}(\mathbf{r}, t) \cdot [\mathbf{E}^*(\mathbf{r}, t) \times \mathbf{E}(\mathbf{r}, t)]$$

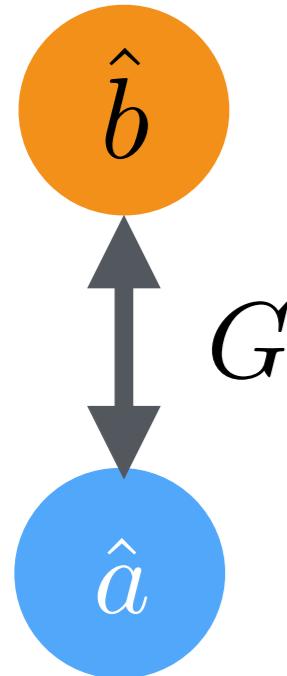
$$\mathbf{m}(\mathbf{r}, t) = \mathbf{m}_0(\mathbf{r}) + \delta\mathbf{m}(\mathbf{r}, t)$$

$$\delta\mathbf{m}(\mathbf{r}, t) \rightarrow \frac{1}{2} \sum_{\gamma} \left(\delta\mathbf{m}_{\gamma}(\mathbf{r}) \hat{b}_{\gamma} e^{-i\omega_{\gamma}t} + \delta\mathbf{m}_{\gamma}^*(\mathbf{r}) \hat{b}_{\gamma}^\dagger e^{i\omega_{\gamma}t} \right)$$

$$\mathbf{E}^{(*)}(\mathbf{r}, t) \rightarrow \sum_{\beta} \mathbf{E}_{\beta}^{(*)}(\mathbf{r}) \hat{a}_{\beta}^{(\dagger)} e^{-(-+)i\omega_{\beta}t}$$

Optomagnonics beyond the Kittel mode

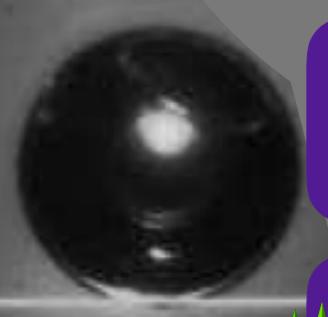
Optomagnetic Hamiltonian in the spin-wave limit



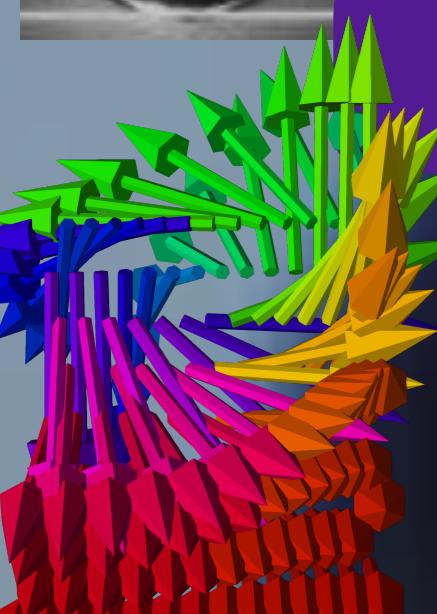
$$\hat{H}_{MO} = \sum_{\alpha\beta\gamma} G_{\alpha\beta\gamma} \hat{a}_\alpha^\dagger \hat{a}_\beta \hat{b}_\gamma + \text{h.c.}$$

Optomagnetic coupling

$$G_{\alpha\beta\gamma} = -i \frac{\theta_F \lambda_n}{4\pi} \frac{\varepsilon_0 \varepsilon}{2} \int d\mathbf{r} \delta \mathbf{m}_\gamma(\mathbf{r}) \cdot [\mathbf{E}_\alpha^*(\mathbf{r}) \times \mathbf{E}_\beta(\mathbf{r})]$$



Introduction: cavity optomagnonics

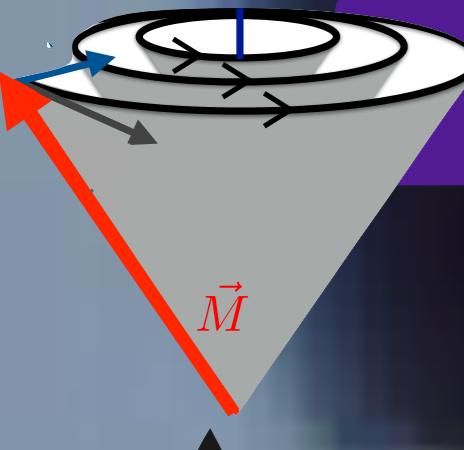


Magnetic textures

Why do they form?

Equilibrium condition

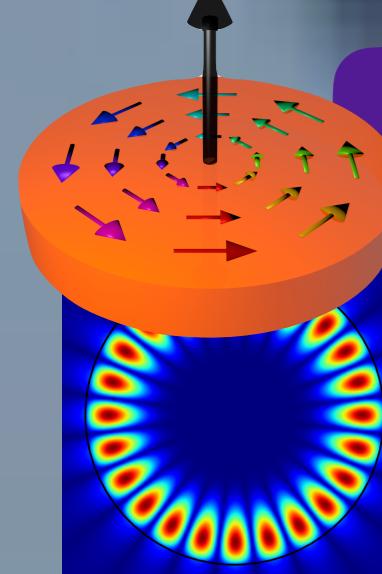
Topology of smooth textures



Dynamics of the magnetization

Landau Lifschitz Gilbert equation

Thiele equation for topological defects



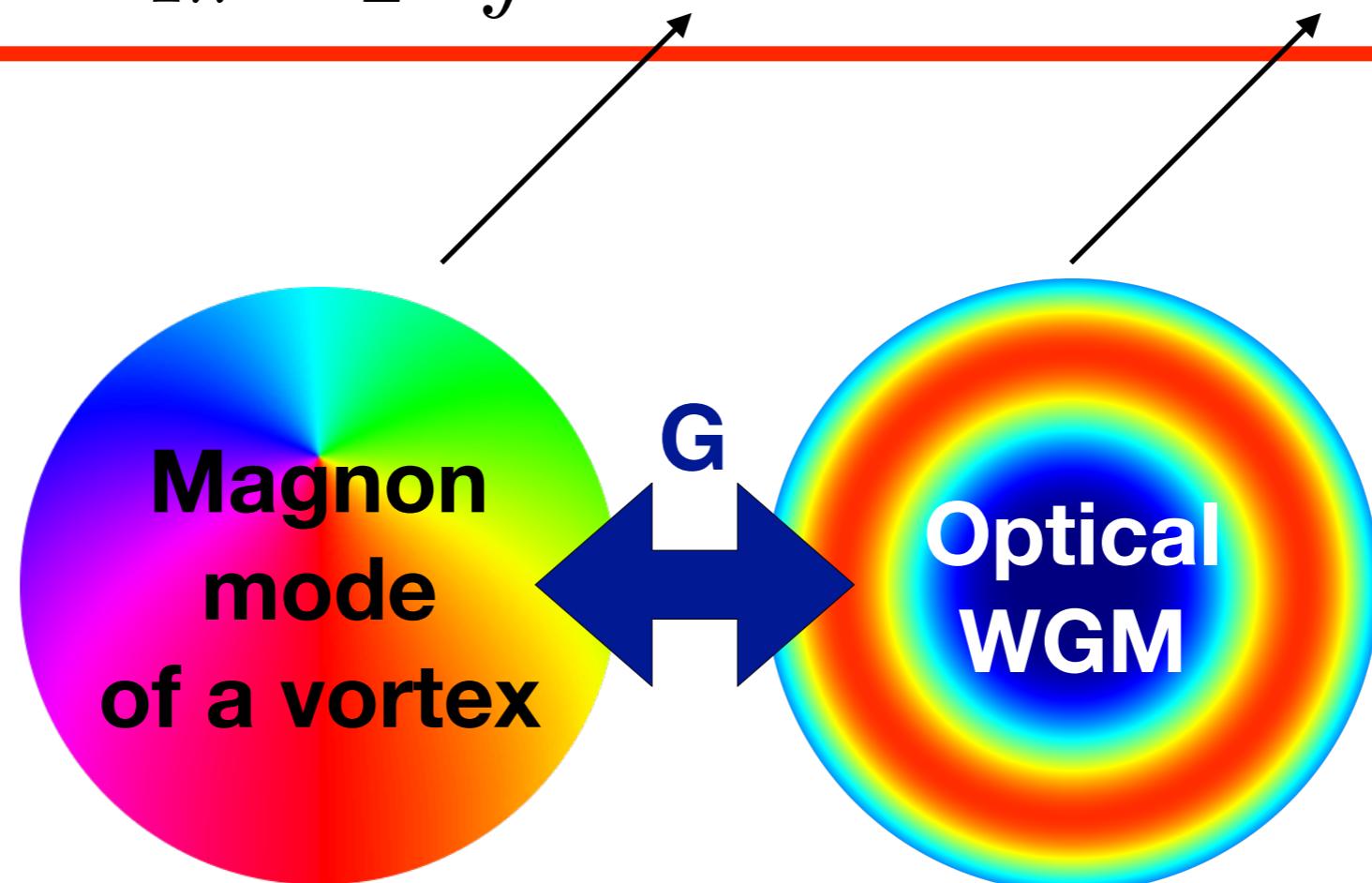
Cavity optomagnonics with magnetic textures

Hamiltonian: Holstein-Primakoff

Optomagnonic coupling: gyrotropic mode

Optomagnonic Coupling

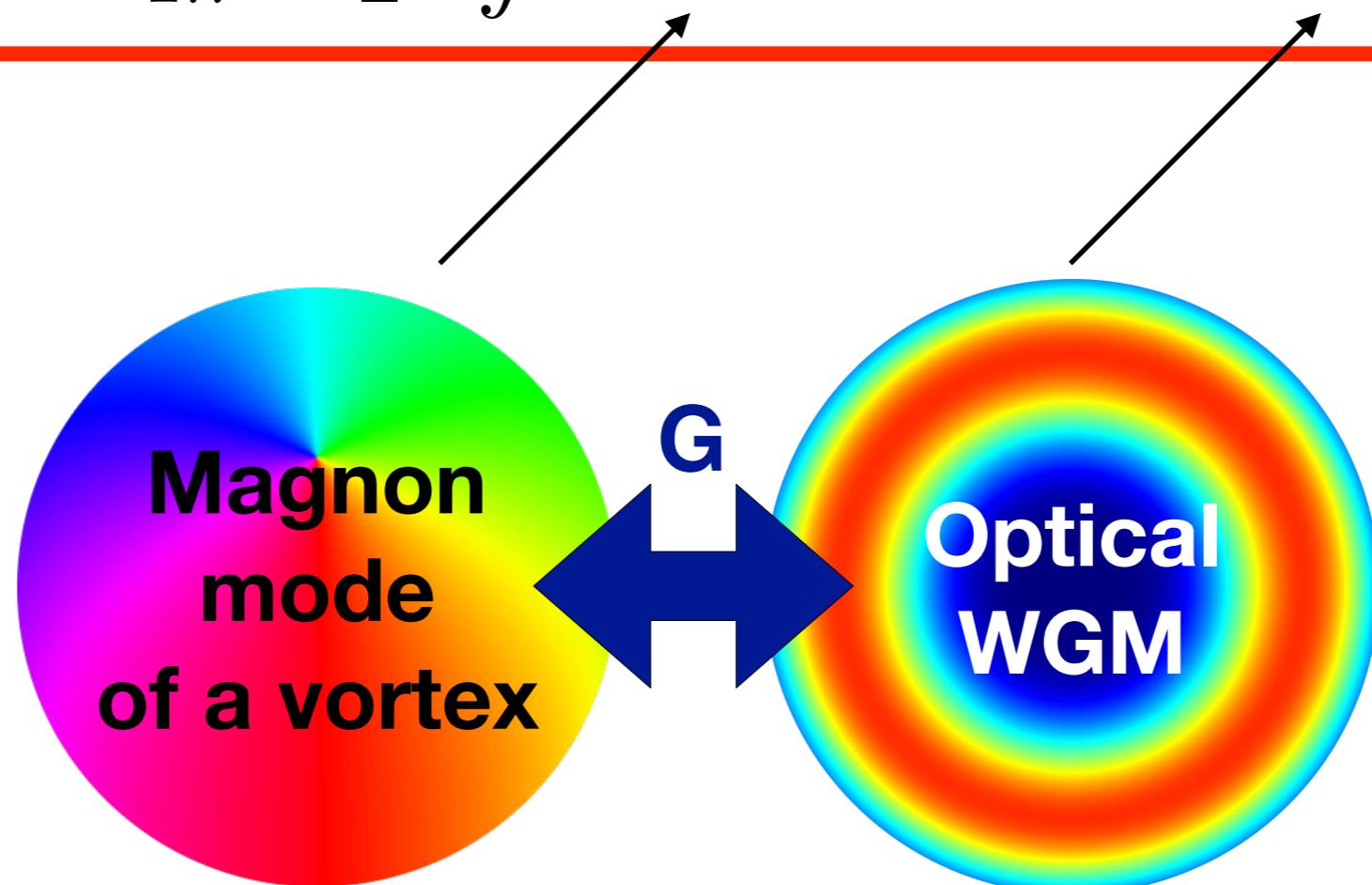
$$G_{\alpha\beta\gamma} = -i \frac{\theta_F \lambda_n}{4\pi} \frac{\epsilon_0 \epsilon}{2} \int d\mathbf{r} \delta \mathbf{m}_\gamma(\mathbf{r}) \cdot [\mathbf{E}_\alpha^*(\mathbf{r}) \times \mathbf{E}_\beta(\mathbf{r})]$$



Analytical results for very thin micromagnetic disk (2D)

Optomagnetic Coupling

$$G_{\alpha\beta\gamma} = -i \frac{\theta_F \lambda_n}{4\pi} \frac{\epsilon_0 \epsilon}{2} \int d\mathbf{r} \delta \mathbf{m}_\gamma(\mathbf{r}) \cdot [\mathbf{E}_\alpha^*(\mathbf{r}) \times \mathbf{E}_\beta(\mathbf{r})]$$



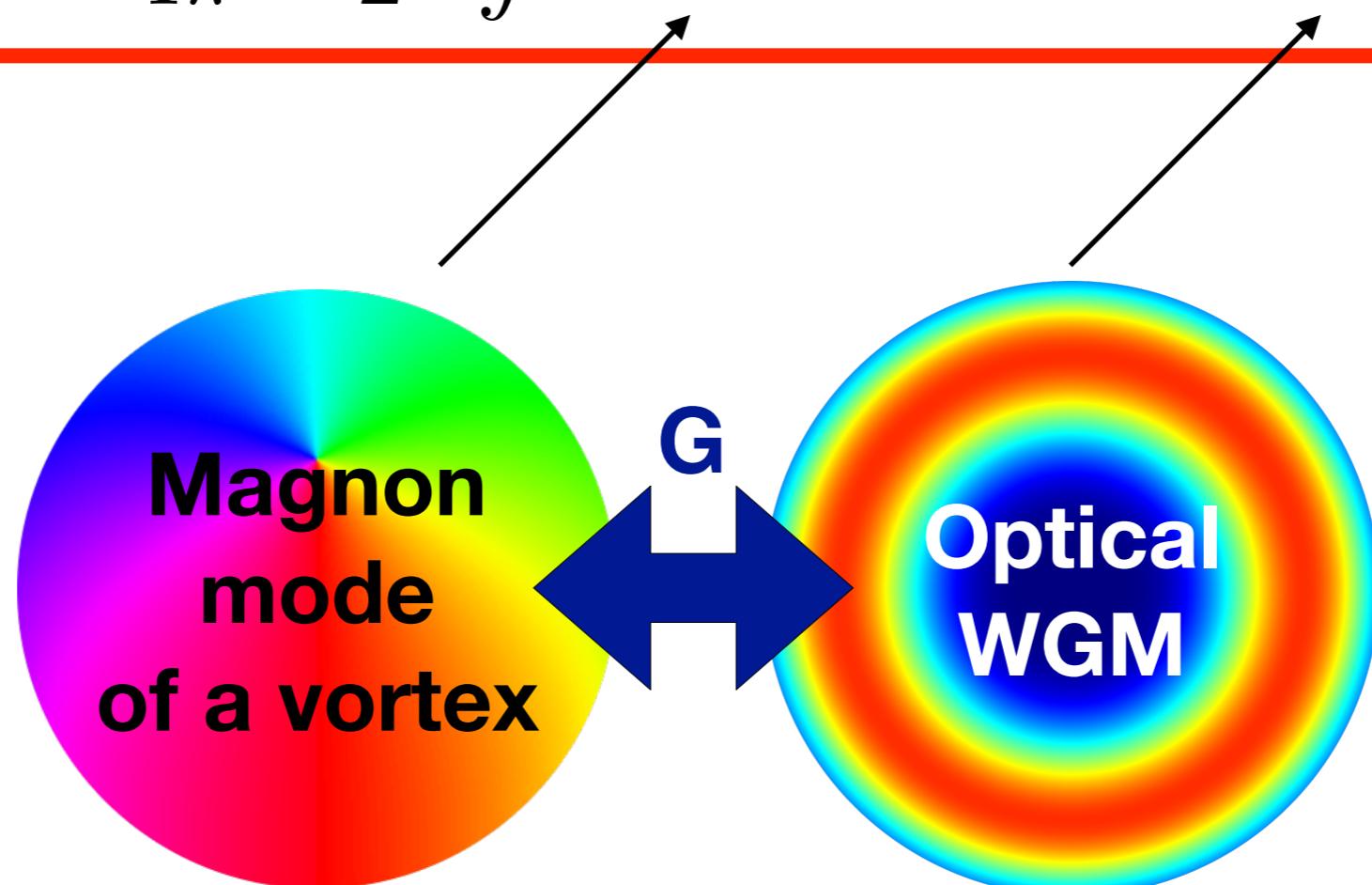
Analytical results for very thin micromagnetic disk (2D)

Coupling proportional to the gradient of the optical spin density

$$|G(s)| \propto |\partial_s [(\mathbf{E}_m^{\text{TE}*} \times \mathbf{E}_m^{\text{TE}})_z]| \quad (s \text{ vortex position})$$

Optomagnonic Coupling

$$G_{\alpha\beta\gamma} = -i \frac{\theta_F \lambda_n}{4\pi} \frac{\epsilon_0 \epsilon}{2} \int d\mathbf{r} \delta \mathbf{m}_\gamma(\mathbf{r}) \cdot [\mathbf{E}_\alpha^*(\mathbf{r}) \times \mathbf{E}_\beta(\mathbf{r})]$$



MuMax3
Vansteenkiste et. al
AIP Advances 4,
107133 (2014)

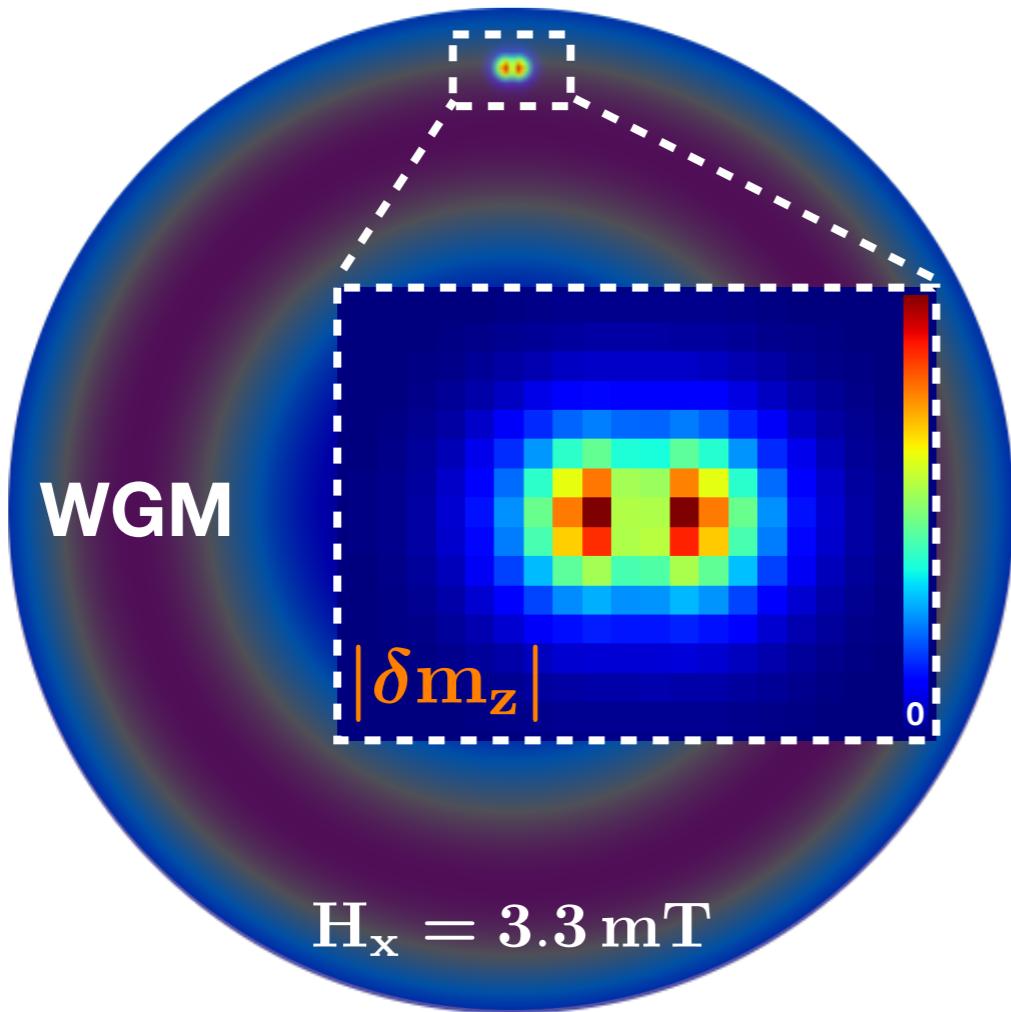
micromagnetics

finite element

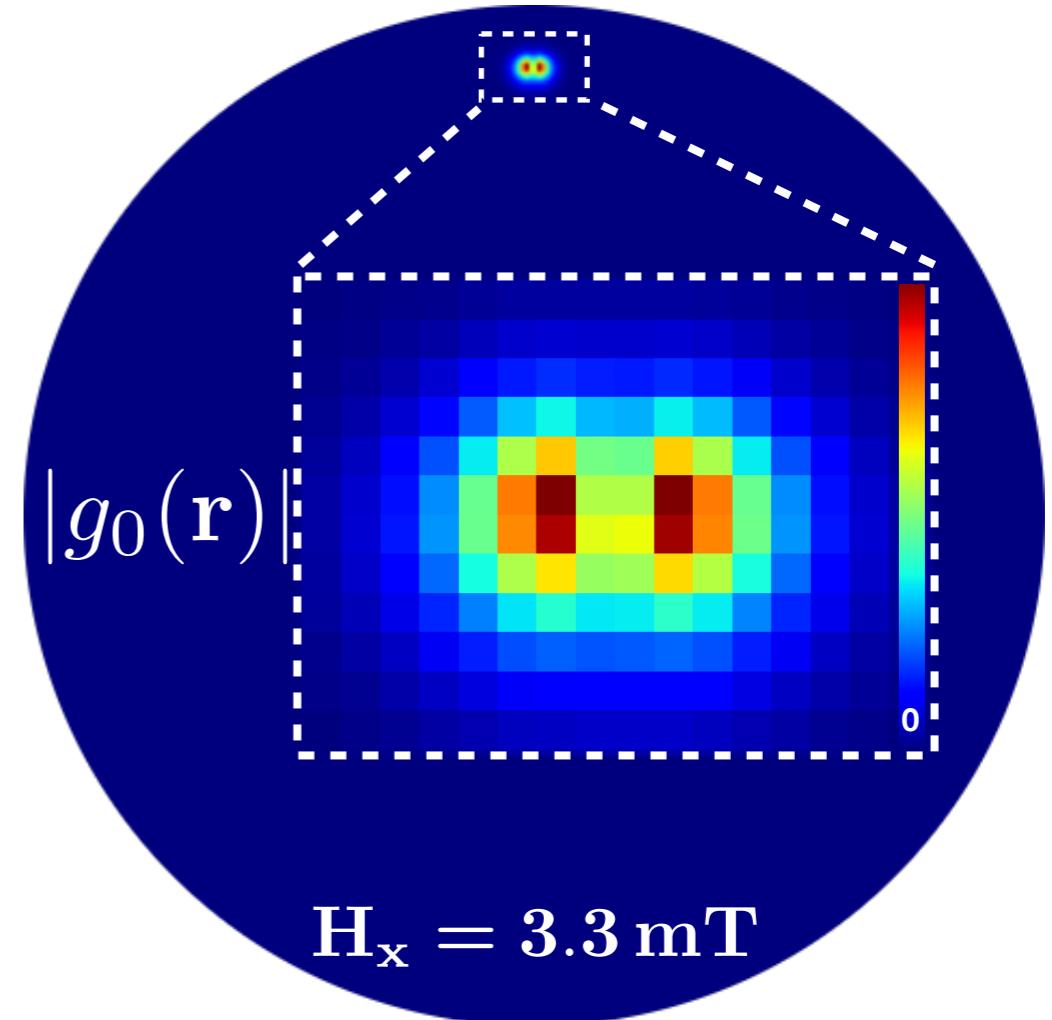
Simulation software

Optomagnetic coupling: gyrotropic mode spatial dependence

Magnon and optical
modes

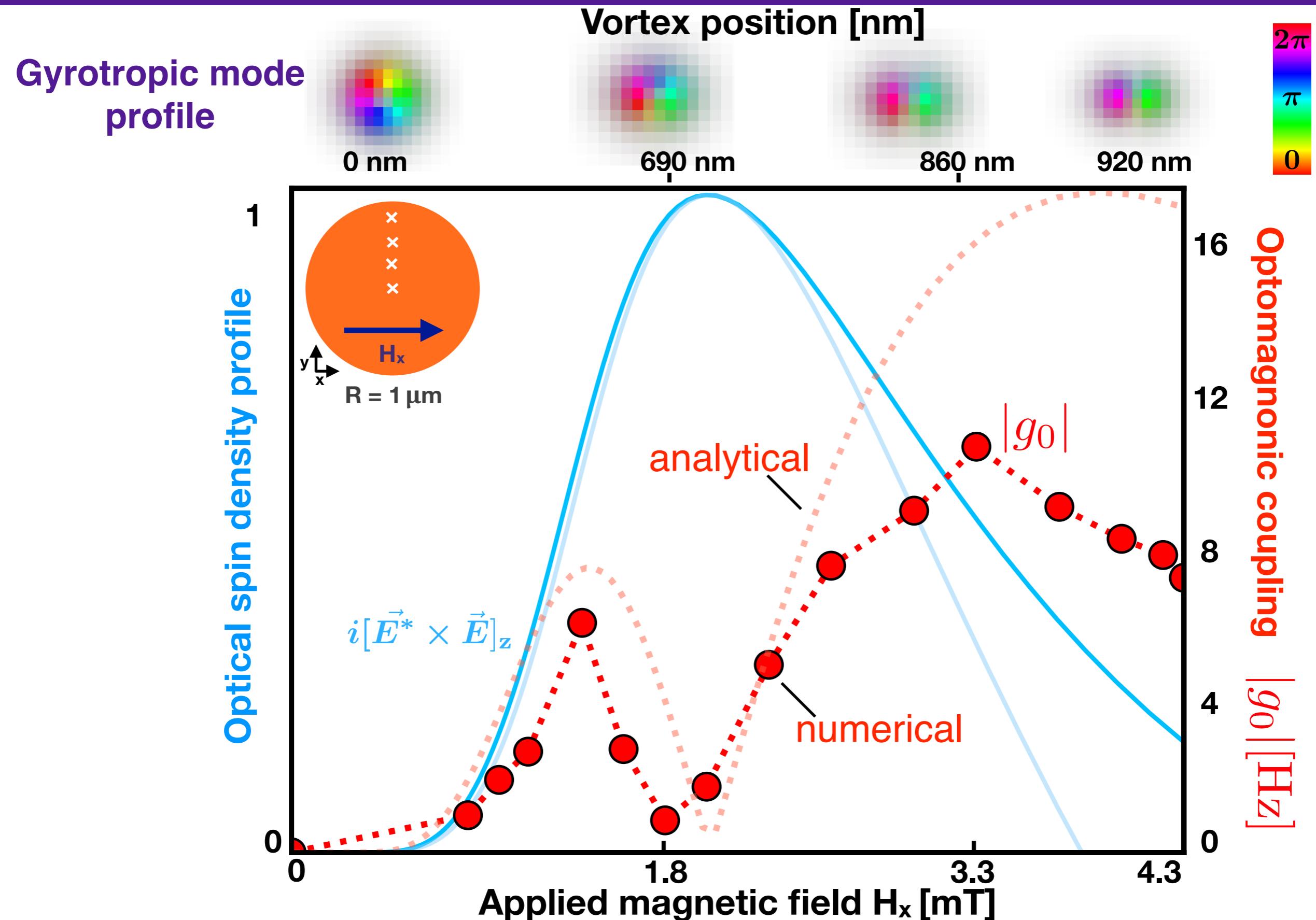


Optomagnetic
coupling



integrate over the
whole volume

Optomagnetic coupling: gyrotropic mode



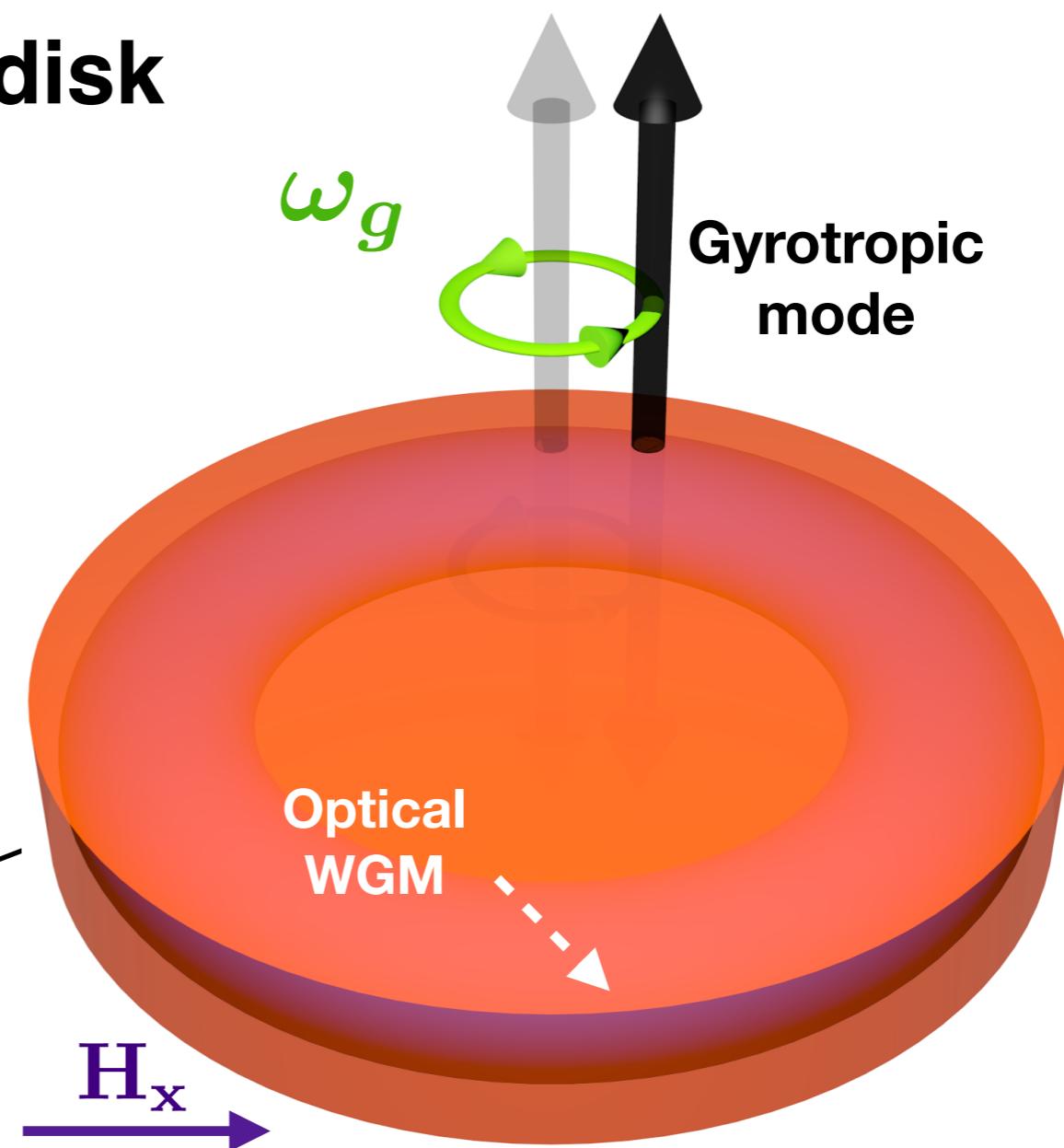
Optomagnetic coupling: beyond 2D

“Thick” micromagnetic disk

$$R = 2\mu\text{m}$$

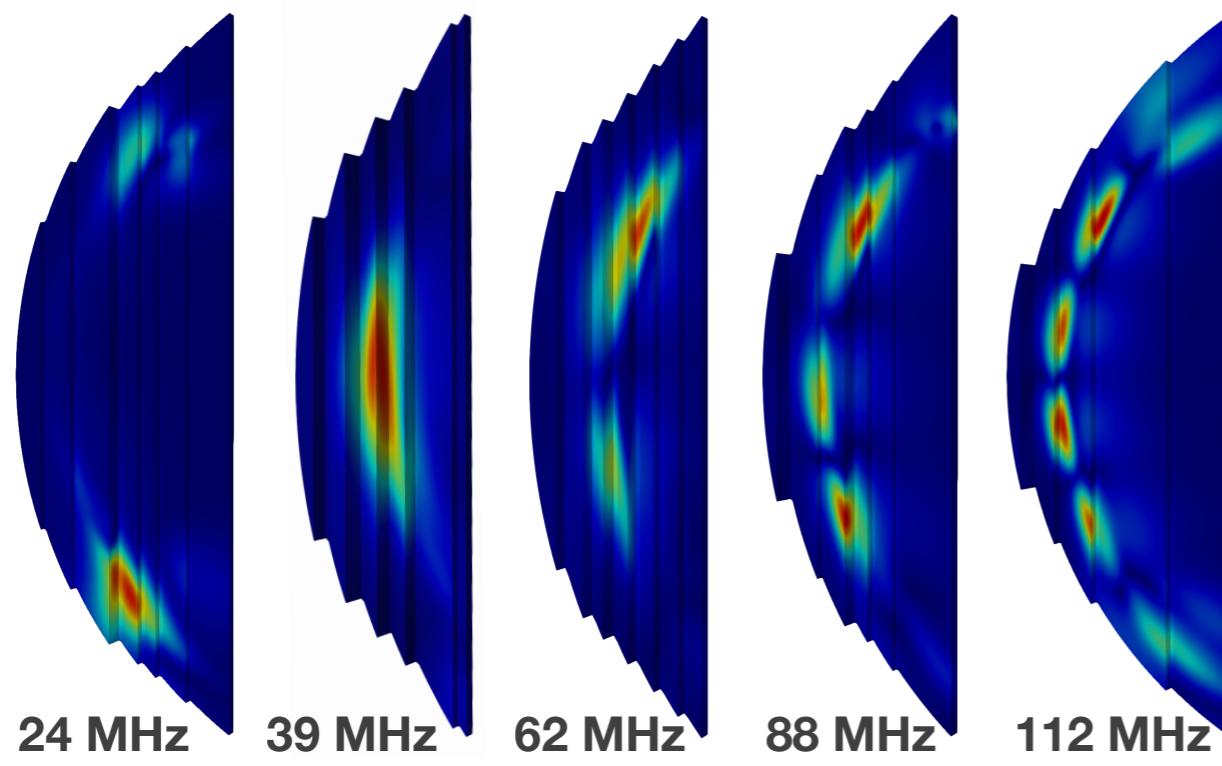
$$h = 500\text{nm}$$

YIG disk: magnons
+ optical cavity



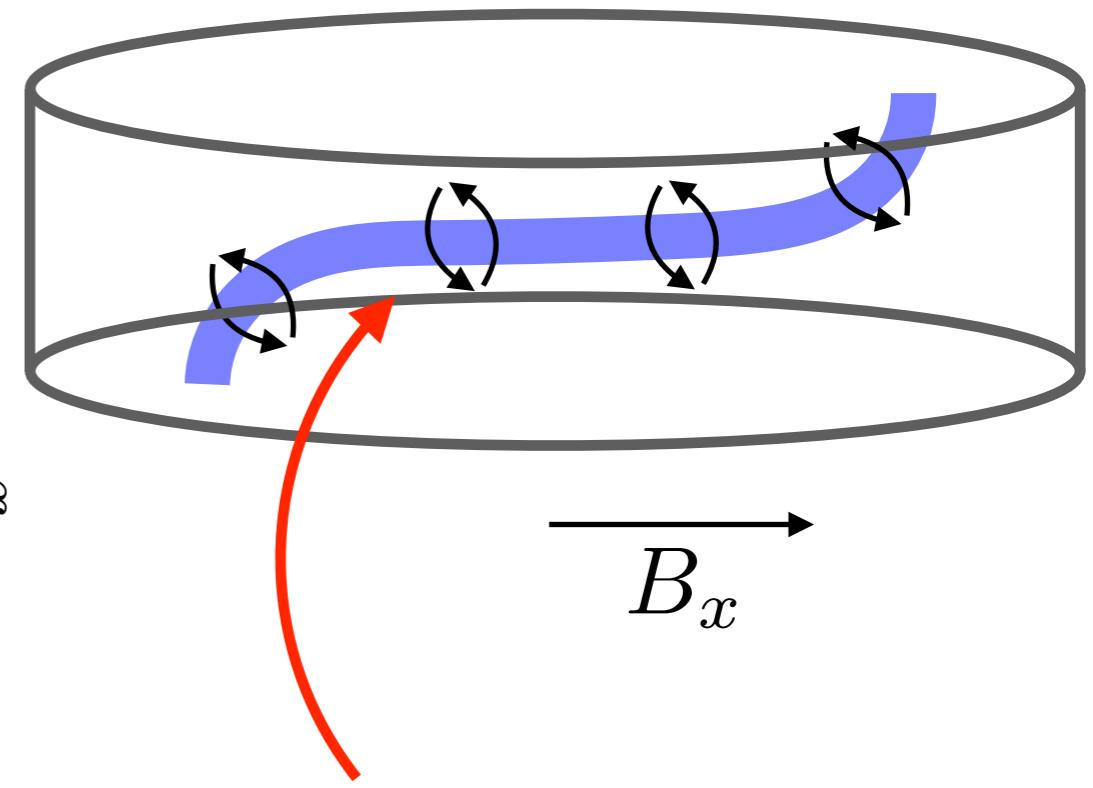
Simple picture of the vortex breaks down:
Non-trivial z dependence

Optomagnetic coupling: beyond 2D



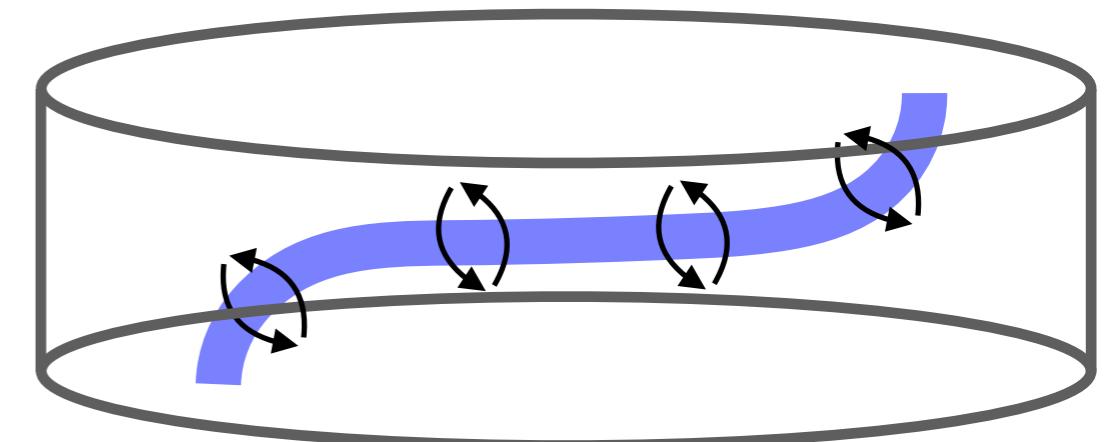
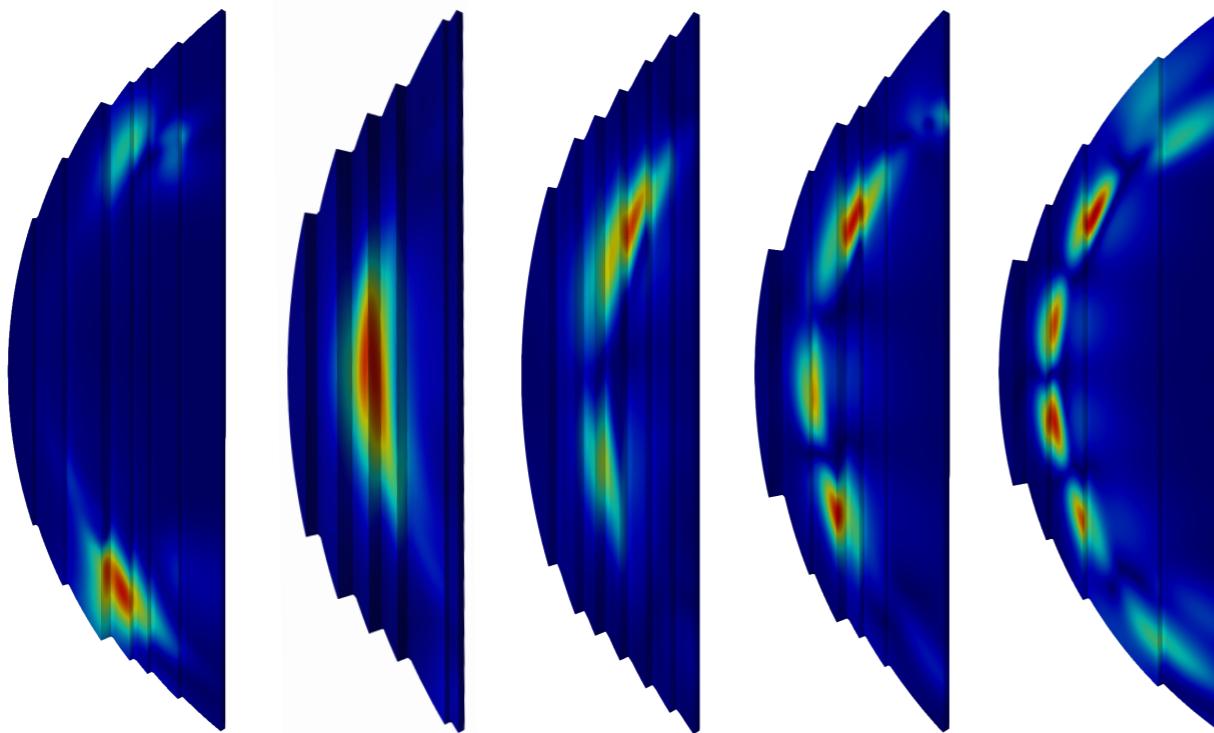
flexural magnon modes

$$B_x = 17 \text{ mT}$$

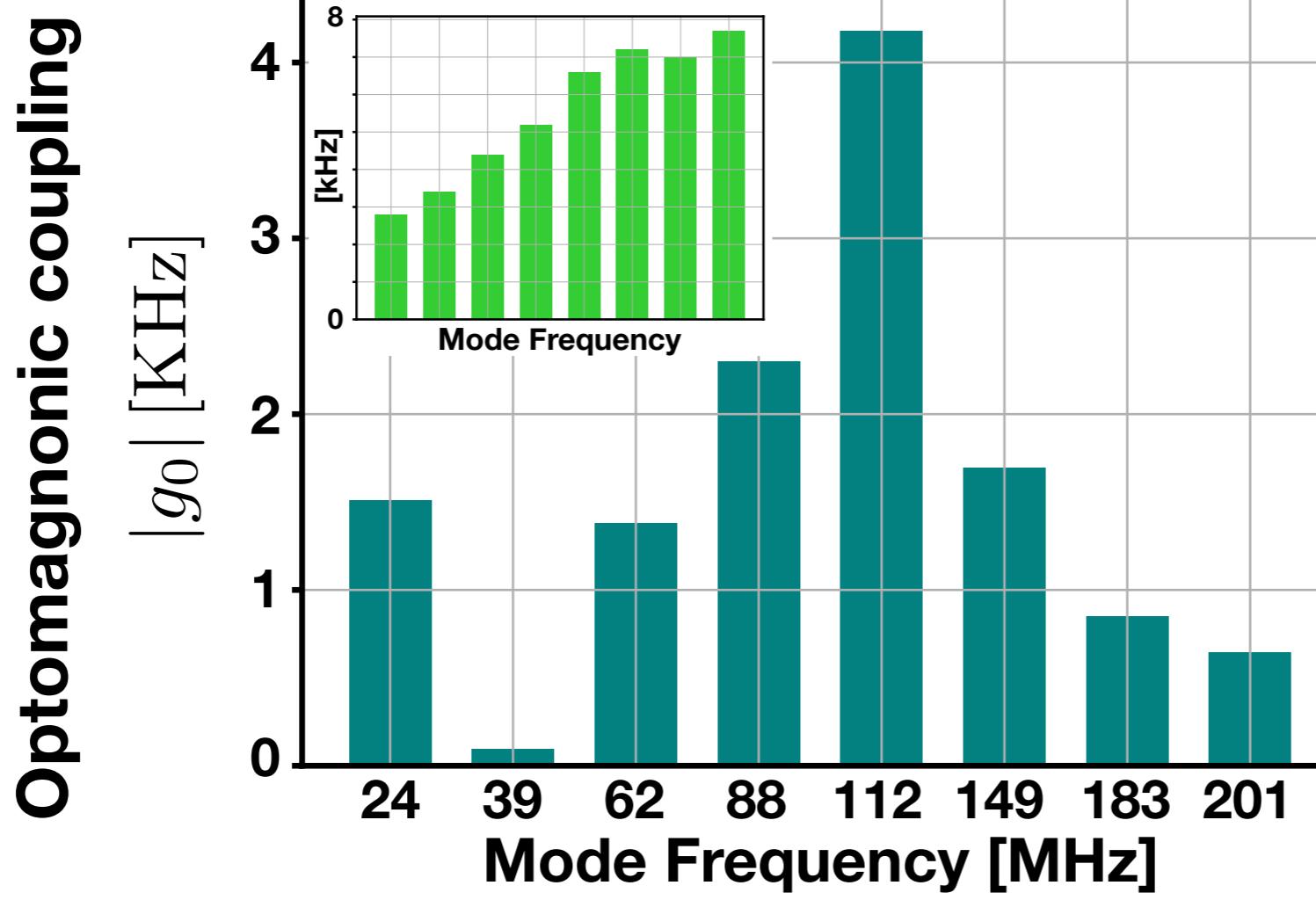


"vibrating string"

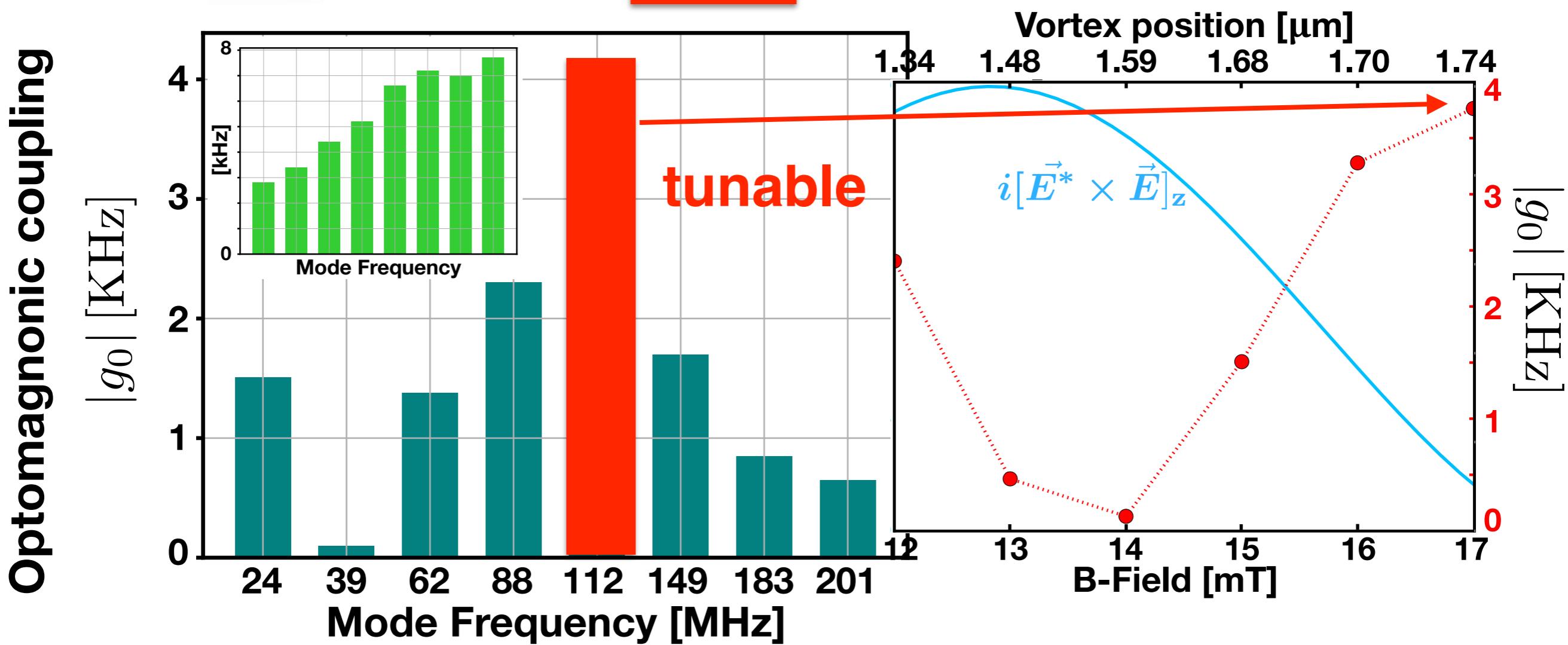
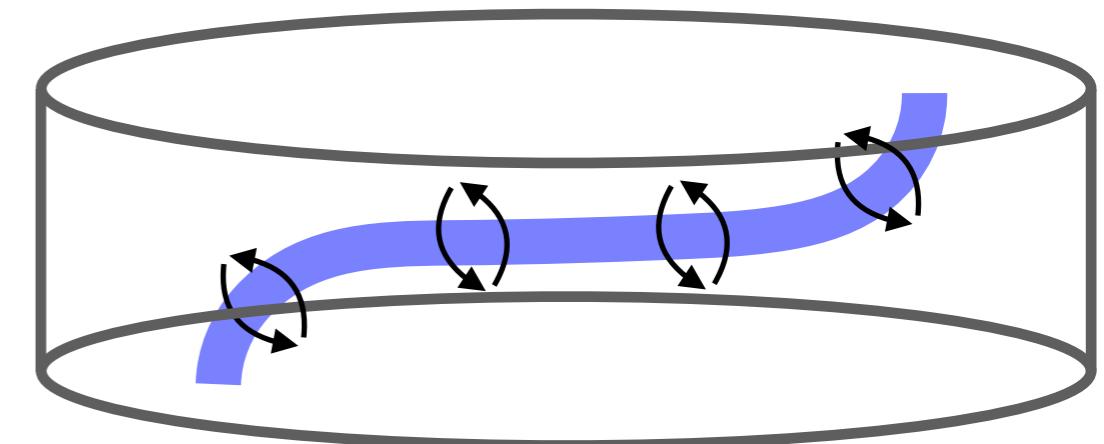
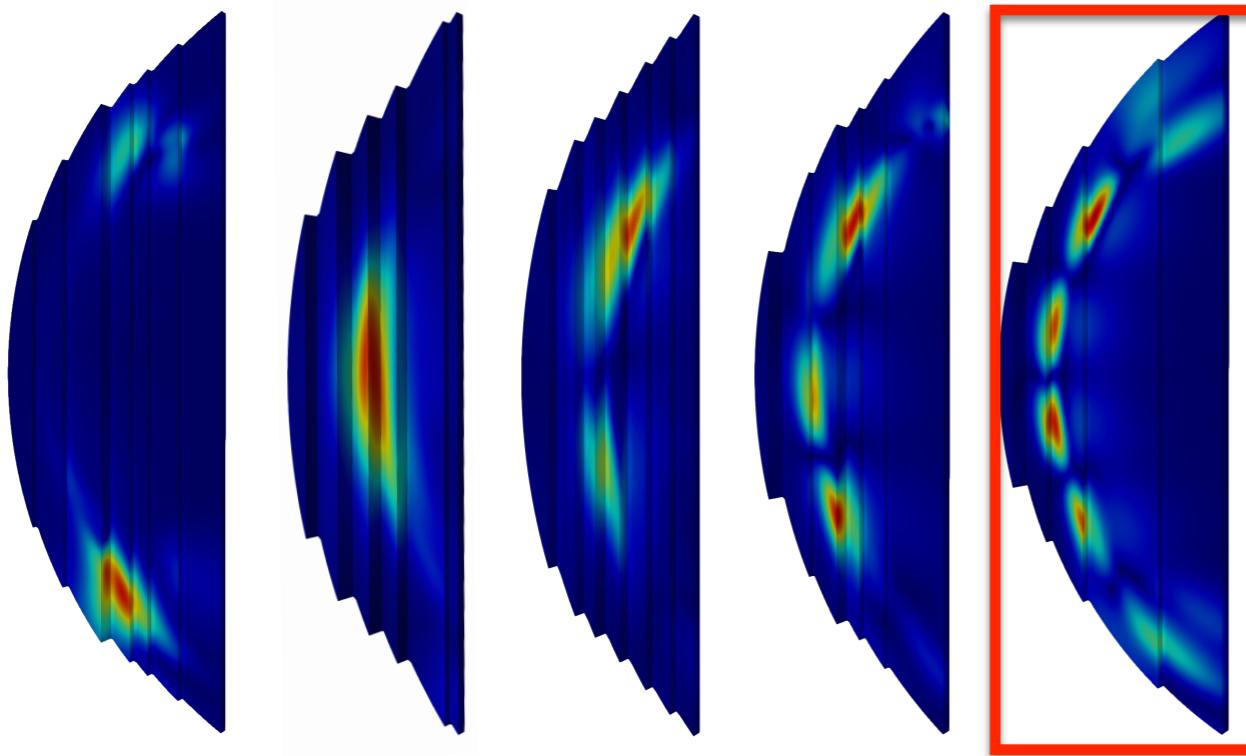
Optomagnetic coupling: beyond 2D



$B_x = 17\text{mT}$

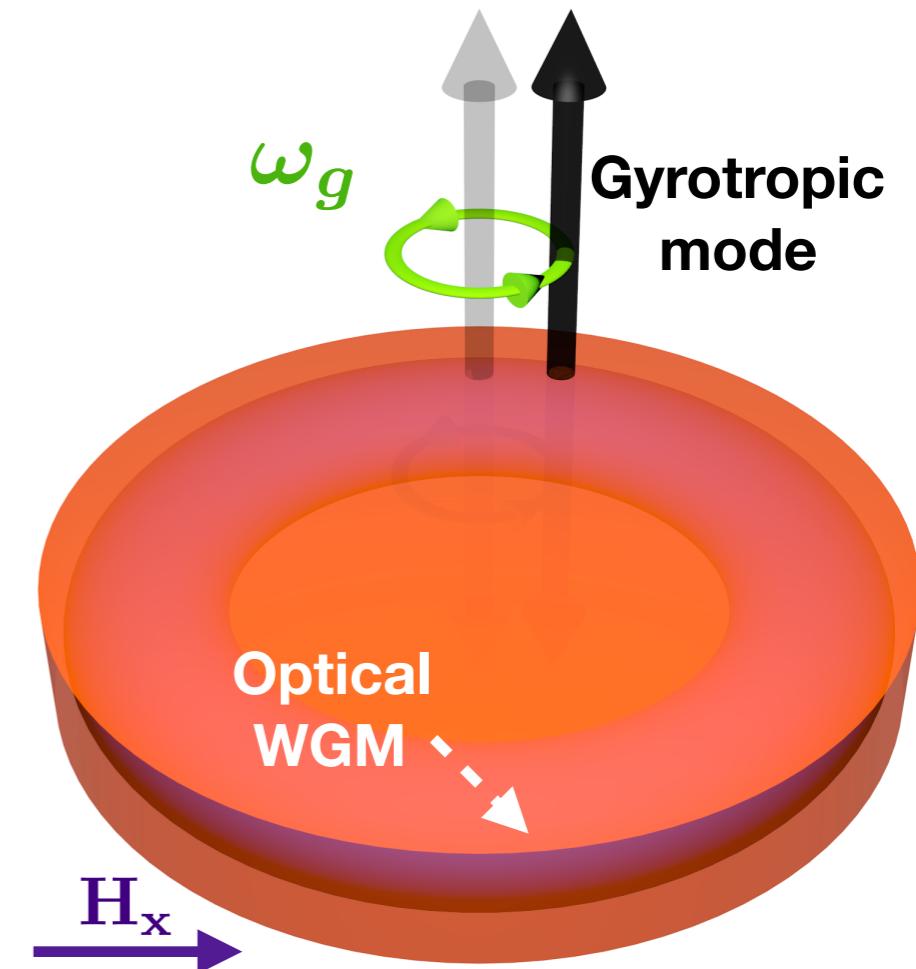
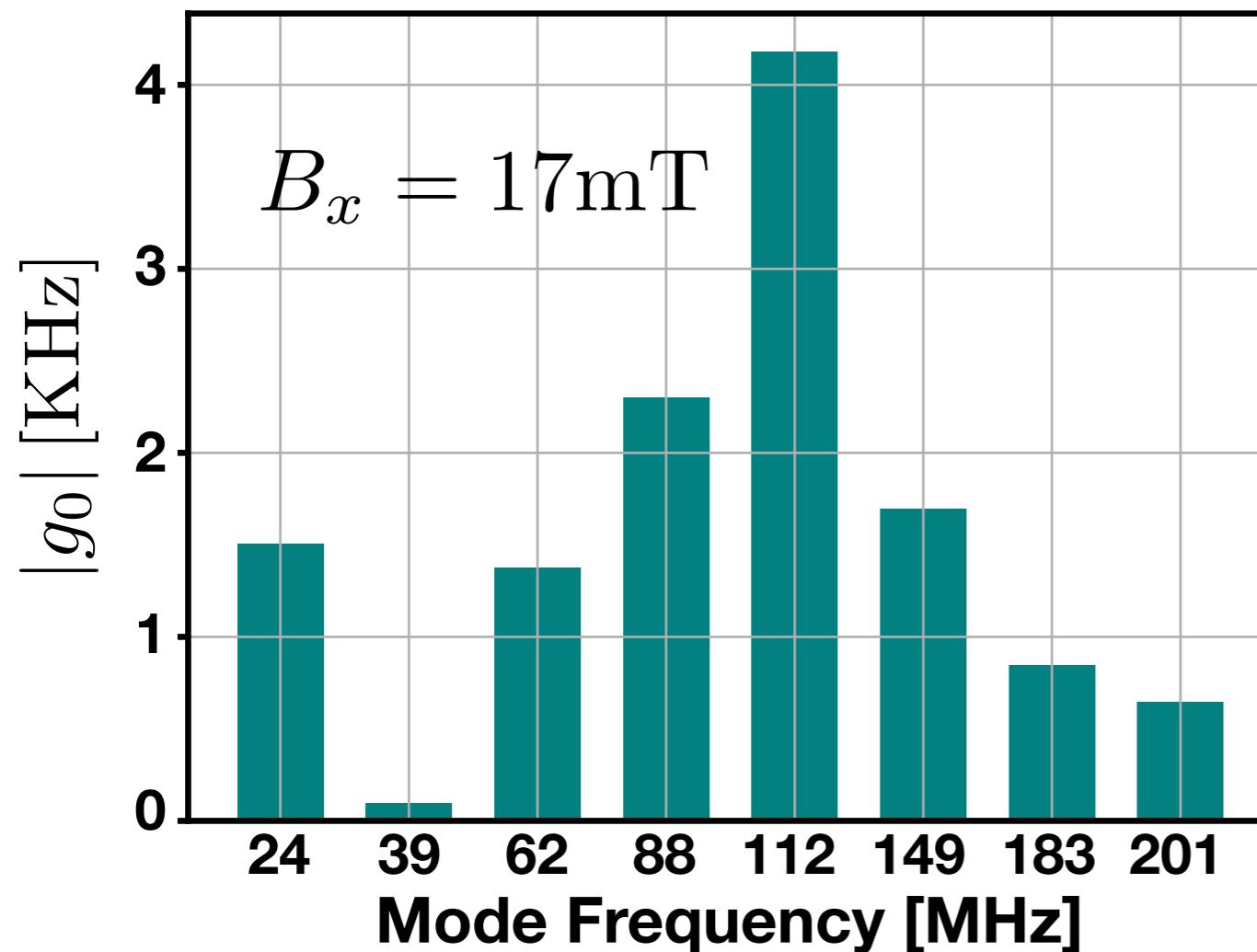


Optomagnonic coupling: beyond 2D



Optomagnetic coupling: beyond 2D

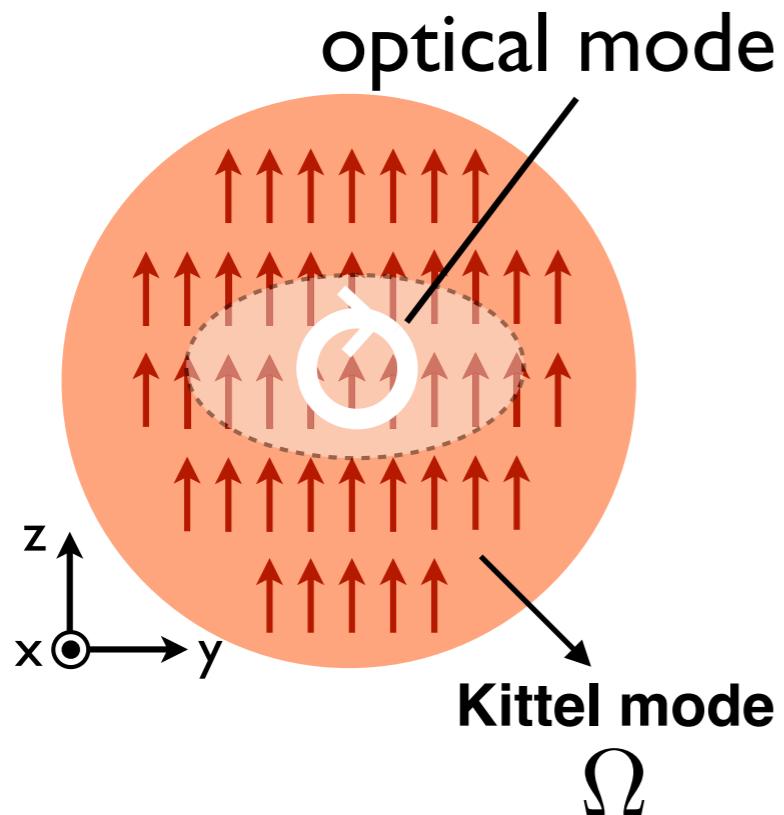
Optomagnetic coupling



- First time optomagnonics with textures
- Promising values for coupling, tunable - design!
- Coupled Dynamics?

Coupled dynamics?

Cavity Optomagnonics: simple model



Simplest Coupling

$$\hbar G \hat{S}_x \hat{a}^\dagger \hat{a}$$

$$G = \frac{1}{S} \frac{c \theta_F}{4\sqrt{\epsilon}}$$

$$H = -\hbar \Delta \hat{a}^\dagger \hat{a} - \hbar \Omega \hat{S}_z + \hbar G \hat{S}_x \hat{a}^\dagger \hat{a}$$

driving laser detuning

$$\Delta = \omega_{las} - \omega_{cav}$$

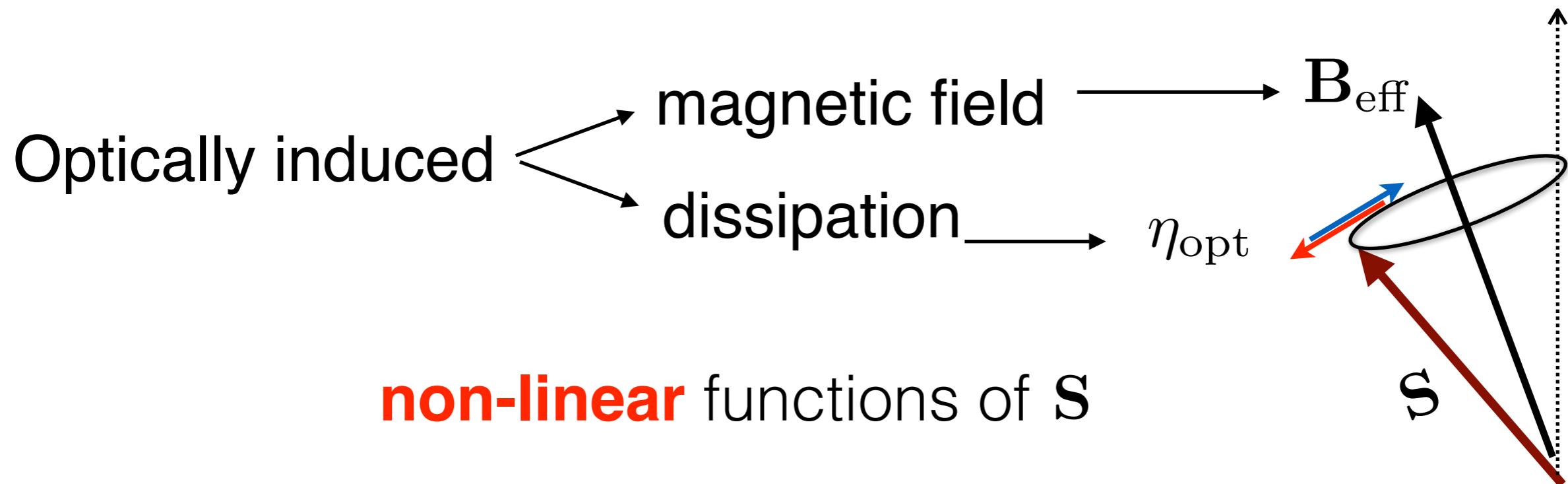
Effective Equation of Motion for a Macrospin

For the homogeneous (Kittel) mode

Fast cavity limit: integrate out the light field

$$\dot{\mathbf{S}} = \mathbf{B}_{\text{eff}} \times \mathbf{S} + \frac{\eta_{\text{opt}}}{S} (\dot{S}_x \mathbf{e}_x \times \mathbf{S})$$

Effective Landau-Lifshitz-Gilbert equation of motion



Fast Cavity Limit

$$\dot{\mathbf{S}} = \mathbf{B}_{\text{eff}} \times \mathbf{S} + \frac{\eta_{\text{opt}}}{S} \left(\dot{S}_x \mathbf{e}_x \times \mathbf{S} \right)$$

effective field

$$\mathbf{B}_{\text{eff}} = -\Omega \mathbf{e}_z + \mathbf{B}_{\text{opt}}$$

$$\mathbf{B}_{\text{opt}} = \frac{G}{[(\frac{\kappa}{2})^2 + (\Delta - GS_x)^2]} \left(\frac{\kappa}{2} \alpha_{\max} \right)^2 \mathbf{e}_x$$

**damping
can change sign**

$$\eta_{\text{opt}} = -2G\kappa S |\mathbf{B}_{\text{opt}}| \frac{(\Delta - GS_x)}{[(\frac{\kappa}{2})^2 + (\Delta - GS_x)^2]^2}$$

tunable by the external laser drive

Fast Cavity Limit

$$\dot{\mathbf{S}} = \mathbf{B}_{\text{eff}} \times \mathbf{S} + \frac{\eta_{\text{opt}}}{S} \left(\dot{S}_x \mathbf{e}_x \times \mathbf{S} \right)$$

effective field

$$\mathbf{B}_{\text{eff}} = -\Omega \mathbf{e}_z + \mathbf{B}_{\text{opt}}$$

$$\mathbf{B}_{\text{opt}} = \frac{G}{[(\frac{\kappa}{2})^2 + (\Delta - GS_x)^2]} \left(\frac{\kappa}{2} \alpha_{\text{max}} \right)^2 \mathbf{e}_x$$

**damping
can change sign**

$$\eta_{\text{opt}} = -2G\kappa S |\mathbf{B}_{\text{opt}}| \frac{(\Delta - GS_x)}{[(\frac{\kappa}{2})^2 + (\Delta - GS_x)^2]^2}$$

**with textures
and defects?**

tunable by the external laser drive